Developing Middle School Number Sense Skills

Second Edition

by

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INTRODUCTION

This number sense handbook is being written to help the middle school teacher (grades 6 - 8) coach the Texas University Interscholastic League (UIL) Number Sense Contest. It is not a complete book on all of the short cuts, but it should help the beginning (novice) teacher/coach get started. I hope it will also help the veteran coach with certain ideas that they might be having trouble with on the number sense test. This handbook I hope will also be used in the average classroom setting when particular ideas can be introduced along with the objectives of the daily lesson plan.

Number sense is mental mathematics. It covers basic arithmetic, algebra, geometry and number theory on the middle school level. Since the fall of 1986, the UIL office has been printing a one page flyer called *Problem Sequencing* for all three levels of the number sense contest. This handbook will be written basically on that format from the junior high level. With that format in mind, hopefully the book will be beneficial to the teacher/sponsor (coach) of the number sense contest.

There will be only three practice tests included in this handbook. The reason being that tests change every year. But the Student Activities Conference (SAC) tests are not included in the test package that you can purchase from the UIL office. Therefore, the test that is in this book you want to make copies of for your students to practice with. The tests are from 1995 and 1996.

I started writing the tests for the UIL in the fall of 1986. Before that time, the middle school tests only had 70 problems. Starting with J-13 they now have 80 problems. Each year you should attend the Student Activities Conference so that you can get a copy of the test and answer key for your students. Otherwise, you will not see the SAC test for that particular year.

CHAPTER ONE

Coaching Number Sense

One day you woke up and found that you were going to be your school's number sense coach/sponsor or maybe you volunteered. You might not have ever seen a number sense test or even competed when you were in school. Don't feel alone, there are a lot of teachers that were in that position once in their lifetime. It even happened to me in the late sixties. Now I am the state director and I never competed as a student.

You might be a veteran teacher but now you are a novice when it comes to coaching a student for the number sense contest. That's okay. Hopefully this book will help you get over that feeling of being a beginner.

The first thing you want to do is read Section 1072 in the *Constitution and Contest Rules*. Your principal should have a copy. Then look at several copies of old tests for the junior high level. These tests are called the 3-Series. A number sense test is a 10-minute test and you can only write down the answer. Therefore, you are going to have to look at the test differently than you would with a test in your classroom and with the teaching of the students for the number sense contest.

A lot of the problems look straightforward and they are, and some might look hard or difficult to do in your head. But that's your first instinct. After carefully looking at a problem, analyzing it and making up several new problems similar to it, hopefully you will find a short cut for it. Every question on a number sense test that I write is either straightforward or can be done easier with a short cut.

A short cut is just a faster way of doing a problem than you might ordinarily do it. A short cut is not "a trick." It is based upon some property or properties of mathematics. It might involve using the distributive property or using an algebraic identity. Here are a couple of examples to show you what I mean by looking at a problem:

The first example might look difficult at first to do in your head.

$7 \times 2 + 7 \times 18 =$ _______.

Whatever you do, you don't want to add 14 to 7×18. Here you would want to think of and use the distributive property.

$7 \times 2 + 7 \times 18 = 7(2 + 18) = 7(20) = 140$.

The next example looks straightforward and it can be worked that way if you like.

$13^2 - 7^2 =$ ________.

Most people think of it as $169 - 49 = 120$. That's okay but if the numbers get a little larger it might not be as easy to do. Another way would be to think of the Difference of Two Squares from algebra. That is, $a^2 - b^2 = (a + b)(a - b)$. Hence, $13^2 - 7^2 = (13 + 7)(13 - 7) = (20)(6) = 120$.

These two examples show what you as a coach of the number sense contest are going to have to do.

Coaching a student to win at number sense is a lot harder than with some of the other UIL contests. But don't give up. A student who competes in the number sense contest whether he/she wins or not, learns a lot. It makes them think faster. You as a coach will also have to think differently and you will have to spend some time on it. You were not trained in school to coach number sense and nobody else was either. All number sense coaches work at it even if they competed as a student. High school number sense coaches are in the same boat as you are if you are just starting out. But they have a larger playing field on which to work.

Now that you are going to coach number sense, you need the students. Try interesting some of your students by showing them how to square two-digit numbers ending in five, multiplying by 11 or squaring a two-digit number. Or talk to the other teachers in your department and have a short meeting before or after school with any students that might be interested in number sense. Starting a number sense program at your school is the hardest part of coaching number sense.

Once you get a program started it will take care of itself. Students will recruit students for you once there is an interest in number sense. Especially when the students start to win at local contests. Working with your students will depend upon how much time you have to give and when you can get together with your students. Before school, during a period of the day or after school will depend upon you, your students and your school day schedule. I found on the high school level before school was best.

Start by meeting once a week or twice. Again this depends upon you and your students. If they really get interested in number sense, later they will want to meet almost daily if possible. Start off by having them know the ideas found on the first 20 problems of the test. Then the ideas found on the next 20 problems. You might not want them to take a test until you have gone over the ideas on the first 20-40 problems of a test. Nothing is worse than having them take a test and having them make a negative score. Before giving them a test to take, you might want to make up a similar test with only 20 questions on it and have them take it during a 10-minute period particularly for beginning students. This way they will get the feel of a test and the amount of time on the test. Then let them take a test, score it and keep a record of it. Each time the student takes a test have them work out the next 10-20 problems before they take another test. You might want them to keep a folder of all the tests they take.

After the students take a test go over it with them. If you don't have time that particular day then the next time you meet. It's important that you go over the test with them each time and discuss the questions with them that they missed.

Some of the things the students should know before taking a test besides the basic operations of arithmetic are the following:

(1) Squares of numbers from 1 to 25.

(2) Decimal and percent equivalents of the most common fractions.

You as a sponsor/coach should make yourself a notebook of short cuts and certain ideas found on the middle school test. There are a few books on the market but they do not cover every idea you need.

If you are a novice number sense sponsor then this is why the book is written the way it is. You can concentrate on the problems 20 at a time. If you are a veteran sponsor, you might want to let your beginning students look at just certain sections of the book. In either case, I hope this book will help you and your students be winners.

Each chapter of the book will have a list of the types of problems that the student can expect to see on the test. The list is very general but it should help you in working with your students. If you decide to make up a practice test for your students follow these guidelines in chapters two through six.

For example, a problem dealing with the least common multiple (LCM) of two or three numbers appears in problems 21 - 40, chapter three. The earliest that this type of question would appear on the test would be problem number 21. But it could appear later on the test like problem number 42.

Chapter two lists nine types of problems that can be found on problems numbered 1 - 20. But I can think of **at least 37** different types of problems that would fit the general guidelines of Chapter Two.

CHAPTER TWO

Problems 1 - 20

1. *Addition, subtraction, multiplication and division of whole numbers, fractions and decimals.*

Guideline number one is straightforward. The student has to be able to mentally compute whole numbers, fractions and decimals with the four basic operations of arithmetic.

2. *Order of Operations.*

Order of operations is another thing. Looking at middle school textbooks I see it is first mentioned on the seventh grade level of the textbooks that I have. It is an important idea and I always have a question of this type on all my tests on both the middle school and high school tests.

An easy way to remember the order of operations is with the saying, <u>P</u>lease <u>M</u>y <u>D</u>ear <u>A</u>unt <u>S</u>ally. First you compute anything that is inside parentheses. You do this first by doing any multiplication or division of the numbers from left to right just like you read a sentence. Then you do any addition or subtraction from left to right of the remaining numbers. Notice it uses the word "or." If a number is raised to a power then you have to evaluate it first. Here are several examples to illustrate the idea of order of operations.

