



UNIVERSITY INTERSCHOLASTIC LEAGUE

Science

State • 2022



GENERAL DIRECTIONS:

- **DO NOT OPEN EXAM UNTIL TOLD TO DO SO.**
- Contestants may take up to two hours to complete the contest. If you are in the process of actually writing an answer when the signal to stop is given, you may finish writing that answer.
- Papers may not be turned in until 30 minutes have elapsed. If you finish the test in less than 30 minutes, remain at your seat and retain your paper until told to do otherwise. You may use this time to check your answers.
- All answers must be written on the answer sheet provided. Indicate your answers in the appropriate blanks provided on the answer sheet. Write clearly and legibly!
- You may place as many notations as you desire anywhere on the test paper but not on the answer sheet, which is reserved for answers only.
- You may use additional scratch paper provided by the contest director.
- All questions have ONE and only ONE correct (BEST) answer. There is a penalty for all incorrect answers.
- If a question is omitted, no points are given or subtracted.
- The back two pages of this test include a copy of the periodic table of the elements, as well as listings of other scientific relationships. You may use this information during the contest and may detach the back page from the test if you wish.
- A simple scientific calculator is sufficient for the high school Science contest. **The UIL provides a list of approved calculators that meet the criteria for use in the Science contest. No other calculators are permitted during the contest.** The Science Contest Approved Calculator List is available in the current Science Contest Handbook and on the UIL website. Contest directors will perform a brief visual inspection to confirm that all contestants are using only approved calculators. Each contestant may use up to two approved calculators during the contest.

- B01. Biological membranes, particularly the plasma membrane, are asymmetrical in both structure and function. Membrane-involved physiological responses and some cellular activities are localized to dense areas of the membrane containing sphingolipids, cholesterol, and protein receptors. These membrane microdomains are called
- exoplasmic leaflets.
 - flippases.
 - lipid rafts.
 - scramblases.
 - floppases.

- B02. Examine the image of a protein motif. Which of the following statements is correct?

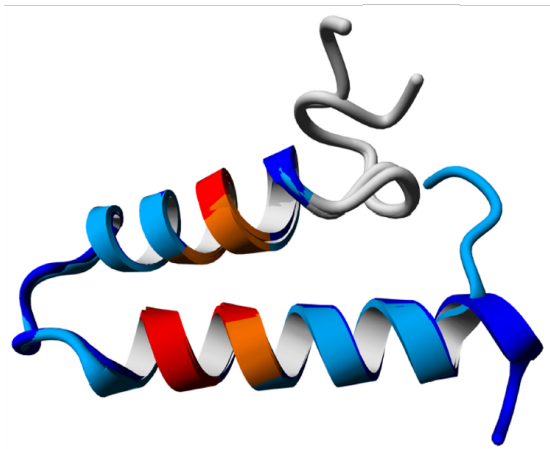
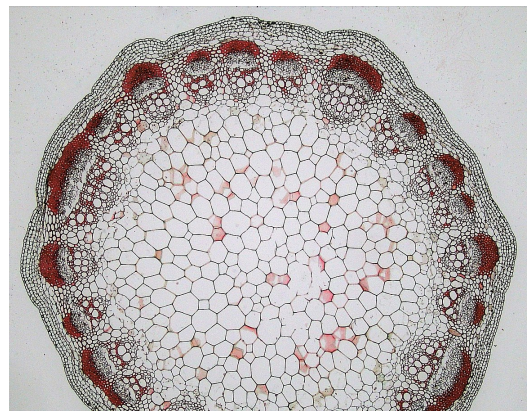


Image by: Johannes van den Boom, et al. (2016) PLoS ONE 11(3):e0151431

- This is a helix-turn-helix motif.
 - There are two α -helices in this image.
 - This is a ribbon model demonstrating secondary structure of protein.
 - This protein likely binds to DNA.
 - All of the above statements are correct.
- B03. Malaria is to _____ as _____ is to *Naegleria fowleri*.
- Plasmodium falciparum*; primary amoebic meningoencephalitis
 - mosquito; tick
 - Plasmodium falciparum*; amoebic dysentery
 - mosquito; snails
 - mosquito; hookworm disease

- B04. Dideoxynucleotide triphosphates (ddNTPs) are similar to deoxynucleotide triphosphates (dNTPs) except that ddNTPs are missing the 3'-OH. Addition of ddATP, along with dTTP, dCTP, and dGTP, to a PCR reaction would
- cause point mutations from ddATP being a base analog of dATP.
 - have no effect on the results of the PCR process.
 - duplicate all nucleic acid more quickly.
 - cause chain termination of the new strand wherever there was a thymine in the template strand.
 - lengthen the sizes of desired PCR products.

- B05. The following image is a cross-section of a vascular plant's stem.



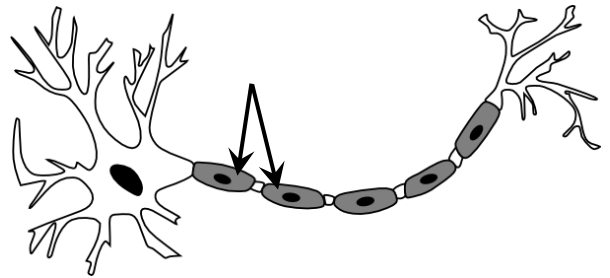
What type of plant would contain this type of arrangement of vascular tissue?

- monocots
 - grasses
 - mosses
 - eudicots
 - gymnosperms
- B06. The technique that uses a labeled DNA or RNA probe to detect the presence of a gene in an organism is called
- Northern blotting.
 - Western blotting.
 - Southern blotting.
 - Polymerase Chain Reaction.
 - DNA fingerprinting.

