

Texas University Interscholastic League

Contest Event: Number Sense

This 80-question mental math contest covers all high school mathematics curricula. All answers must be derived without using scratch paper or a calculator.

The Texas Essential Knowledge and Skills are categorized by course area and grade level. The following are course area abbreviations used for the TEKS in Mathematics:

Algebra 1 = Alg1; Algebra 2 = Alg2; Geometry = Geo; Precalculus = Pc; Mathematical Models = MM.

Each TEK begins with the outline number for the appropriate course area.

Texas Essential Knowledge and Skills	Contest Knowledge and Skills
<p><u>Foundations for Functions Knowledge and Skills:</u></p> <p>A1. Understands that a function represents a dependence of one quantity on another and can be described in a variety of ways. (Alg1) A2 & 2A1. Uses the properties and attributes of functions. (Alg1, Alg2) A3. Understands how algebra can be used to express generalizations and recognizes and uses the power of symbols to represent situations. (Alg1) A4 & 2A2. Understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. (Alg1, Alg2)</p>	<p><u>Foundations for Functions Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Given the value of x find the value of $F(x)$ or given the value of $F(x)$ find x. - Find the domains and/or ranges of various functions. - Factor polynomials and use factors to solve algebraic equations. - Simplify algebraic expressions. - Use algebraic concepts to solve equations and inequalities. - Use direct variations and indirect variations to solve problems. - Find missing elements of real number properties such as commutative, associative, closure, distributive, etc.
<p><u>Linear Functions Knowledge and Skills:</u></p> <p>A5. Understands that linear functions can be represented in different ways and translates among their various representations. (Alg1) A6. Understands the meaning of the slope and intercepts of the graphs of linear functions and zeros of linear functions and interprets and describes the effects of changes in parameters of linear functions in real-world and mathematical situations. (Alg1) A7 & 2A3. Formulates equations and inequalities based on linear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. (Alg1, Alg2) A8 & 2A3. Formulates systems of linear equations from problem situations, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. (Alg1, Alg2)</p>	<p><u>Linear Functions Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Find the slope of a line from an equation or a pair of points. - Find intercepts of equations of lines given in standard form, point-slope form, and slope-intercept form. - Solve linear equations or inequalities using algebraic transformations, substitution, etc. - Find slopes of parallel and perpendicular lines. - Solve systems of equations using substitution, equation transformations, etc.

<p><u>Other Nonlinear Functions - Quadratic, Square Root, Rational, Exponential, Logarithmic, Trigonometric - Knowledge and Skills:</u></p> <p>A9. understands that the graphs of quadratic functions are affected by the parameters of the function and can interpret and describe the effects of changes in the parameters of quadratic functions. (Alg1)</p> <p>A10. Understands there is more than one way to solve a quadratic equation and solves them using appropriate methods. (Alg1)</p> <p>A11. Understands there are situations modeled by functions that are neither linear nor quadratic and models the situations. (Alg1)</p> <p>2A6. Understands that quadratic functions can be represented in different ways and translates among their various representations. (Alg2)</p> <p>2A7. Interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations. (Alg2)</p> <p>2A8, 2A9, 2A10, & 2A11. Formulates equations and inequalities based on other nonlinear functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. (Alg2)</p> <p>P1. Defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions. (Pc)</p>	<p><u>Other Nonlinear Functions - Quadratic, Square Root, Rational, Exponential, Logarithmic, Trigonometric - Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Solve quadratic functions using factoring, substitution, the quadratic formula, completing the square, etc. - Determine the nature of the roots of a quadratic function based on its discriminant. - Determine the nature of the roots of higher degree polynomials using Descartes Rule of Signs. - Find solutions to other non-linear functions. - Find the inverses of other non-linear functions and evaluate the inverses at specified points. - Evaluate composite functions. - Find sums and products of the roots of polynomial equations. - Find the amplitude, period, phase shift, and/or vertical displacement of trigonometric functions. - Find limits, derivatives, anti-derivatives, asymptotes, and points of continuity.
<p><u>Algebra and Geometry Connections Knowledge and Skills:</u></p> <p>2A4. Connects algebraic and geometric representations of functions. (Alg2)</p>	<p><u>Algebra and Geometry Connections Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Find complementary or supplementary angle measures.
<p><u>Geometric Structure Knowledge and Skills:</u></p> <p>G1. Understands the structure of, and relationships within, an axiomatic system. (Geo)</p> <p>G2. Analyzes geometric relationships in order to make and verify conjectures. (Geo)</p> <p>G3. Applies logical reasoning to justify and prove mathematical statements. (Geo)</p> <p>G4. Uses a variety of representations to describe geometric relationships and solve problems. (Geo)</p>	<p><u>Geometric Structure Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Identify real number properties. - Solve problems by using geometric properties, axioms and theorems.
<p><u>Geometric Patterns Knowledge and Skills:</u></p> <p>G5. Uses a variety of representations to describe geometric relationships and solve problems. (Geo)</p>	<p><u>Geometric Patterns Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Use numeric and/or symbolic patterns to find missing terms. - Find missing attributes of various polygons based