(1) $12 \div 3 \times 2 + 1 = 9$

(2) $12 \times 4 \div 2 - 1 = 23$

(3) $(10 - 7)3 + 1 = 10$

(4) $(10 - 7) \div 3 + 1 = 2$

(5) $4 + 2^2 - 3 = 5$

(6) $16 - 8 \div 2 + 1 = 13$

3. *Use of the distributive property.*

Use of the distributive property is an important idea that is introduced to students in the sixth grade. That is, if a, b and c are any three real numbers then a x (b + c) = a x b + a x c or equivalently written a (b + c) = ab + ac. It also is true if the operation of addition is replaced with the operation of subtraction. It is in my opinion one of the most important ideas that students should learn in junior high school. It they don't understand it, they will have trouble with algebra in high school. It is used on the number sense test to compute problems that might at first look difficult. Here are several examples of how it could appear on a test.

(1) $6 \times 15 + 14 \times 15 = (20)(15) = 300$

(2) $(3 \times 9) + (9 \times 17) = (9)(20) = 180$

(3) $1.2 \times 6 + 4 \times 1.2 = (1.2)(10) = 12$

(4) $\frac{3}{4} \times \frac{2}{7} + \frac{5}{7} \times \frac{3}{4} = \left(\frac{3}{4}\right)(1) = \frac{3}{4}$

4. *Comparison of fractions and decimals.*

Guideline number 4 on comparison of fractions and decimals is a question that has several variations to it. If the two numbers are decimals then the student should have no difficulty in determining which is larger or smaller. If one number is a decimal and the other is a fraction, then this is where the student should know the decimal and percent equivalents of the most common fractions that I mentioned in chapter one. You as a number sense sponsor/coach should have a one-page flyer that lists the decimal and percent equivalents of the most common fractions. Here is where you need for yourself, as a coach, a little notebook that I also mentioned in chapter one.

If both numbers are fractions then this problem might seem difficult on the test. First you don't want the students to try and get common denominators of the two fractions and then compare the numerators. If $\frac{a}{b} = \frac{c}{d}$ then $ad = bc$ is how we can tell if two fractions are equal. Of course we assume that b and d are not zero. We can use this idea to compare two fractions. Suppose we are to compare two fractions, $\frac{a}{b}$ and $\frac{c}{d}$. All we need to do is look at the products of ad and bc.

(1) If $ad > bc$, then $\frac{a}{b} > \frac{c}{d}$.

(2) If $bc > ad$, then $\frac{c}{d} > \frac{a}{b}$.

Here is an example to show this idea.

The larger of $\frac{4}{5}$ or $\frac{7}{9}$ is ______ .

Since $4(9) = 36$ and $5(7) = 35$, then $\frac{4}{5} > \frac{7}{9}$ and you don't have to change both fractions to a decimal or get a common denominator.

5. *Multiplication short cuts.*

On multiplication short cuts, students should be able to multiply by 11, 12, 15, 25, 50 and 75. Most of these methods you should know. Multiplication by 12 is the only short cut that will be explained and proved. To multiply a number by 12, the following is the short cut:

(1) Double the units digit and write down the product if it is a one digit number. If it is a two-digit number, write down the units digit and remember the one.

(2) Double the remaining digits and add one to this product if your product in step (1) was a two-digit number.

(3) Take the number being multiplied by 12 and add it to the product or sum in step (2).

(4) Place your answer in step (3) to the left of your answer in step (1) and you will have your answer to the problem.

Examples:

12 x 21 = ?

(1) 2 x 1 = 2 write itdown

(2) 2 x 2 = 4

(3) 21 + 4 = 25

(4) Therefore, 12 x 21 = 252

12 x 127 = ?

(1) 2 x 7 = 14, write down the 4 and remember the l

(2) $2 \times 12 = 24$ and $24 + 1 = 25$

(3) $25 + 127 = 152$

(4) Therefore, $12 \times 127 = 1524$

Proof:

$12 \times ab = ?$

$12 \times (10a + b) = 120a + 12b$

$100a + 20a + 10b + 2b$

$100a + 10(2a + b) + 2b$

$10(10a + 2a + b) + 2b$

$10[(10a + b) + 2a] + 2b$

$10[(ab) + 2a] + 2b$

$13 \times 16 = ?$

(1) $3 \times 6 = 18$, write down 8 and remember the 1

(2) $3 \times 1 = 3$ and $3 + 1 = 4$

(3) $4 + 16 = 20$

(4) Therefore, $13 \times 16 = 208$

6. *Squaring numbers.*

On squaring numbers, students should know:

(1) their squares from 1 to 25.

(2) how to square numbers ending in five and

(3) how to square a two-digit number by the foil method or an equivalent method.

7. *Roman numerals/Arabic numerals.*

I agree Roman numerals are not used much today; however, they are introduced in the sixth grade textbook that I have. The book calls the numbers decimal numerals but they are really called Arabic numerals. You can check any mathematics dictionary for this terminology. For instance the *Mathematics Dictionary* edited by James and James and published by D. Van Nostrand Company, Inc. Most school libraries have a copy of this book or they should in their reference section of the library. On the tests we will only be considering the additive and subtractive methods of Roman numerals. We will not use the bar representation of Roman numerals.

Roman Numeral	I	V	X	L	C	D	M
Arabic Numeral	1	5	10	50	100	500	1000

The following examples will illustrate this idea.

(1)	XXI = 21	(4)	32 = XXXII
(2)	XIV = 14	(5)	25 = XXV
(3)	CXV = 115	(6)	201 = CCI

8. *Mean, Median and mode.*

Guideline number eight has to do with statistics. When we count or measure things, the set of numbers is called data. The *mean* (arithmetic mean) or commonly called the *average* is the sum of all the numbers in a set divided by the number of addends. The *median* is the middle number in a set of numbers arranged from the smallest to the largest or from the largest to the smallest.

When you have an even number of numbers, the median is the average of the two middle numbers. The *mode* is the number that appears the most times in a set of numbers. The following two examples will illustrate these ideas.

(1) Given the numbers 10, 17 and 15, the average is 14, the median is 15 and there is no mode in this set of numbers.

(2) Given the numbers 23, 14, 17 and 14, the average is 17, the median is 15.5 and the mode is 14.

9. *Sums of whole numbers.*

The last guideline in this chapter has to do with sums of whole numbers.

There are several variations to this problem. The following examples will illustrate the variations.

(1) $1 + 2 + 3 + 4 + 5 =$ _______.

(2) $4 + 6 + 8 + 10 + 12 =$ _______.

(3) $2 + 4 + 6 + ... + 20 =$ _______.

Example number one can be found

(1) by adding $(1 + 5) + (2 + 4) + 3 = 6 + 6 + 3 = 15$ or

(2) by knowing that $1 + 2 + 3 + ... + n = \frac{n(n + 1)}{2}$; that is, the sum $\frac{5(6)}{2} = 15$.

Example number two can be found by noticing that $(4 + 12) = (6 + 10)$ and you have 8 all by itself so the sum is $2(16) + 8 = 40$.

Example number three can be found:

(1) by noticing that you have 10 even numbers and that $(2 + 20) = (4 +$

18) = 22. Hence there are 5 groups that each sum up to 22. Therefore, the sum is 5(22) = 110, or

(2) by knowing that $2 + 4 + 6 + ... + 2n = n(n+ 1)$; that is, the sum is 10(11) = 110.

CHAPTER THREE

Problems 21 - 40

1. *Addition, subtraction, multiplication and division of whole numbers, fractions and decimals.*

Guideline number one is straightforward. The student has to be able to compute mentally with mixed numbers and integers. The student also has to be aware of the type of answer the problem is asking for. The problem might ask for the answer to be a mixed number, a fraction, a decimal or a percent. All fractions have to be reduced to lowest terms. If a problem does not state a particular type of answer then leave it as an improper fraction if the answer is greater than one. I know that in a classroom you would rather have your students write the answer to a problem as $1\frac{2}{7}$ instead of as $\frac{9}{7}$. But a number sense test is different. Also you have only 10 minutes to work on the test. On the answer key if an answer can be written in several ways, then the first answer is what your students should be trying to arrive at. Suppose the answer key has the following: $3\frac{1}{4}$ or $\frac{13}{4}$ or 3.25. Your students should be writing the answer as $3\frac{1}{4}$. It will be the fastest and most efficient way of working that particular problem. If your students are getting $\frac{13}{4}$ then you should try to figure out a short cut that will give your students the answer of $3\frac{1}{4}$.