- B07. In human blood typing, the alleles for type A glycoproteins (I_A) and type B glycoproteins (I_B) are codominant to each other but each is dominant over the O (i) allele. These alleles determine the type of glycoproteins that can be made by the individual. Another gene, designed as the H allele, is responsible for whether or not the glycoproteins are attached (H) or not attached (hh) to red blood cells. In the following cross, what percent of the progeny will have type O blood.

$$I_A i H h \times I_B i h h$$

- A) 0%
B) 25%
C) 50%
D) 75%
E) 100%
- B08. Which of the following statements about ribosomes is not correct?
A) Cytosolic ribosomes synthesize proteins designated for secretion from the cell.
B) Mitochondria have 70S ribosomes.
C) Ribosomes have three binding sites for tRNAs—E site, P site, and A-site.
D) Bacterial ribosomes contain a 50S large subunit and a 30S small subunit.
E) Antibiotics that target bacterial ribosomes impact the molecular process of translation.
- B09. An organism that gets its energy from the oxidation of inorganic chemicals and its carbon from carbon dioxide fixation would be called
A) chemoheterotrophs.
B) chemolithoautotrophs.
C) photoheterotroph.
D) photoautotroph.
E) chemoorganoautotroph.
- B10. Amoeba, Entamoeba, and some slime molds belong to Supergroup
A) Excavata.
B) Opisthokonta.
C) Archaeplastida.
D) Amoebozoa.
E) Chromalveolata.
- B11. The Centers for Disease Control and Prevention issued a Food Safety Alert in March 2022 for processed cantaloupe from Liberty Fruit Company. Which organism was identified as the culprit?
A) *Escherichia coli*
B) *Listeria* sp.
C) *Salmonella* sp.
D) *Vibrio* sp.
E) *Campylobacter* sp.
- B12. Which of the following cell structures is not common to both plant and animal cells?
A) nucleus
B) mitochondria
C) Golgi apparatus
D) rough endoplasmic reticulum
E) plasmodesmata
- B13. Examine this image of a neuron. The cell structures identified by the arrows function in



- A) release of neurotransmitters, such as acetylcholine.
B) more quick and efficient transmission of electrical impulses.
C) receiving synaptic inputs from axons of other neurons.
D) the daily “housekeeping” processes of the neuron, such as metabolism and protein synthesis.
E) slowing down the speed of an electrical impulse.
- B14. The structures in the respiratory system where gas exchange occurs are called the
A) alveoli.
B) bronchioles.
C) bronchi.
D) trachea.
E) vesicles.

- B15. Limpkins are non-migratory birds that are native to South and Central America, and Florida but are becoming more widespread in the Southern United States, including a few at Brazos Bend State Park in Texas. Limpkins love consuming Apple snails, which are invasive but present in considerable numbers in Southeast Texas. Apple snails aggressively consume large amounts of native vegetation and agricultural crops, such as rice, and destroy the banks of wetland areas by burrowing into the soil. They are also known vectors of a deadly parasite called rat lung worms, which can be transmitted to humans and other mammals after consumption of raw or undercooked snails. Which of the following scenarios would be the most likely given the information provided in this question?
- A) Limpkins are following their food source and therefore expanding their range. Permanent populations will likely become established in Southeast Texas.
 - B) The incidence of rat lung worm disease will increase as the number of Apple snails increase in the area due to humans swimming in the same environment as the snails.
 - C) The use of indiscriminate pesticides against the snails would have no impact on native snail populations or the ecosystem.
 - D) The native vegetation is able to recover with no major loss due to the presence of invasive Apple snails.
- B16. The ability to interbreed in nature and produce viable, fertile offspring is the foundation of the
- A) phenotypic species concept.
 - B) phylogenetic species concept.
 - C) ecological species concept.
 - D) morphological species concept.
 - E) biological species concept.
- B17. Examine the answer choices. For a eukaryotic cell undergoing meiosis, relative to the other phases listed, when would you expect to first find haploid cells?
- A) Telophase/Cytokinesis II
 - B) Prophase I
 - C) Metaphase I
 - D) Telophase/Cytokinesis I
 - E) Metaphase II
- B18. The rate limiting step of the citric acid cycle is the reaction catalyzed by isocitrate dehydrogenase, which performs the oxidative decarboxylation of isocitrate into α -ketoglutarate and CO_2 . Cellular phosphorylation of a serine residue at position 113 (Ser113) inhibits the enzymatic activity of isocitrate dehydrogenase. If a researcher was able to mutagenize the gene sequence of isocitrate dehydrogenase to change the amino acid at position 113 from serine (polar) to leucine (nonpolar), what would be the expected outcome?
- A) No effect would be observed on the citric acid cycle rate because the mutagenized isocitrate dehydrogenase contains the same number of amino acid residues as the wild-type.
 - B) Isocitrate dehydrogenase with Leu113 would never become phosphorylated and the citric acid cycle would constantly be running.
 - C) Cellular phosphorylation would likely still occur at the Leu113 residue in the mutated isocitrate dehydrogenase and there would be no effect on the rate of the citric acid cycle.
 - D) Phosphorylation would not occur because the amino acid residues have completely different properties, and this would completely shut down the citric acid cycle.
 - E) Phosphorylation of Leu113 would not occur and the citric acid cycle would not continue.
- B19. Microevolution is driven by
- A) natural selection.
 - B) genetic drift.
 - C) gene flow.
 - D) genetic variation.
 - E) all of the above.
- B20. In a population at Hardy-Weinberg equilibrium, 364 out of 592 individuals in the population exhibit the recessive phenotype. What percent of the population are heterozygotes?
- A) 4.7%
 - B) 21.6%
 - C) 33.9%
 - D) 61.4%
 - E) 78.4%