	<p>on geometric properties and definitions of polygons.</p> <ul style="list-style-type: none"> - Find missing attributes of similar polygons including common ratios, angle measures, and side lengths. - Use Pythagorean relations to find missing parts of right triangles
<p><u>Two and Three Dimensional Figures Knowledge and Skills:</u></p> <p>G6. Analyzes the relationship between three-dimensional geometric figures and related two-dimensional representations and uses these representations to solve problems. (Geo) G7. Understands that coordinate systems provide convenient and efficient ways of representing geometric figures and uses them accordingly. (Geo)</p>	<p><u>Two and Three Dimensional Figures Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Find points of intersections of points, lines, planes, and various geometric figures. - Use properties of parallel lines, perpendicular lines, intersecting lines to determine slopes, x & y intercepts, and points of intersection. - Find the number of and length of distinct diagonals or polyhedra and Platonic Solids
<p><u>Properties and Relationships of Figures Knowledge and Skills:</u></p> <p>G8. Uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations. (Geo, PK8) G9. Analyzes properties and describes relationships in geometric figures. (Geo) G10. Applies the concept of congruence to justify properties of figures and solve problems. (Geo) G11. Applies the concepts of similarity to justify properties of figures and solve problems. (Geo)</p>	<p><u>Properties and Relationships of Figures Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Find side lengths, heights, perimeters, and areas of various geometric figures. - Use theorems to find measures of angles, arcs, sides, etc. - Use vocabulary about circles to determine radius/diameter/chord lengths. - Find circumference, area, and other missing parts of circles. - Find surface area, volume, and missing parts of various polyhedra and Platonic Solids. - Find missing parts of congruent and/or similar of figures.
<p><u>Symbolic Representation Knowledge and Skills:</u></p> <p>P2. Interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems. (Pc)</p>	<p><u>Symbolic Representation Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Evaluate functions and inverse functions at specific points. - Evaluate composite functions at specific points. - Evaluate logarithmic functions, exponential functions, trigonometric functions, etc.

<p><u>Probability and Statistics Knowledge and Skills:</u></p> <p>M4. Uses probability models to describe everyday situations involving chance. (MM)</p>	<p><u>Probability and Statistics Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Find the probability and/or odds of experiments and situations. - Use the fundamental counting principle to determine the number of possible outcomes.
<p><u>Problem Solving Knowledge and Skills:</u></p> <p>P3. Uses functions and their properties, tools and technology, to model and solve meaningful problems. (Pc)</p> <p>P4. Uses sequences and series as well as tools and technology to represent, analyze, and solve real-life problems.</p> <p>M1. Uses a variety of strategies and approaches to solve both routine and non-routine problems. (MM)</p> <p>M2. Uses graphical and numerical techniques to study patterns and analyze data. (MM).</p> <p>M8. Uses algebraic and geometric models to describe situations and solve problems. (MM)</p> <p>M9. Uses algebraic and geometric models to represent patterns and structures. (MM)</p>	<p><u>Problem Solving Knowledge and Skills:</u></p> <ul style="list-style-type: none"> - Find values of the basic trigonometric functions given an angle or radian measure. - Apply trigonometric functions to analyze and solve problems and to make predictions. - Convert degree measures to radian measures and vice versa. - Find missing terms of sequences and series. - Find the sums and limits of sequences and series. - Use sequences and series to find solve problems. - Use the binomial expansion to find missing coefficients of polynomials. - Find means, medians, modes, and ranges from given data. - Use means, medians, modes, and ranges to solve problems. - Use linear functions, direct variations, indirect variations, etc. to solve problems. - Use common trigonometric identities to solve expressions.

Appendix - UIL Number Sense Test Problem Sequencing

Problems 1 - 20 ***

- 1) Addition, subtraction, multiplication, & division of Integers, Mixed Numbers, Fractions, and Decimals
- 2) Order of Operations
- 3) Use of the Distributive Property
- 4) Comparison of Fractions and Decimals
- 5) Multiplication Short-Cuts
- 6) Squaring Numbers
- 7) Conversion Problems (either way):
Percent/Fractions, English/Metric,
Roman Numerals/Arabic Numerals
- 8) Greatest Common Divisor (GCD) & Least Common Multiple (LCM)
- 9) Percent Problems
- 10) Mean, Median, & Mode
- 11) Sums of Integers
- 12) Remainder Problems
- 13) Consumer Type Problems
- 14) Number Theory Problems Involving: Prime Numbers, Divisors, Sums of Divisors, etc.

Problems 21 - 40

- 1) Powers of Numbers
- 2) Substitution
- 3) Word Problems
- 4) Inverses
- 5) Absolute Value
- 6) Ratio/Proportion
- 7) Square Roots/Cube Roots
- 8) Sets
- 9) Base System Problems
- 10) Solving Simple Equations
- 11) Simultaneous Equations
- 12) Repeating Decimals to Fractions
- 13) More Remainder Type Problems
- 14) Perimeter & Area Problems of Polygons
- 15) Sequences
- 16) Quadratic & Cubic Equation Problems

Problems 41 - 50

- 1) Laws of Exponents
- 2) Right Triangle Problems
- 3) Coordinate Geometry Problems

- 4) Regular Polygon Problems
- 5) Inequalities

Problems 51 – 60

- 1) Applications of Theorems from Geometry
- 2) Direct and Inverse Variation
- 3) Sequences & Series (Finite & Infinite)

Problems 51 - 70

- 1) Complex Numbers
- 2) Logarithms & Logarithmic Equations
- 3) Permutations & Combinations
- 4) Probability
- 5) Conics
- 6) Binomial Theorem (Expansion)

Problems 61 - 70

- 1) Volume & Surface Area
- 2) Greatest Integer
- 3) Application of Remainder Theorem
- 4) Trigonometry
- 5) Determinants
- 6) Matrices
- 7) Vectors
- 8) Composite Functions

Problems 71 - 80

- 1) Value of Domain of a Given Function
- 2) Bases Involving Decimal Fractions
- 3) Polar/Rectangular Coordinates
- 4) Modular Arithmetic
- 5) Limits
- 6) Derivative
- 7) Slopes of Tangent Lines
- 8) Horizontal & Vertical Asymptotes
- 9) Determining Critical Values
- 10) Maximum & Minimum Problems
- 11) Definite Integration

*** A type of problem from a particular section could appear later in the test. Example: A GCD problem could appear as problem #43, but not any earlier than problem #21.