2. *More multiplication short cuts.*

On more multiplication short cuts I mean new ways or a combination of ideas. The following short cuts might he new to you.

(1) Multiplication by 125

Think of 125 as being $\frac{500}{4}$; 24 x 125 = 24 x $\frac{500}{4}$ = 6(500) = 3000.

(2) Double and Half Method

This idea is useful when trying to multiply

(i) a multiple of five times an even number,

(ii) a multiple of eleven times a number or

(iii) a mixed number that has $\frac{1}{2}$ as its fractional part.

The short cut is:

(1) double one factor and take one-half of the other factor.

(2) Multiply together the two new factors and you will have the answer.

Examples:

15 x 28 = ?

(1) Think 30 x 14 = 420

(2) Therefore, 15 x 28=420

22 x 21 = ?

(1) Think 11 x 42 = 462

(2) Therefore, 22 x 21 = 462

$6\frac{1}{2}$ x 18 = ?

(1) Think 13 x 9 = 117

(2) Therefore, $6\frac{1}{2}$ x 18 = 117

This method also works if you triple a number and take one third of its other factor or etc.

(3) Difference of Two Squares, $a^2 - b^2 = (a + b)(a - b)$. I mentioned this method in Chapter One.

24 x 36 = ?

(1) Think 24 = 30 - 6 and 36 = 30 + 6

(2) 24 x 36 = $30^2 - 6^2$ = 900 - 36 = 864

(4) Multiplication of Two 2-Digit Numbers Whose Tens Digits Are the Same and the Units Digits Add Up to Ten.

The short cut is:

(1) multiply together the units digits and write down the product. If the product is not a two digit number, place a zero to the left of the product.

(2) Add 1 to one of the tens digits and multiply this sum times the original tens digit.

(3) Place this product to the left of the product of the units digits and you will have the product of the original two numbers.

Examples:

41 x 49 = ?

(1) 1 x 9 = 09

(2) 4 x 5 = 20, since 5 = 4 + 1

(3) Therefore, 41 x 49 = 2009

74 x 76 = ?

(1) 4 x 6 = 24

(2) 7 x 8 = 56 since 8 = 7 + 1

(3) Therefore, 74 x 76 = 5624

This method also works if the numbers are decimals or mixed numbers.

Examples:

A) 4.1 x 4.9 = 20.09

B) $4\frac{3}{10} \times 4\frac{7}{10} = 20\frac{21}{100}$

(5) Multiplying Two Numbers That Are Close to But Less Than 100.

The short cut is:

(1) subtract each number from 100.

(2) Multiply these two numbers together and write down the product. If the product is not a two digit number, place a zero to the left of the product.

(3) Take either one of the differences in step (1) and subtract it from the other number.

(4) Place this difference from step (3) to the left of your answer in step (2) and you will have your answer.

96 x 95 = ?

(1) 100 - 96 = 4 and 100 - 95 = 5

(2) 4 x 5 = 20

(3) 96 - 5 = 91 or 95 - 4 = 91

(4) Therefore, 96 x 95 = 9120

3. *Percent problems.*

Guideline number three on percent problems is straightforward. The following examples will illustrate the different types that you can expect to see on a test.

(1) 40% of 128 = _________.

(2) 18 is what percent of 72? _________.

(3) 16 is 25% of _________.

(4) 60% = __________ (fraction).

(5) $\frac{1}{20}$ = _______ %.

(6) 20 less 10% of 20 is ________.

4. *Conversion problems (either way): English/metric, length, weight, area, capacity, or time.*

Conversion problems are problems where the student is asked to change from one unit to another. Here's where another one-page flyer would be helpful for the students. It should contain both the metric and English (customary) system of length, weight, area, capacity and time.

5. *Consumer type problems.*

They are short story (verbal) problems. Here are three examples to illustrate this type of problem.

(1) Your bill is $14.86. How much change will you receive from a $20.00 bill?

(2) The cost of two apples is 37 cents. How much does a half-dozen cost?

(3) A car travels 162 miles in 3 hours. The average speed was ______ mph.

6. *Substitution problems.*

These are problems where the student has to substitute numbers for variables/letters of the alphabet. The most important thing that the student has to watch for is the concept of order of operations of real numbers. Generally they will be given in terms of the variables A, B and C. Check the idea given in section 2 of Chapter Two.

7. *Solving simple equations.*

Solving simple equations means solving a linear equation for x using one or two basic operations. The following two examples should illustrate this idea.

(1) If $x + 17 = 45$, $x =$ ________.

(2) Find x, if $3x + 4 = 19$. ________.

8. *Square roots/cube roots.*

Here again is where the student should know his/her squares from 1 to 25. I have not written a middle school test with any cube roots and I will probably not in the future.

9. *Greatest common factor (GCF) and least common multiple (LCM).*

The Greatest Common Factor (GCF) and the Least Common Multiple (LCM) type problems appear on every test. Before the fall of 1988 (tests J-13 to J-20) I used the terminology, the greatest common divisor (GCD). To me a factor and a divisor mean the same thing. The number 2 is a divisor of 6 as well as saying that 2 is a factor of 6. But I have now changed the terminology to match the middle school textbooks. But on the high school level it will be called the greatest common divisor.

I assume that you know how to find the GCF and the LCM of two or three numbers. The following idea you might not know. Let GCF (a,b) and LCM (a,b) denote the greatest common factor and the least common multiple of two whole numbers a and b, respectively. Then

$$\text{GCF}(a,b) \times \text{LCM}(a,b) = a \times b.$$

Try this idea out and you will see that it works every time.

10. *Number Theory - Prime numbers and divisors.*

Number Theory questions on the test relate only to the set of positive integers. Here is where the student should know:

(1) some of the divisibility rules for numbers,

(2) what a prime number is, and

(3) what we mean by the remainder in a division problem.

Generally, some of these types of problems have appeared later on in the test.

The Fundamental Theorem of Arithmetic states that every composite number can be factored into prime factors. An idea that you might not know is how to find the number of positive integral divisors of a number.

The short cut is:

(1) prime factor the number.

(2) Add one to each of the exponents.

(3) Multiply together the numbers in step (2) and you will have the number of positive integral divisors of the number.

The number of positive integral divisors of 24 is ?

(1) $24 = 2^3(3)$

(2) $3 + 1 = 4$ and $1 + 1 = 2$

(3) $(4)(2) = 8$

Therefore, there are 8 positive integral divisors of 24. There are a lot of other ideas that deal with the prime factorization of a number.

11. *Perimeter/Area of a: square, rectangle, or circle.*

The students should know some elementary ideas about a rectangle, a square and a circle. In particular they should know the perimeter and area of each. This is a straightforward question.

12. *Ratio/Proportion.*

Guideline number 12 deals with ratios or a proportion. A proportion is where two fractions are equal or are assumed to be equal. An example of this type of question is the following:

$$\text{Find x if } \frac{x}{35} = \frac{4}{5}.$$

Here the student has to know what we mean by the cross product or equivalent fractions.

13. *Inverses.*

The last idea in this chapter deals with the idea of inverses. The student should know what we mean by the additive inverse (opposite) and multiplicative inverse (reciprocal) of a number. If they do, then this is straightforward.

CHAPTER FOUR

Problems 41 - 60

1. *Sets.*

This topic is held over from the *New Math Era* of the sixties. I agree most books do not have sets in them. My textbooks have them as a challenge problem. Here again you can make up a one-page flyer on sets, subsets, intersection and union of two sets, and finite and infinite sets. You cannot talk about numbers without using this terminology. The numbers 2, 4, 6, 8,... is a subset of the counting numbers. The set of integers is the union of two sets; the set of whole numbers and their opposites (additive inverses). The least common multiple of 4 and 6 is the smallest positive number in the intersection of the two sets 4, 8, 12,... and 6, 12, 18, ... If a set contains n elements (objects) then the total number of subsets is 2^n.