- C01. What is the mass percent oxygen in the fictional compound sodium überbromate, NaBrO_5 ?
- A) 34.99%
B) 38.35%
C) 43.74%
D) 47.94%
E) 52.49%
- C02. A 10.0 L container holds a mixture of H_2 and C_2H_4 gas at 25° and has a total pressure of 5.66 atm. If the mole fraction of H_2 in the container is 0.675, how many grams of oxygen would be required to completely combust the gas in the container?
- A) 79.0 g
B) 84.3 g
C) 89.9 g
D) 97.2 g
E) 105 g
- C03. In 1983 the meter was defined as the distance light travels in a vacuum in $1/299,792,458$ second. Before that, in 1960 the meter was defined as 1650763.73 wavelengths of the orange-red emission line of the krypton-86 atom in a vacuum. What is the energy of one mole of photons of that orange-red light?
- A) 198 kJ
B) 122 kJ
C) 215 kJ
D) 302 kJ
E) 166 kJ
- C04. You add 300.0 grams of dry ice to an open 100.0-liter, rigid-walled container at 1.00 atm pressure and 25.0°C and then close the container. The dry ice sublimates to gas and the container returns to its initial temperature. What is the final pressure inside the container?
- A) 3.33 atm
B) 2.67 atm
C) 2.33 atm
D) 1.67 atm
E) 1.33 atm
- C05. What bond angles are present around the central atom in a xenon tetrafluoride molecule?
- A) 109.5°
B) 180°
C) 120°
D) 90° , 120° , and 180°
E) 90° and 180°
- C06. Containers A and B are at the same pressure. Which image below best represents Container B?
-
- A) 1.0 L B) 2.0 L C) 3.0 L
- D) 4.0 L E) 5.0 L
- C07. You need to make up 10.0 L of water at 65°C , but all you have available is water at 88°C and ice at 0°C . What mass of each should you use to end up with 10.0 L of water at 65°C ? Assume the density of water is 1.000 g/mL. (You don't need the density of ice to solve this.)

Answer	Mass of water (g)	Mass of ice (g)
A	8030	1970
B	8330	1670
C	8630	1370
D	8930	1070
E	9230	770

- C08. What are the strongest intermolecular forces present in a sample of liquid SF₄?

- A) dispersion
B) dipole-dipole
C) S-F bonding
D) Hydrogen bonding
E) Covalent bonds

- C09. Given the following thermodynamic data,

Reaction	ΔH° (kJ)
$\text{A}_2\text{X}(\text{g}) + 2 \text{B}_2(\text{g}) \rightarrow \text{A}_2\text{XB}_4(\ell)$	-235.9
$\text{XB}_3(\text{g}) + \text{A}_2\text{B}(\text{g}) \rightarrow \text{A}_2\text{XB}_4(\ell)$	-67.5
$\text{XB}_3(\text{g}) + \text{A}_2\text{B}(\ell) \rightarrow \text{A}_2\text{X}(\text{g}) + 2 \text{B}_2(\text{g})$	225

$$S^\circ \text{ A}_2\text{B} (\ell) = 75.5 \text{ J/mol}\cdot\text{K}$$

$$S^\circ \text{A}_2\text{B}(\text{g}) = 239.8 \text{ J/mol}\cdot\text{K}$$

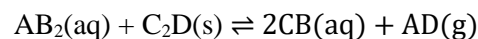
What is the normal boiling point of A_2B ?

- A) 71.5 °C
B) 84.4 °C
C) 96.3 °C
D) 114 °C
E) 121 °C

- C10. Our current periodic table ends with element 118. The periodic table shown below goes 92 elements further, to element 210. Which letter in the table indicates the position of element 165?

A 10x10 grid with letters D and E. D is at row 6, column 3. E is at row 3, column 9.

- C11. You are the production manager in a chemical plant that produces AD gas. The exothermic reaction you are using to produce the gas is



Your yield of AD gas is the amount of AD produced by the reaction when the reaction reaches equilibrium. Given the pricing information below, which of the answer choices would be the cheapest way to improve your yield of AD gas at equilibrium from this reaction?

Resource	Cost
AB ₂ (aq)	\$3/M
C ₂ D(s)	\$2/kg
CB(aq)	\$1.50/M
AD(g)	\$5/kg
heat	\$0.50/kJ

- A) Increase the concentration of AB_2 in the reaction chamber by 2M
- B) Add 2 kg of C_2D to the reaction chamber
- C) Increase the concentration of CB in the reaction chamber by 2M
- D) Add 1 kg of AD to the reaction chamber
- E) Add 10 kJ of heat energy to the reaction chamber

- C12. What is the pH of a 0.0050 M solution of hydrofluoric acid? $K_a = 6.2 \times 10^{-4}$.

- A) 2.25
B) 2.30
C) 2.75
D) 2.83
E) 3.49

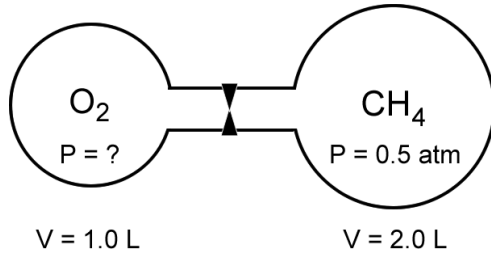
- C13. If you mix 500 mL of 0.25 M $\text{Ba}(\text{NO}_3)_2$ with 500 mL of 0.50 M Na_2MoO_4 , what will the equilibrium concentration of Ba^{2+} be? $K_{\text{sp}} = 3.5 \times 10^{-8}$

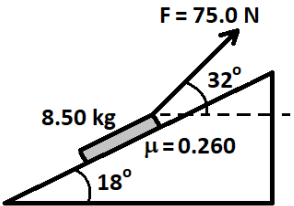
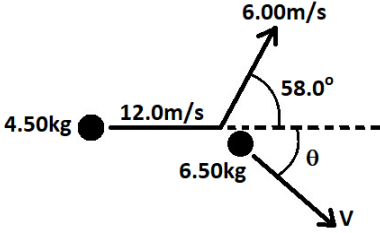
- A) 0.00 M
B) 3.7×10^{-6} M
C) 2.8×10^{-7} M
D) 1.9×10^{-4} M
E) 0.125 M
F) 0.25 M

- C14. The initial rate of the reaction $A + B \rightarrow 2Y + Z$ was determined for different initial conditions, with the results listed in the following table:

Run #	$[A]_0, M$	$[B]_0, M$	Initial rate, M/s
1	0.185	0.133	3.37×10^{-4}
2	0.185	0.266	1.35×10^{-3}
3	0.370	0.133	6.74×10^{-4}
4	0.200	0.275	?

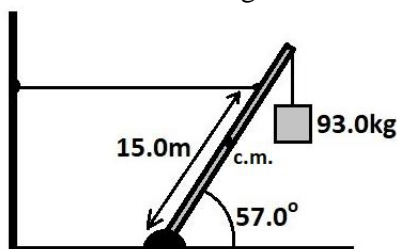
Determine the initial rate of reaction (in M/s) for the fourth run in the above experiment.

- A) 8.14×10^{-4}
 B) 9.72×10^{-4}
 C) 1.49×10^{-3}
 D) 2.02×10^{-3}
 E) 1.56×10^{-3}
- C15. Compound A has a vapor pressure of 18 torr and compound B has a vapor pressure of 36 torr. A solution made up of 170 mL of A and 315 mL of B has a vapor pressure of 30.0 torr. The density of A is 0.925 g/mL and the density of B is 0.815 g/mL. Compound A has a molar mass of 95.0 g/mol. What is the molar mass of Compound B?
- A) 105.6 g/mol
 B) 85.7 g/mol
 C) 82.3 g/mol
 D) 77.5 g/mol
 E) 57.0 g/mol
- C16. If you have 1.0 L of 1.0 M solutions of each of the following ions and you were to pass a 1.5 amp current through each solution for 3.0 hours, which one would result in the greatest amount of solid metal reducing out of the solution?
- A) Cu^{2+}
 B) Sn^{2+}
 C) Au^{3+}
 D) Fe^{3+}
 E) Sn^{4+}
 F) Pb^{4+}
- C17. Two gas bulbs at 400 K are connected by a closed valve. One bulb has a volume of 2.0 L and contains methane gas at a pressure of 0.5 atm. The other bulb has a volume of 1.0 L and contains O_2 gas at an unknown pressure. The valve is opened and a combustion reaction occurs which completely consumes the CH_4 . If the final pressure in the two-bulb system is 1.25 atm, what was the original pressure of the O_2 in the 1.0 L bulb?
- 
- A) 1.00 atm
 B) 2.00 atm
 C) 2.25 atm
 D) 2.50 atm
 E) 2.75 atm
- C18. What is the deBroglie wavelength of a helium atom at SATP?
- A) 0.0732 nm
 B) 0.0400 nm
 C) 1.12 nm
 D) 13.6 nm
 E) 0.512 nm
- C19. What is the mole fraction of water in 500 mL of a 2.70 M solution of $CaCl_2$? The density of the solution is 1.22 g/mL.
- A) 1.05
 B) 0.950
 C) 0.905
 D) 0.863
 E) 0.814
- C20. Metal ion M^+ is added to a solution that contains 0.0050 M X^- and 0.0025 M Z^- . MX has a K_{sp} of 8.5×10^{-9} , and MZ has a K_{sp} of 1.7×10^{-10} . What percentage of the Z^- is still in solution (not precipitated) when the MX just begins to precipitate out?
- A) 5% B) 4% C) 3% D) 2% E) 1%

- P01. According to Kaku, satellite photographs of the cosmic microwave background show tiny, minuscule ripples in the background radiation. These miniscule ripples are due to...
- the distribution of dark matter.
 - the distribution of dark energy.
 - the symmetry-breaking principle of gravitons.
 - the quantum uncertainty principle.
 - the Pauli exclusion principle.
- P02. According to Kaku, in 1997 physicist Juan Maldacena discovered that there is a duality (a mathematical equivalence) between a string theory in ten dimensions and _____ in four dimensions.
- the quantum uncertainty principle
 - Einstein's theory of general relativity
 - a supersymmetric Yang-Mills theory
 - a symmetric electroweak theory
 - a supersymmetric membrane theory
- P03. According to Kaku, the supersymmetric partner of the photon, the photino, is likely a weakly interacting massive particle (WIMP). This class of particles is a leading candidate for explaining...
- dark matter
 - dark energy
 - graviton interactions with matter
 - proton decay
 - Hawking radiation
- P04. During the formation of the solar system, protoplanets formed from the gas and dust in a disk surrounding the young Sun. The inner planets did not grow as large, or as fast, as the gas giants because the areas in which the inner planets formed lacked an important group of solid raw materials. Which solid materials did the inner planets lack?
- metal oxides
 - silicate minerals
 - hydrated minerals (containing water)
 - sulfates
 - ices (water, ammonia, methane)
- P05. What is the result of this calculation to the correct number of significant digits?
- | | | |
|----|---------|--------------------------------|
| A) | 2.1 | |
| B) | 2.07 | $\frac{(8.94)^2 - 73.42}{\pi}$ |
| C) | 2.070 | |
| D) | 2.0702 | |
| E) | 2.07016 | |
- P06. Today, you are taking a ride in the basket of a hot air balloon. Starting from rest, the basket lifts off of the ground with a constant acceleration of 2.55 m/s^2 directly upward (vertical). Exactly 3.00 seconds after lifting off of the ground, you throw a ball out of the basket. You throw the ball horizontally with a velocity of 8.00 m/s (relative to you). How far from the launch point does the ball land on the ground? Assume the ground is level and that the launch point is directly beneath the basket. Also, ignore air resistance.
- 7.50 m
 - 12.2 m
 - 14.2 m
 - 20.0 m
 - 24.6 m
- P07. An 8.50 kg sled is being pulled up an inclined plane that is angled at 18.0° above the horizontal. The sled is pulled by a force of 75.0 N that is itself angled at 32.0° above the horizontal (as shown). The coefficient of friction between the incline and the sled is 0.260 . What is the acceleration of the sled?
- 
- 3.11 m/s^2
 - 3.66 m/s^2
 - 5.53 m/s^2
 - 6.28 m/s^2
 - 7.40 m/s^2
- P08. A ball with a mass of 4.50 kg was rolling East with a velocity of 12.0 m/s when it impacted a second ball. The second ball has a mass of 6.50 kg and was initially at rest. After the impact, the first ball is observed moving at 6.00 m/s at an angle of 58.0° North of East (as shown). What is the velocity of the second ball after the impact?
- 
- 5.27 m/s at an angle of 24.7° South of East
 - 5.27 m/s at an angle of 65.3° South of East
 - 7.05 m/s at an angle of 30.0° South of East
 - 7.05 m/s at an angle of 60.0° South of East
 - 10.2 m/s at an angle of 30.0° South of East
 - 10.2 m/s at an angle of 60.0° South of East