2. *Word problems.*

On the middle school level these are just a written fill in the blank statement. They usually have something to do with geometry or a question about a number. The following two examples should illustrate this idea.

(1) The volume of a cube is 64 cu. in. The edge of the cube is ______ in.

(2) Two times a number plus 1 is 37. The number is ________.

3. *Pythagorean theorem.*

I find that this idea is not introduced until grade eight in my books. This is a very important idea that I believe should be introduced earlier. This is why it is listed by itself and not under the topic of basic geometry facts. If your students are now working in this interval of problems and your book does not introduce it, then you need to show them this idea.

Every book I have seen states it in some form like the following:

If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$.

This idea has been known since about 540 B.C. It has been proven over 300 different ways. The 20th president of the United States, James A. Garfield, had a unique proof of the Pythagorean theorem. The book, *The Pythagorean Proposition* by Elisha Scott Loomis and published by the National Council of Teachers of Mathematics (NCTM) contains 256 different proofs.

Besides finding one side of a right triangle when given two sides, it can be used to find the diagonal of a square or rectangle. Any prime number of the form 4n + 1 where n is a positive integer can be represented as the sum of two squares.

4. *Sequences.*

A sequence is a set of numbers in one-to-one correspondence with the set of positive integers (an infinite sequence). The members of the sequence are called terms.

On the test an infinite sequence is given and the student is asked to find the next term in the given sequence. The following examples will illustrate this idea and the answer is in the set braces.

The next term in the sequence

(1) 4, 6, 8, 10,... is ____________. {12}

(2) 9, 7, 5, 3, ... is ____________. {1}

(3) 7, 8, 10, 13, 17,... is ____________. {22}

(4) 5, 4, 7, 6,...is ________. {9}

The most famous sequence known is the Fibbonacci sequence:

1, 1, 2, 3, 5, 8, 13,….

5. *Volume/surface area of rectangular solid/cube.*

Guideline number five is a straightforward question. Let e be the edge, V the volume and S the surface area of a cube. Then the following ideas are known:

(1) $S = 6e^2$ (2) $V = e^3$

6. *Base systems.*

This is another topic held over from the *New Math Era.* If a student really understands our decimal system, base ten, then they should not have any trouble with other base systems. Here again you might want to have a one-page flyer on base systems.

The numeral 34 in base five is usually written in most books as 34_{five}. But on the number sense tests I will write it as 34_5. My reason being that I think students can compute the problem faster if the base is written as a numeral than by having it spelled out. In any base system, the largest number is always one less than the base. In base two, the smallest base system, only two numbers are used: the numbers 0 and 1. On the number sense test, base system problems will only use bases 2 through 9.

The value of a number in some other base system is found similar to base ten numerals. Let b be a base system greater than or equal to two. Suppose we have a three-digit numeral in base b with digits x, y and z respectively. The value of xyz_b is found by the following:

$xyz_b = x(b^2) + y(b) + z$

Examples:

(1) $111_2 = 1(2^2) + 1(2) + 1 = 7$

(2) $143_5 = 1(5^2) + 4(5) + 3 = 48$

The following examples will illustrate the types of problems that will be found on the middle school tests.

(1) $46_8 =$ ________ $_{10}$.

(2) $31_6 + 42_6 =$ ________ $_6$.

(3) $13_4 =$ ________ $_2$.

(4) 1f $45_b = 33$, then b = ________.

(5) If $3b_5 = 18$ then b = ________.

7. *Area of a: parallelogram, rhombus, trapezoid, or triangle.*

The questions of this guideline are straightforward. The student should be able to find the area of a parallelogram or a triangle given the base and the height (altitude) of the polygon.

They should know the difference between a parallelogram and a rhombus. The area of a parallelogram (rhombus) is equal to the product of the base and the height. The following might be a new idea to you. Let x be a diagonal and y be the other diagonal of a rhombus with A denoting its area. Then the following is known:

$$A = \frac{1}{2}xy$$

Students should know the difference between the following types of triangles.

(1) equilateral

(2) isosceles

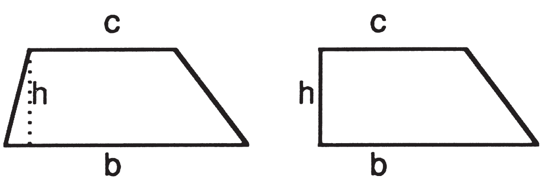
(3) right

(4) scalene

I noticed in my books they only mention trapezoids but do not give the area of a trapezoid. Let b be a base, c the other base, h the altitude (height) and A the area of a trapezoid. Then the following is known:

$$A = \frac{1}{2}(b + c)h$$

Here are two drawings of a trapezoid that might be helpful.



8. *Solving inequalities.*

Here the student will be asked to solve a simple linear inequality. This topic is near the end of my eighth grade textbook. But if a student can solve a linear equation, see section 7 of chapter three, then they should not have any difficulty with this topic. The following two examples should illustrate this idea.

(1) The smallest integer x, such that $x + 4 > 9$ is x = _________ .

(2) The largest integer x, such that $2x + 1 < 11$ is __________.

9. *Basic geometry facts.*

Guideline number nine is a rather broad topic. Here the student should know what a polygon is and the different types of polygons that have not already been discussed. They should know what a regular polygon is and what we mean by a diagonal in a polygon.

Students should also know something about angles and the difference between the following types of angles.

(1) acute

(2) complimentary

(3) obtuse

(4) right

(5) supplementary

10. *Remainder problems.*

This question is straightforward on the middle school test. The student basically needs to know what a remainder is in a division problem. The following two examples should illustrate this idea.

(1) $(1 + 3 \times 6) \div 4$ has a remainder of __________ .

(2) $(3 + 14 \times 6) \div 6$ has a remainder of _________ .

CHAPTER FIVE

Problems 61 - 80

1. *Repeating decimals.*

This topic involves changing a repeating decimal to a rational number (fraction). There are several variations to this idea. The two simplest ones will be discussed here. The proof of the first short cut can be found in most books and the proof of the second short cut would be similar.

Examples: $\overline{.24}$ $\overline{.015}$ $\overline{.234}$

The short cut is:

(1) think of a fraction whose numerator is the repeating digits and the denominator contains as many 9's as the number of repeating digits.

(2) Reduce to lowest terms if possible.

Examples:

.242424... = ?

(1) Think $\overline{.24} = \frac{24}{99}$ and reduce

(2) Therefore, $\overline{.24} = \frac{8}{33}$

.015015... = ?

(1) Think $\overline{.015} = \frac{15}{999}$ and reduce

(2) Therefore, $\overline{.015} = \frac{5}{333}$

Examples: $.2\bar{7}$ $.4\bar{5}$ $.3\bar{2}$

The short cut is:

(1) take the repeating decimal and think of it as a two-digit number.

(2) Subtract the tens digit from the number.

(3) Divide the difference from step (2) by 90 and you will have the rational number.

(4) Reduce to lowest terms if possible.

Examples:

.27777... =_______ (fraction).

(1) Think of $.2\bar{7}$ as being 27

(2) 27 - 2 = 25

(3) $25 \div 90 = \frac{25}{90}$

(4) Therefore, $.2\bar{7} = \frac{5}{18}$

.32222... =________ (fraction).

(1) Think of $.3\bar{2}$ as being 32

(2) 32 - 3 = 29

(3) $29 \div 90 = \frac{29}{90}$

(4) Therefore, $.3\bar{2} = \frac{29}{90}$

2. *More number theory.*

This is a continuation of section 10 in chapter three. The problems get a little harder with larger numbers.

3. *Powers of numbers.*

The question is straightforward. The student is asked to raise a number to a certain power.

4. *Volume of a: circular cylinder, cone, or sphere.*

These are found mostly in eighth grade textbooks. The questions are straightforward but the formulas are given here if you do not have them all in your book.

The volume of a solid is the number of cubic units it contains. In general the volume is found by multiplying the area of the base times its height.