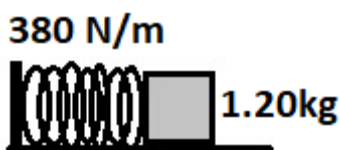
- P09. A 20.0m long, uniform steel beam with a mass of 60.0kg is used to hold a 93.0kg crate up off of the ground. The beam is angled at 57.0° above the horizontal, and the crate is hanging from a cable attached to the end of the beam. A second cable is attached to the beam at a point that is 15.0m up the length of the beam; this cable runs horizontally and is attached to a nearby wall. The diagram is shown below. The entire system is in static equilibrium; what is the magnitude of the net force on the base of the beam where it attaches to the ground?

- A) 1044 N
B) 1499 N
C) 1827 N
D) 2543 N
E) 3202 N



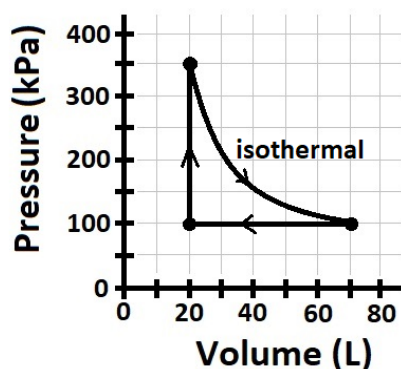
- P10. A 1.20kg mass is attached to a horizontal spring that has a spring constant of 380N/m. The mass is pulled out 20.0cm from equilibrium and released from rest. This starts the mass moving with simple harmonic motion. Because of friction between the mass and the floor, the amplitude of the harmonic motion slowly decreases. After one period, the mass slides out only 16.0cm from equilibrium. What is the coefficient of friction between the mass and the floor?

- A) 0.084
B) 0.11
C) 0.16
D) 0.23
E) 0.32



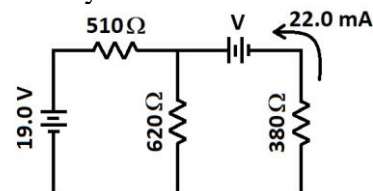
- P11. An engine follows the cycle illustrated in the PV-diagram below. If 1.50 moles of an ideal gas go through one complete engine cycle, how much work is done by the gas?

- A) 13770 J
B) 8770 J
C) 7000 J
D) 5000 J
E) 3770 J
F) 2000 J



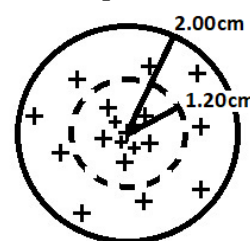
- P12. For the DC circuit shown: what is the voltage, V , of the unknown battery?

- A) 11.3 V
B) 15.2 V
C) 19.1 V
D) 22.0 V
E) 24.9 V



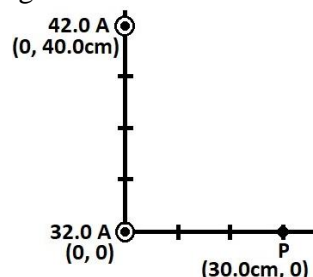
- P13. A sphere of charge has a radius of 2.00cm and a total charge of 1.608 nC. The charge density in the sphere varies radially according to the equation: $\rho(r) = (0.0300)(4 - r^2)$ where r is in centimeters and the charge density is in nC/cm³. What is the magnitude of the electric field at a distance of 1.20cm from the center of the sphere?

- A) 2.17×10^4 N/C
B) 3.61×10^4 N/C
C) 4.25×10^4 N/C
D) 6.02×10^4 N/C
E) 1.00×10^5 N/C



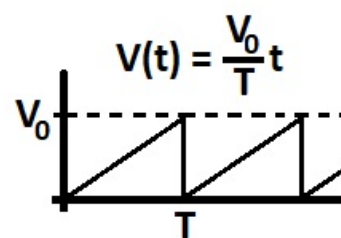
- P14. Two long, straight, wires carry currents in the +z-direction, as shown. The first wire carries a current of 42.0A, and passes through the x-y plane at the point (0, 40.0cm). The second wire carries a current of 32.0A, and passes through the x-y plane at the origin (0, 0). What is the magnitude of the magnetic field at the point P (30.0cm, 0) due to the two current carrying wires?