By a circular cylinder we mean a right circular cylinder. That is, where the height is perpendicular to the base. For a cylinder the base is a circle. Let r be the radius of the base, B the area of the base, h the height and V the volume of a right circular cylinder. Then the following is known:

$$V = Bh = \pi r^2 h$$

By a cone, we mean a right circular cone where the height is perpendicular to the base. The base is a circle. Let r be the radius of the base, B the area of the base, h the height and V the volume of a right circular cone. Then the following is known:

$$V = \frac{1}{3}Bh = \frac{1}{3}\pi r^2 h$$

The volume of a sphere is another thing. My textbook gave an intuitive explanation for the surface area and volume. The student has to just accept this formula for the present until he/she studies integral calculus. Let r be the radius of a sphere and V the volume. Then the following is known:

$$V = \frac{4}{3}\pi r^3$$

If the reader is interested, there is a single formula that will give the volume of a cylinder, cone, sphere, cube and rectangular and triangular prisms all in one. It is called the prismoidal formula. Look in the mathematics dictionary that I mentioned in section 7 of chapter two.

5. *Sequences and series.*

This topic is a continuation of section 4 in chapter four. A series is just a sequence where the commas are replaced by addition signs and you are asked to find the sum. The problems become a little more difficult and different than those mentioned earlier.

6. *Multiplication by 101, 111.*

Multiplication of 101 by a two-digit number is straightforward. Multiplication of 111 by a two-digit number will only be discussed here. The short cut is similar to multiplying by 11.

The short cut for a two-digit number is:

(1) write down the units digit in the number being multiplied by 111.

(2) Add the tens digit to the units digit and write down this sum. If the sum is greater than or equal to 10, write down the units digit and add the tens digit to your next sum.

(3) Repeat step (2) and write down the sum.

(4) The number being multiplied by 111 can be thought of as having a zero in front of the number. Now write down the tens digit if you didn't carry one from step (3).

Examples:

111 x 45 = ?

(1) Write down the 5

(2) 4 + 5 = 9, write down the 9

(3) 4 + 5 = 9, write down the 9

(4) 0 + 4 = 4, write down the 4

Therefore, 111 x 45 = 4995

111 x 67 = ?

(1) Write down the 7

(2) 6 + 7 = 13, write down the 3 and carry the 1

(3) 6 + 7 + 1 = 14, write down the 4 and carry the 1

(4) 0 + 6 + 1 = 7, write down the 7

Therefore, 111 x 67=7437

7. *Factorial.*

The idea of a factorial is used throughout mathematics especially in systematic counting and probability. Let n be a counting number, then n! (read n factorial) is defined as <u>the product of consecutive integers</u> from n down to one; that is,

$$n! = n(n-1)(n-2)(n-3) \ldots (3)(2)(1)$$

By definition, $0! = 1$. From the definition of $n!$, we see that $n! = n(n-1)!$.

The first few values are:

$1! = 1$, $2! = 2$, $3! = 6$, $4! = 24$, and $5! = 120$

8. *Coordinate geometry.*

Here the student should know the following two forms for the equation of a line.

(1) general form – $ax + by = c$

(2) slope-intercept form – $y = mx + b$

They should know what the slope of a line is and how to find it. Also, they should know how to find the x and y-intercept of a line.

9. *Probability.*

I know this topic is near the end of most books. Some teachers tell me they never have time to teach it. If a student is going to get this far on a number sense test then you will have to teach them some basic concepts of probability.

They should know what we mean by the following:

(1) anoutcome,

(2) a sample space,

(3) the probability of an event happening (definition),

(4) the sample space for rolling a die,

(5) the sample space for rolling a pair of dice, and

(6) what a regular deck of cards consists of.

10. *More percent type problems.*

A continuation of section 3 from chapter three.

11. *More remainder* type *problems.*

A continuation of section 10 from chapter four.

12. *More multiplication short cuts.*

A continuation of section 2 from chapter three.

CHAPTER SIX

ESTIMATION PROBLEMS

Every tenth problem on the number sense test is an estimation problem. You do not have to get the exact answer on these problems. Your answer though, starting in the fall of 1988, has to be an integer that is within 5% of the correct answer. When writing your answer on any question on the number sense test never write in the comma for an answer. For example, if the answer is 12,564 then just write the answer ast 12564. Writing in the comma takes up time which you cannot afford to lose on a number sense test.

Suppose the exact answer to one of these problems was 594. The 5% interval would be from 564.3 to 623.7. The answer key would read: An integer between 565 - 623 inclusive.

1. *Rounding off numbers to multiples of ten, hundred.*

Since your answer has to be within 5% of the correct answer, round off the numbers.

Examples:

(1) 398 + 201 - 4 = ________.

Think of the problem as being 400 + 200 = ________ and write down 600. The subtracting off of the 4 is negligible. The exact answer is 595 and the

5% interval on the answer key would be 566-624.

(2) 201 x 199 = ________

Think of the problem as being 200 x 200 = ________ and write down 40000. The exact answer is 39,999 and the 5% interval would be 38,000 - 41,998.

2. *The distributive property.*

Use the distributive property and the idea of rounding off.

Example:

(22 x 12) + (28 x 12) + 4 = ________.

Think of it as (22 x 12) + (28 x 12) = __________. Thus you should get 12(22 + 28) = 12(50) = 600. The exact answer is 604 and the 5% interval on the answer key would be 574-634.

3. *Division problems.*

The short cut is:

(1) round off the dividend to the nearest multiple of thousand.

(2) Round off the divisor to the nearest multiple of ten.

(3) Divide the answer in step (1) by the answer in step (2) and round off your answer. This is your answer.

Example:

25012 ÷ 148 = ________.

(1) Round off 25012 to 25000.

(2) Round off 148 to 150.

(3) Divide 25000 by 150 and round off. 25000 ÷ 150 = 167.

Therefore, 25012 ÷ 148 = 167; the exact answer is 169 and the 5% interval would be 161 - 177.

CHAPTER SEVEN

UIL AND TMSCA

The UIL offers a test for the middle school during the school year besides the SAC test and the district test. During the rest of the year an organization called the Texas Math and Science Coaches Association (TMSCA) offers a middle school championship contest and a monthly newsletter.

It is an organization that started here in the state during the early eighties. The main objective of TMSCA is to improve the competition in all four contests (calculator applications, mathematics, number sense and science) on the high school and middle school levels, besides having an excellent newsletter. The newsletter gives the dates of contests that will be held around the state, the results of the contests held, coaching tips on the contests and has a resource file. The resource file lists practice materials and books that you can purchase for the number sense contest.

For more information about TMSCA, contact the UIL office at:

UIL Academics
Box 8028, University Station
Austin, TX 78713-8028

Someone there will know who to put you in contact with at TMSCA. The UIL and TMSCA have had a very good working relationship over the years.

The University Interscholastic League
Number Sense Test, Series J956-SAC

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant's Number ____________________

Read Directions Carefully
Before Beginning Test

Do Not Unfold This Sheet
Until Told To Begin

DIRECTIONS: Do not turn this page until the person conducting this test gives the signal to begin. This is a ten-minute test. There are 80 problems. Solve accurately and quickly as many as you can in the order in which they appear. ALL PROBLEMS ARE TO BE SOLVED MENTALLY. Make no calculations with paper and pencil. Write only the answer in the space provided at the end of each problem. Problems marked with a (*) require approximate integral answers; any answer to a starred problem that is within five per cent of the exact answer will be scored correct; all other problems require exact answers.

The person conducting this contest should explain these directions to the contestants.

Stop - Wait for Signal!

(1) 36 - 9 + 14 = ____________________ .

(2) 3(11 + 17) = ____________________ .

(3) 1442 ÷ 7 = ____________________ .

(4) Which is larger, $\frac{3}{8}$ or $\frac{5}{11}$? ____________ .

(5) 25 x 140 = ____________________ .

(6) 1.24 x 5 = ____________________ .

(7) 572 – 275 = ____________________ .

(8) $\frac{3}{8} + \frac{3}{4}$ = ____________ (Mixed Number).