- A) 27.2 μ T
B) 34.2 μ T
C) 36.2 μ T
D) 38.1 μ T
E) 44.9 μ T

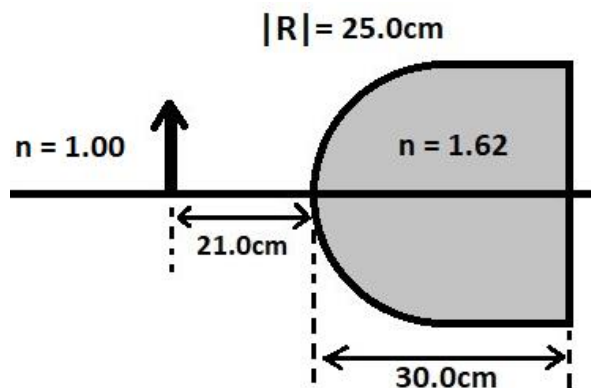


- P15. Determine the relationship between the amplitude voltage, V_0 , and the root-mean-square (rms) voltage, V_{rms} , for the sawtooth waveform shown.

- A) $V_{rms} = V_0$
B) $V_{rms} = \frac{V_0}{\sqrt{2}}$
C) $V_{rms} = \frac{V_0}{\sqrt{3}}$
D) $V_{rms} = \frac{V_0}{2}$
E) $V_{rms} = \frac{V_0}{\sqrt{5}}$



- P16. An object is placed 21.0cm to the left of a thick lens, as shown. The side of the lens nearest the object is convex and has a radius of curvature of 25.0cm. The far side of the lens is flat. The lens is 30.0cm thick and is made of a glass that has an index of refraction of 1.62. Where is the final image located?

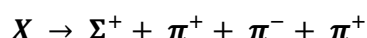


- A) 32.3 cm left of the curved surface
 B) 43.8 cm left of the curved surface
 C) 62.3 cm left of the curved surface
 D) 25.3 cm right of the flat surface
 E) 71.0 cm right of the flat surface
- P17. The normalized wavefunction for a quantum system is given by:

$$\Psi = \frac{\sqrt{3}}{x^2} \quad 1 \leq x < \infty$$

What is the expectation value of position, $\langle x \rangle$, for this quantum system?

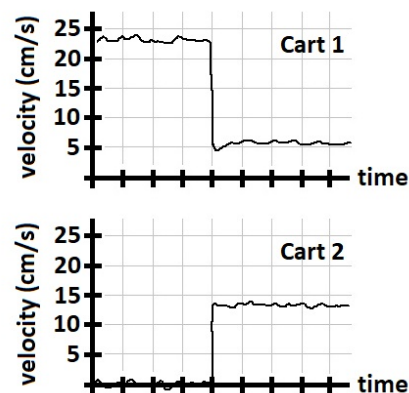
- A) $\langle x \rangle = 0.75$ units
 B) $\langle x \rangle = 0.866$ units
 C) $\langle x \rangle = 1.00$ units
 D) $\langle x \rangle = 1.50$ units
 E) $\langle x \rangle = 1.73$ units
- P18. An unknown subatomic particle has a lifetime of 6.0×10^{-12} seconds and a dominant decay mode shown below. What is the most likely quark structure of this unknown particle?



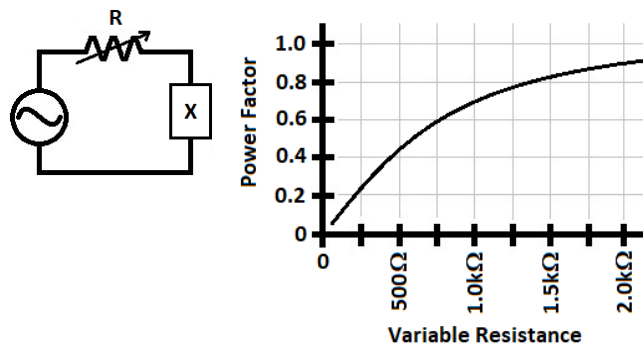
- A) uus
 B) uuc
 C) usc
 D) $u\bar{s}$
 E) $u\bar{c}$

- P19. Two carts are set up on a frictionless, horizontal track. The first cart is given a small initial velocity, and the second cart is initially at rest. The two carts collide head-on. Using a velocimeter, you measure the velocities of the carts before, during, and after the collision. The data is shown below. Based on this data, what is the ratio of the masses, $\frac{M_2}{M_1}$, of the two carts?

- A) 1.3
 B) 1.8
 C) 2.6
 D) 3.6
 E) 4.6



- P20. An AC circuit consists of a voltage source, a variable resistor, and an unknown reactance, all in series (as shown). As you change the variable resistance, you measure the power factor of the circuit. This data is plotted below. Based on this data, what is the approximate magnitude of the unknown reactance?