(9) 11 x 23 = ____________________ .

*(10) 17 + 129 + 3454 = ____________________ .

(11) 9 x 13 + 9 x 27 = ____________________ .

(12) 24 + 12 ÷ 6 – 2 = ____________________ .

(13) 24 ÷ $\frac{4}{7}$ = ____________________ .

(14) 50 x 39 = ____________________ .

(15) 17^2 = ____________________ .

(16) Reduce $\frac{24}{28}$ to lowest terms. ____________ .

(17) XXI = ____________________ (Arabic Numeral).

(18) 45 x 45 = ____________________ .

(19) 204 ÷ .4 = ____________________ .

*(20) 139 x 261 = ____________________ .

(21) 9 + 12 + 15 + 18 + 21 = ____________________ .

(22) 13 x 32 = ____________________ .

(23) –16 – 4 + 13 = ____________________ .

(24) (–3) (4) (–5) = ____________________ .

(25) $3\frac{1}{4} - 1\frac{5}{8}$ = ____________ (Mixed Number).

(26) The remainder when 131 is divided by 4 is _____ .

(27) 12% of 210 is ____________________ .

(28) $4\frac{1}{2}$ x 16 = ____________________ .

(29) $\frac{3}{8}$ = ____________________ (decimal).

*(30) 30003 ÷ 219 = ____________________ .

(31) 2.16 meters = ____________________ centimeters.

(32) The mean (average) of 14, 28 and –6 is ________ .

(33) If 4 pencils cost 89 cents, then 1 dozen pencils cost $ ____________________ .

(34) The square root of 121 is ____________________ .

(35) 1 square foot = ____________ square inches.

(36) If x = –3 then 4x – 5 = ____________________ .

(37) 34% = ____________________ (fraction).

(38) The GCF of 32 and 18 is ____________________ .

(39) If 3x +7 = 2x then x = ____________________ .

*(40) $11 \times 27 + 11 \times 22 =$ ______________________.

(41) $213 \div 9 =$ ______________ (Mixed Number).

(42) A car averages 32 mph for 2.5 hours. The distance traveled in miles is ____________________ .

(43) If $\frac{2x}{7} = \frac{9}{14}$ then x = ____________________ .

(44) 10% of 14% of 400 is ______________ (decimal).

(45) $(32 + 5 \times 7) \div 6$ has a remainder of __________ .

(46) The number 30 has ____________ distinct positive prime factors.

(47) $2\frac{1}{4} \times 6\frac{1}{4} =$ ______________ (Mixed Number).

(48) $34_7 =$ ______________________________ $_{10}$.

(49) A right triangle has sides of 5, 13, and 12 inches. Its area is ________________________ square inches.

*(50) $\sqrt{22400} =$ ______________________________ .

(51) The next term of 5, 8, 6, 9, 7, . . . is _________ .

(52) $6^2 + 12^2 =$ ______________________________ .

(53) If $2x + 1 < 17$ then $x <$ ____________________ .

(54) The total number of degrees in a right triangle is ____ .

(55) 24 is 6% of ______________________________ .

(56) 2 hours 15 minutes = ________________ seconds.

(57) If A = 3, B = 4 and C = 6 then $AB \div C =$ _____ .

(58) 80 is ____________________________ % of 250.

(59) What number is halfway between –8 and 12? ____ .

*(60) $219 \times 621 + 1200 =$ ______________________ .

(61) $37 \times 43 =$ ______________________________ .

(62) A diagonal of a rectangle divides the rectangle into ______________________________ triangles.

(63) .121212 . . . = ____________________ (fraction)

(64) $3^4 =$ ______________________________ .

(65) The smallest prime number greater than 37 is ____ .

(66) $97 \times 99 =$ ______________________________ .

(67) The distance between the points (2,3) and (2,7) is ____ .

(68) $101 \times 13 =$ ______________________________ .

(69) $111 \times 12 =$ ______________________________ .

*(70) $(13)^3 =$ ______________________________ .

(71) $19^2 - 18^2 =$ ______________________________ .

(72) Two dice are rolled. What is the probability that the sum is less than 4? ____________________ .

(73) $1211 \div 9 =$ ______________ (Mixed Number).

(74) $104 \times 105 =$ ______________________________ .

(75) Divide 36 into 2 parts such that the larger number exceeds the smaller number by 4. Find the smaller number. ______________________________ .

(76) $17316 \div 111 =$ ______________________ .

(77) If x = 3 then $x^2 - 6x + 9 =$ __________________ .

(78) The number 292 is a palindrome. What is the smallest palindrome larger than 319? ______________ .

(79) $13^2 + 29^2 =$ ______________________________ .

*(80) $(1 + 3 + 5 + 7)^3 =$ ______________________ .

The University Interscholastic League
Number Sense Test, Series J956C

	Score	Initials
Final	______	____
2nd	______	____
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Contestant's Number ________________

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Stop - Wait for Signal!

(1) $12 + 32 =$ ________________.

(2) $59 - 37 =$ ________________.

(3) $.3 \times 14 =$ ________________.

(4) $1456 \div 7 =$ ________________.

(5) $342 - 243 =$ ________________.

(6) $\frac{5}{8} - \frac{1}{4} =$ ________________.

(7) $5 \times 3.28 =$ ________________.

(8) Which is larger, $\frac{2}{5}$ or $\frac{3}{7}$? ________________.

(9) $11 \times 14 =$ ________________.

*(10) $28 + 149 + 1171 + 3199 =$ ________________.

(11) $123 \times 8 + 3 =$ ________________.

(12) $33 \div \frac{3}{5} =$ ________________.

(13) $25 \times 16 =$ ________________.

(14) $456 + 654 =$ ________________.

(15) Reduce $\frac{16}{28}$ to lowest terms. ________________.

(16) $24 \div 6 + 2 \times 3 =$ ________________.

(17) $15^2 =$ ________________.

(18) XXV = ________________ (Arabic Numeral).

(19) $\frac{3}{4} \times 14 =$ ________________ (Mixed Number).

*(20) $119 \times 199 =$ ________________.

(21) $13 \times 19 =$ ________________.

(22) $5 + 8 + 11 + 14 + 17 =$ ________________.

(23) $6^2 + 12^2 =$ ________________.

(24) $14 \times 12 + 14 \times 18 =$ ________________.

(25) The mean (average) of 16, 24 and 20 is ________________.

(26) 3 hours 12 minutes = ________________ minutes.

(27) The remainder when 194 is divided by 4 is ________________.

(28) $2\frac{1}{2} \times 32 =$ ________________.

(29) $311 \div 9 =$ ________________ (Mixed Number).

*(30) $139005 \div 129 =$ ________________.

(31) 6% of 140 is ________________.

(32) 42 inches = ________________ feet.

(33) The GCF of 35 and 84 is ________________.

(34) $\sqrt{144}$ = ______________________.

(35) $\frac{3}{4} + 1\frac{5}{8}$ = ______________ (Mixed Number).

(36) $\frac{2}{5}$ = ______________________ %.

(37) How many positive prime numbers are less than 8? ____ .

(38) 10% of 210 plus 13 is ______________ .

(39) 16 is what percent of 64? ______________ %.

*(40) 12 x 17 + 14 x 34 = ______________ .

(41) If x = –2 then $x^2 + 4x$ = ______________ .

(42) $\frac{1}{8}$ = ______________ (decimal).

(43) A circle has a circumference of 12 π inches. Its diameter is ______________ inches.

(44) 32 – 16 – 2(–7) = ______________ .

(45) If 2x – 8 = 3x – 14 then x = ______________ .

(46) The sum of the positive integer divisors of 7 is ________ .

(47) If $\frac{x}{4} = \frac{3}{10}$ then x = ______________ .

(48) The cube root of 27 is ______________.

(49) 1 square meter = ______________ square centimeters.

*(50) $\frac{3}{4}$ x 5810 ÷ 3 = ______________ .

(51) (32 x 8 + 3) ÷ 5 has a remainder of ______________ .

(52) The next term of 6, –8, 10, –12, ... is ______________ .

(53) 32 is 4% of ______________ .

(54) If two angles are complementary, then their sum equals ______________ degrees.

(55) If 3x – 8 < x then x < ______________ .

(56) The legs of a right triangle are 3 and 4. Its perimeter is ______________ .

(57) The surface area of a cube with edges of 2 inches is ______________ square inches.

(58) If A = 3, B = 4 and C = 8 then $AB^2 \div C$ = ______________ .

(59) 24 x 25 + 28 x 50 = ______________ .

*(60) 29 x 131 x 39 = ______________ .

(61) The multiplicative inverse of $\frac{3}{2}$ is ______________ .

(62) 5^3 = ______________ .

(63) $3\frac{1}{3} \times 6\frac{1}{3}$ = ______________ (Mixed Number).

(64) 13_5 = ______________ $_{10}$.

(65) .121212... = ______________ (Fraction).

(66) The smallest prime number greater than 19 is ________ .

(67) 28 x 32 = ______________ .

(68) A die is tossed. What is the probability that an even number comes up? ______________ .

(69) 97 x 96 = ______________.

*(70) $(2 + 4 + 6 + 8 + 12)^2 - 13$ = ______________ .

(71) 111 x 12 = ______________ .

(72) 1111_2 = ______________ $_4$.

(73) 102 x 104 = ______________ .

(74) Divide 32 into 2 parts such that the larger number exceeds the smaller number by 8. Find the smaller number.

______________ .

(75) $33^2 - 30^2$ = ______________ .

(76) 25641 ÷ 111 = ______________ .

(77) If x = –3 then $-x^2 - 3x$ = ______________ .

(78) Find the number of proper fractions in lowest terms with a denominator of 12. ______________ .

(79) $23^2 + 28^2$ = ______________ .

*(80) $(14)^3$ = ______________ .

The University Interscholastic League
Number Sense Test, Series J9561

	Score	Initials
Final	______	____
2nd	______	____
1st	______	____

Contestant's Number ________________

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Stop - Wait for Signal!

(1) $95 + 138 =$ ________________ .

(2) $.5 \times 1.7 =$ ________________ .

(3) $87 - 49 =$ ________________ .

(4) $532 - 235 =$ ________________ .

(5) $\frac{7}{12} + \frac{1}{3} =$ ________________ .

(6) $2472 \div 8 =$ ________________ .

(7) Which is smaller, $\frac{4}{9}$ or $\frac{3}{11}$? ________________ .

(8) $11 \times 37 =$ ________________ .

(9) $42 \div \frac{3}{7} =$ ________________ .

*(10) $1492 + 688 + 71 + 9 =$ ________________ .

(11) $50 \times 23 =$ ________________ .

(12) $13 \times 15 + 15 \times 27 =$ ________________ .

(13) $18 \times 6 \div 12 - 1 =$ ________________ .

(14) $3 + 8 + 13 + 18 + 23 + 28 =$ ________________ .

(15) $1234 \times 8 + 4 =$ ________________ .

(16) Reduce $\frac{15}{35}$ to lowest terms. ________________ .

(17) XC = ________________ (Arabic Numeral).

(18) $18^2 =$ ________________ .

(19) $3456 + 6543 =$ ________________ .

*(20) $209 \times 169 - 1200 =$ ________________ .

(21) $\frac{7}{8} \times 12 =$ ________________ (Mixed Number).

(22) $12 \times 21 =$ ________________ .

(23) The average of 18, 24 and 27 is ________________ .

(24) 1 hour 10 seconds = ________________ seconds.

(25) $8^2 + 16^2 =$ ________________ .

(26) $3\frac{1}{4} \times 36 =$ ________________ .

(27) 72 ounces = ________________ quarts.

(28) $214 \div 9 =$ ________________ (Mixed Number).

(29) The remainder when 1651 is divided by 4 is ________ .

*(30) $41400 \div 229 =$ ________________ .

(31) $1\frac{3}{4} + 1\frac{5}{8} =$ ________________ (Mixed Number).

(32) The GCF of 28 and 42 is ________________ .

(33) 8% of 132 is ________________ .

(34) $\sqrt{289}$ = ______________________ .

(35) $\frac{3}{8}$ = ______________________ (decimal).

(36) 24 is what percent of 80? ______________ %.

(37) The LCM of 12 and 20 is ______________ .

(38) The negative cube root of 64 is ______________ .

(39) If $\frac{5}{2x} = \frac{7}{2}$ then x = ______________ .

*(40) $2\frac{1}{4}$ x 49824 ÷ 9 = ______________ .

(41) The area of a rectangle is 4 times its length. Its width is ______________ units.

(42) A hexagon has ______________ sides.

(43) If x = –3 then $x^2 - 2x$ = ______________ .

(44) How many even numbers are greater than 3 and less than 15? ______________ .

(45) 3 (–9) – 4 (–6) = ______________ .

(46) The additive inverse of $\frac{3}{4}$ is ______________ .

(47) The sum of the positive integer divisors of 8 is ______ .

(48) 26 x 34 = ______________ .

(49) (16 x 7 + 9) ÷ 4 has a remainder of ______________ .

*(50) 24 x 32 + 48 x 9 = ______________ .

(51) If two angles are supplementary then their sum equals ______________ degrees.

(52) 6^3 = ______________ .

(53) The legs of a right triangle are 5 and 12. Its area is ______ square units.

(54) The next term of 4,6,9,13,18,... is ______________ .

(55) A circle has a diameter of 12 inches. Its area is kπ square inches and k = ______________ .

(56) 34_6 = ______________ $_{10}$.

(57) If 2x + 7 < x – 9 then x < ______________ .

(58) If $x^2 + 7^2 = 25^2$ then x^2 = ______________ .

(59) If 4 apples cost 89 cents then 16 apples cost $ ______ .

*(60) $\sqrt{34200}$ = ______________ .

(61) .212121... = ______________ (fraction).

(62) 13 minus 20% of 10% of 150 is ______________ .

(63) $2\frac{1}{4}$ x $6\frac{1}{4}$ = ______________ (Mixed Number).

(64) 101 x 15 = ______________ .

(65) Two dice are rolled. What is the probability that the sum is a 6? ______________ .

(66) 111 x 14 = ______________ .

(67) The number of positive integral factors of 12 is ______ .

(68) 93 x 98 = ______________ .

(69) $42^2 - 38^2$ = ______________ .

*(70) 13^3 = ______________ .

(71) 103 x 109 = ______________ .

(72) Divide 43 into 2 parts such that the larger number exceeds the smaller number by 9. Find the larger number.

______________ .

(73) 63603 ÷ 111 = ______________ .

(74) 3211 ÷ 9 = ______________ (Mixed Number).

(75) The distance between the points (2,3) and (6,3) is ____ .

(76) 1001_2 = ______________ $_4$.

(77) Find the number of proper fractions in lowest terms with a denominator of 16. ______________ .

(78) 1 + 3 + 5 + ... + 13 = ______________ .

(79) $42^2 + 16^2$ = ______________ .

*(80) 39 x 40 x 41 = ______________ .

University Interscholastic League Number Sense Answer Key

(Jr. High School) Series J956-SAC

*Number) x - y means an integer between x and y inclusive.

1)	41	21)	75	*40)	513 - 565	61)	1591
2)	84	22)	416	41)	23 2/3	62)	2
3)	206	23)	-7	42)	80	63)	4/33
4)	5/11	24)	60	43)	9/4 or 2.25 or 2 1/4	64)	81
5)	3500	25)	1 5/8	44)	5.6	65)	41
6)	6.2	26)	3	45)	1	66)	9603
7)	297	27)	25.2	46)	3	67)	4
8)	1 1/8	28)	72	47)	14 1/16	68)	1313
9)	253	29)	.375 not 0.375	48)	25	69)	1332
*10)	3420 - 3780	*30)	131 - 143	49)	30	*70)	2088 - 2306
11)	360	31)	216	*50)	143 - 157	71)	37
12)	24	32)	12	51)	10	72)	1/12
13)	42	33)	$2.67	52)	180	73)	134 5/9
14)	1950	34)	11	53)	8	74)	10920
15)	289	35)	144	54)	180	75)	16
16)	6/7	36)	-17	55)	400	76)	156
17)	21	37)	17/50	56)	8100	77)	0
18)	2025	38)	2	57)	2	78)	323
19)	510	39)	-7	58)	32	79)	1010
*20)	34,466 38,092			59)	2	*80)	3892 - 4300
				*60)	130,340 - 144,058		

NOTE: If an answer is of the type like 2/3 it cannot be written as .666... or $.\overline{6}$

University Interscholastic League Number Sense Answer Key

(Jr. High School) Series J956C

*Number) x - y means an integer between x and y inclusive.