- A) 1800 Ω
 B) 1500 Ω
 C) 1300 Ω
 D) 1000 Ω
 E) 700 Ω

Chemistry																		1A 1	2A 2											3A 13	4A 14	5A 15	6A 16	7A 17	8A 18
1 H 1.01	2A 2																	3A 13	4A 14	5A 15	6A 16	7A 17	2 He 4.00												
3 Li 6.94	4 Be 9.01																5 B 10.81	6 C 12.01	7 N 14.01	8 O 16.00	9 F 19.00	10 Ne 20.18													
11 Na 22.99	12 Mg 24.31	3B 3	4B 4	5B 5	6B 6	7B 7	8B 8 9 10			1B 11	2B 12	13 Al 26.98	14 Si 28.09	15 P 30.97	16 S 32.07	17 Cl 35.45	18 Ar 39.95																		
19 K 39.10	20 Ca 40.08	21 Sc 44.96	22 Ti 47.87	23 V 50.94	24 Cr 52.00	25 Mn 54.94	26 Fe 55.85	27 Co 58.93	28 Ni 58.69	29 Cu 63.55	30 Zn 65.38	31 Ga 69.72	32 Ge 72.64	33 As 74.92	34 Se 78.96	35 Br 79.90	36 Kr 83.80																		
37 Rb 85.47	38 Sr 87.62	39 Y 88.91	40 Zr 91.22	41 Nb 92.91	42 Mo 95.94	43 Tc (98)	44 Ru 101.07	45 Rh 102.91	46 Pd 106.42	47 Ag 107.87	48 Cd 112.41	49 In 114.82	50 Sn 118.71	51 Sb 121.76	52 Te 127.60	53 I 126.90	54 Xe 131.29																		
55 Cs 132.91	56 Ba 137.33	57 La 138.9	72 Hf 178.49	73 Ta 180.95	74 W 183.84	75 Re 186.21	76 Os 190.23	77 Ir 192.22	78 Pt 195.08	79 Au 196.97	80 Hg 200.59	81 Tl 204.38	82 Pb 207.20	83 Bi 208.98	84 Po (209)	85 At (210)	86 Rn (222)																		
87 Fr (223)	88 Ra (226)	89 Ac (227)	104 Rf (261)	105 Db (262)	106 Sg (266)	107 Bh (264)	108 Hs (277)	109 Mt (268)	110 Ds (281)	111 Rg (281)	112 Cn (285)	113 Nh (286)	114 Fl (289)	115 Mc (289)	116 Lv (293)	117 Ts (293)	118 Og (294)																		

58	Ce	140.1	59	Pr	140.9	60	Nd	144.2	61	Pm	(145)	62	Sm	150.4	63	Eu	152.0	64	Gd	157.3	65	Tb	158.9	66	Dy	162.5	67	Ho	164.9	68	Er	167.3	69	Tm	168.9	70	Yb	173.0	71	Lu	175.0
90	Th	232.0	91	Pa	231.0	92	U	238.0	93	Np	(237)	94	Pu	(244)	95	Am	(243)	96	Cm	(247)	97	Bk	(247)	98	Cf	(251)	99	Es	(252)	100	Fm	(257)	101	Md	(258)	102	No	(259)	103	Lr	(262)

Water Data

T_{mp}	$= 0^{\circ}\text{C}$
T_{bp}	$= 100^{\circ}\text{C}$
c_{ice}	$= 2.09 \text{ J/g}\cdot\text{K}$
c_{water}	$= 4.184 \text{ J/g}\cdot\text{K}$
c_{steam}	$= 2.03 \text{ J/g}\cdot\text{K}$
ΔH_{fus}	$= 334 \text{ J/g}$
ΔH_{vap}	$= 2260 \text{ J/g}$
K_{f}	$= 1.86 ^{\circ}\text{C}/m$
K_{b}	$= 0.512 ^{\circ}\text{C}/m$

Definitions

STP	$= 0^{\circ}\text{C}$ and 1 atm
SATP	$= 25^{\circ}\text{C}$ and 1 atm

Electrochemical constants

1 amp	$= 1 \text{ Coulomb/sec}$
1 \mathcal{F}	$= 96,485 \text{ Coulombs}$
1 \mathcal{F}	$= 1 \text{ mole of electrons}$

Constants

R	$= 0.08206 \text{ L}\cdot\text{atm/mol}\cdot\text{K}$
R	$= 8.314 \text{ J/mol}\cdot\text{K}$
R	$= 62.36 \text{ L}\cdot\text{torr/mol}\cdot\text{K}$
e	$= 1.602 \times 10^{-19} \text{ C}$
N_{A}	$= 6.022 \times 10^{23} \text{ mol}^{-1}$
k	$= 1.38 \times 10^{-23} \text{ J/K}$
h	$= 6.626 \times 10^{-34} \text{ J}\cdot\text{s}$
c	$= 3.00 \times 10^8 \text{ m/s}$
R_{H}	$= 2.178 \times 10^{-18} \text{ J}$
m_{e}	$= 9.11 \times 10^{-31} \text{ kg}$

Other Data

All other necessary data for this exam is included in the problems themselves.

Physics

Useful Constants

quantity	symbol	value
Free-fall acceleration	g	9.80 m/s^2
Permittivity of Free Space	ϵ_0	$8.854 \times 10^{-12} \text{ C}^2/\text{Nm}^2$
Permeability of Free Space	μ_0	$4\pi \times 10^{-7} \text{ Tm/A}$
Coulomb constant	k	$8.99 \times 10^9 \text{ Nm}^2/\text{C}^2$
Speed of light in a vacuum	c	$3.00 \times 10^8 \text{ m/s}$
Fundamental charge	e	$1.602 \times 10^{-19} \text{ C}$
Planck's constant	h	$6.626 \times 10^{-34} \text{ Js}$
Electron mass	m_e	$9.11 \times 10^{-31} \text{ kg}$
Proton mass	m_p	$1.67265 \times 10^{-27} \text{ kg}$ 1.007276 amu
Neutron mass	m_n	$1.67495 \times 10^{-27} \text{ kg}$ 1.008665 amu
Atomic Mass Unit	amu	$1.66 \times 10^{-27} \text{ kg}$ $931.5 \text{ MeV}/c^2$
Gravitational constant	G	$6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$
Stefan-Boltzmann constant	σ	$5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$
Universal gas constant	R	$8.314 \text{ J/mol} \cdot \text{K}$ $0.082057 \text{ L} \cdot \text{atm/mol} \cdot \text{K}$
Boltzmann's constant	k_B	$1.38 \times 10^{-23} \text{ J/K}$
Speed of Sound (at 20°C)	v	343 m/s
Avogadro's number	N_A	$6.022 \times 10^{23} \text{ atoms/mol}$
Electron Volts	eV	$1.602 \times 10^{-19} \text{ J/eV}$
Distance Conversion	miles \rightarrow meters	$1.00 \text{ mile} = 1609 \text{ meters}$
Rydberg Constant	R_∞	$1.097 \times 10^7 \text{ m}^{-1}$
Standard Atmospheric Pressure	1 atm	$1.013 \times 10^5 \text{ Pa}$
Density of Pure Water	ρ_{water}	1000.0 kg/m^3