1)	44	17)	225	34)	12	57)	24
2)	22	18)	25	35)	2 3/8	58)	6
3)	4.2	19)	10 1/2	36)	40	59)	2000
4)	208	*20)	22,497 - 24,865	37)	4	*60)	140,753 - 155,569
5)	99	21)	247	38)	34	61)	2/3
6)	3/8	22)	55	39)	25	62)	125
7)	16.4 not 16.40	23)	180	*40)	646 - 714	63)	21 1/9
8)	3/7	24)	420	41)	- 4	64)	8
9)	154	25)	20	42)	.125 not 0.125	65)	4/33
*10)	4320 - 4774	26)	192	43)	12	66)	23
11)	987	27)	2	44)	30	67)	896
12)	55	28)	80	45)	6	68)	1/2 or .5 not 0.5
13)	400	29)	34 5/9	46)	8	69)	9312
14)	1110	*30)	1024 - 1131	47)	1.2 or 6/5 or 1 1/5	*70)	961 - 1061
15)	4/7	31)	8.4 or 8 2/5	48)	3	71)	1332
16)	10	32)	3.5 or 3 1/2 or 7/2	49)	10,000	72)	33
		33)	7	*50)	1380 - 1525	73)	10608
				51)	4	74)	12
				52)	14	75)	189
				53)	800	76)	231
				54)	90	77)	0
				55)	4	78)	4
				56)	12	79)	1313
						*80)	2607 - 2881

NOTE: If an answer is of the type like 2/3 it cannot be written as .666... or $.\overline{6}$

University Interscholastic League Number Sense Answer Key

(Jr. High School) Series J9561

*Number) x - y means an integer between x and y inclusive.

1)	233	18)	324	34)	17	57)	-16
2)	.85 not 0.85	19)	9999	35)	.375 not 0.375	58)	576
3)	38	*20)	32,415 - 35,827	36)	30	59)	$3.56
4)	297	21)	10 1/2	37)	60	*60)	176 - 194
5)	11/12	22)	252	38)	-4	61)	7/33
6)	309	23)	23	39)	5/7	62)	10
7)	3/11	24)	3610	*40)	11,834 - 13,078	63)	14 1/16
8)	407	25)	320	41)	4	64)	1515
9)	98	26)	117	42)	6	65)	5/36
*10)	2147 - 2373	27)	9/4 or 2.25 or 2 1/4	43)	15	66)	1554
11)	1150	28)	23 7/9	44)	6	67)	6
12)	600	29)	3	45)	-3	68)	9114
13)	8	*30)	172 - 189	46)	-3/4	69)	320
14)	93	31)	3 3/8	47)	15	*70)	2088 - 2306
15)	9876	32)	14	48)	884	71)	11227
16)	3/7	33)	10.56	49)	1	72)	26
17)	90			*50)	1140 - 1260	73)	573
				51)	180	74)	356 7/9
				52)	216	75)	4
				53)	30	76)	21
				54)	24	77)	8
				55)	36	78)	49
				56)	22	79)	2020
						*80)	60,762 - 67,158

NOTE: If an answer is of the type like 2/3 it cannot be written as .666... or $.\overline{6}$

Problem Sequencing
Elementary Number Sense Test

Problem 1 - 20:*

1) Addition, Subtraction, Multiplication, & Division of Whole Numbers
2) Recognizing Place Value
3) Rounding Off Whole Numbers
4) Multiplication Short-Cuts
5) Remainder Type Problems
6) Even & Odd Number Type Problems
7) Expanded Notation
8) Sums of Whole Numbers
9) Roman Numerals/Arabic Numerals

Problems 21 - 40:

1) Addition/Subtraction of Fractions with Common Denominators
2) Addition, Subtraction, Multiplication, & Division of Decimal Fractions
3) Comparing Decimal Fractions & Common Fractions
4) Conversion Problems (either way):
   Fraction/Decimal, Percent/Fraction, Percent/Decimal
5) Order of Operations
6) More Multiplication Short-Cuts
7) Ratio/Proportion
8) Consumer Type Problems
9) Problems About Prime Numbers
10) Greatest Common Factor(GCF) & Least Common Multiple(LCM)
11) Conversion Problems (either way):
    Length, Weight, Volume

Problems 41 - 60:

1) Addition, Subtraction, Multiplication & Division of Fractions and Mixed Numbers
2) Substitution Problems
3) Perimeter/Area of:
   Square, Rectangle, Triangle
4) Radius/Diameter of a Circle
5) Powers & Roots of Numbers
6) Solving Simple Equations
7) Sequences
8) Sets
9) Word Problems
10) Volume of Cube/Rectangular Box
11) Right Triangle Problems
12) More Multiplication Short-Cuts
13) Base Systems

Problems 61 - 80:

1) Addition, Subtraction, Multiplication & Division of Integers
2) Inverses
3) Basic Geometry Facts
4) More Area Problems
5) Squaring 2-Digit Numbers
6) More Multiplication Short-Cuts
7) Powers of Numbers
8) More Consumer Type Problems
9) Inequalities
10) Probability
11) More Area Problems:
    Parallelogram, Rhombus, Trapezoid
12) Coordinate Geometry - Number Line
13) More Percent Type Problems

* A type of problem from a particular section could appear later in the test. Example: A GCF problem could appear as problem #43, but not any earlier than problem #21.

Problem Sequencing
Junior High School Number Sense Test

Problems 1 - 20:*

1) Addition, Subtraction, Multiplication & Division of Whole Numbers, Fractions, and Decimals
2) Order of Operations
3) Use of the Distributive Property
4) Comparison of Fractions & Decimals
5) Multiplication Short-Cuts
6) Squaring Numbers
7) Roman Numerals/Arabic Numerals
8) Mean, Median, Mode
9) Sums of Whole Numbers

Problems 21 - 40:

1) Addition, Subtraction, Multiplication & Division of Mixed Numbers and Integers
2) More Multiplication Short-Cuts
3) Percent Problems
4) Conversion Problems (either way):
   English/Metric, Length, Area, Capacity, Time
5) Consumer Type Problems
6) Substitution Problems
7) Solving Simple Equations
8) Square Roots/Cube Roots
9) Greatest Common Factor(GCF) & Least Common Multiple(LCM)
10) Number Theory - Prime Numbers and Divisors
11) Perimeter/Area of:
    Square, Rectangle, Circle
12) Ratio/Proportion
13) Inverses

Problems 41 - 60:

1) Sets
2) Word Problems
3) Pythagorean Theorem
4) Sequences
5) Volume/Surface Area of Rectangular Solid/Cube
6) Base Systems
7) Area of:
   Parallelogram, Rhombus, Trapezoid, Triangle
8) Solving Inequalities
9) Basic Geometry Facts
10) Remainder Problems

Problems 61 - 80:

1) Repeating Decimals
2) More Number Theory
3) Powers of Numbers
4) Volume of:
   Circular Cylinder, Cone, Sphere
5) Sequences & Series
6) Multiplication of 101, 111
7) Factorial
8) Coordinate Geometry
9) Probability
10) More Percent Type Problems
11) More Remainder Type Problems
12) More Multiplication Short-Cuts

* A type of problem from a particular section could appear later in the test. Example: A GCF problem could appear as problem #43, but not any earlier than problem #21.