Science Contest Answer Sheet

Conference _____

Grade Level _____

Contestant # _____

Biology

B01 _____

B02 _____

B03 _____

B04 _____

B05 _____

B06 _____

B07 _____

B08 _____

B09 _____

B10 _____

B11 _____

B12 _____

B13 _____

B14 _____

B15 _____

B16 _____

B17 _____

B18 _____

B19 _____

B20 _____

Chemistry

C01 _____

C02 _____

C03 _____

C04 _____

C05 _____

C06 _____

C07 _____

C08 _____

C09 _____

C10 _____

C11 _____

C12 _____

C13 _____

C14 _____

C15 _____

C16 _____

C17 _____

C18 _____

C19 _____

C20 _____

Physics

P01 _____

P02 _____

P03 _____

P04 _____

P05 _____

P06 _____

P07 _____

P08 _____

P09 _____

P10 _____

P11 _____

P12 _____

P13 _____

P14 _____

P15 _____

P16 _____

P17 _____

P18 _____

P19 _____

P20 _____

B Score

C Score

P Score

Grader Initials _____

OVERALL SCORE

**UIL HIGH SCHOOL SCIENCE CONTEST
ANSWER KEY
2022 STATE**

Biology

B01. C
B02. E
B03. A
B04. D
B05. D
B06. C
B07. C
B08. A
B09. B
B10. D
B11. B
B12. E
B13. B
B14. A
B15. A
B16. E
B17. D
B18. B
B19. E
B20. C

Chemistry

C01. C
C02. D
C03. A
C04. B
C05. E
C06. D
C07. C
C08. B
C09. A
C10. B
C11. A
C12. D
C13. C
C14. E
C15. D
C16. C
C17. E
C18. A
C19. D
C20. B

Physics

P01. D
P02. C
P03. A
P04. E
P05. A
P06. D
P07. B
P08. C
P09. C
P10. E
P11. E
P12. E
P13. C
P14. B
P15. C
P16. A
P17. D
P18. B
P19. A
P20. D

CHEMISTRY SOLUTIONS – UIL STATE 2022

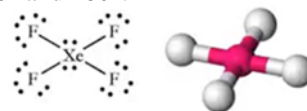
- C01. (C) Molar mass of $\text{NaBrO}_5 = 22.99 + 79.90 + 16.00 \times 5 = 182.89 \text{ g/mol}$. Mass of oxygen $= 5 \times 16.00 = 80.00$.
Mass percent $= (80.00/182.89) \times 100 = 43.74\%$
- C02. (D) Total moles of gas: $n = PV/RT = (5.66)(10.0)/(0.08206)(298) = 2.3146 \text{ moles}$.
Moles of $\text{H}_2 = 0.675 \times 2.3146 = 1.5623 \text{ mol H}_2$. Moles of $\text{C}_2\text{H}_4 = 2.3146 - 1.5623 = 0.7523 \text{ mol C}_2\text{H}_4$.
The combustion reactions are: $2 \text{H}_2 + \text{O}_2 \rightarrow 2 \text{H}_2\text{O}$ and $\text{C}_2\text{H}_4 + 3 \text{O}_2 \rightarrow 2 \text{CO}_2 + 2 \text{H}_2\text{O}$
 O_2 needed to combust the H_2 : $1.5623 \text{ mol H}_2 \times (1 \text{ mol O}_2/2 \text{ mol H}_2) = 0.7812 \text{ mol O}_2$
 O_2 needed to combust the C_2H_4 : $0.7523 \text{ mol C}_2\text{H}_4 \times (3 \text{ mol O}_2/1 \text{ mol C}_2\text{H}_4) = 2.257 \text{ mol O}_2$
Total moles O_2 needed $= 3.038 \text{ mol O}_2$. $3.038 \text{ mol O}_2 \times 32.00 \text{ g/mol} = 97.2 \text{ g}$.
- C03. (A) For one photon, $E = h\nu = E = hc/\lambda$. $1650763.73 \text{ wavelengths} = 1 \text{ meter}$, so each wavelength is $1/1650763.73 \text{ meter} = 6.0578 \times 10^{-7} \text{ m}$. $E = (6.626 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s})/(6.0578 \times 10^{-7} \text{ m}) = 3.2814 \times 10^{-19} \text{ J}$. To get the energy of one mole of photons, multiply this energy by Avogadro's number: $(3.2814 \times 10^{-19} \text{ J})(6.022 \times 10^{23}) = 197,600 \text{ J} = 198 \text{ kJ}$
- C04. (B) Dry ice is CO_2 , with a molar mass of 44.01 g/mol , so 300.0 g of dry ice is $6.817 \text{ moles of CO}_2$. The CO_2 pressure inside the container is therefore $P = nRT/V = (6.817)(0.08206)(298)/100.0 = 1.67 \text{ atm}$. But before you added the dry ice, the open container already had 1.00 atm pressure from the atmosphere, so assuming the 300 g of dry ice takes up a negligible volume in the 100 L container, the total pressure in the container is $1.00 + 1.67 = 2.67 \text{ atm}$. (Yes, it's kind of a tricky question, that's why I tried to tip you off on the District exam with the question about how much gas is inside an "empty" two liter bottle.)
- C05. (E) Xenon tetrafluoride is a square planar molecule, so the F-Xe-F bond angles are 90° and 180° .
- C06. (D) Since the pressure in Container B is the same as that in Container A but the temperature of Container B is higher, Container B must have either a larger volume or a lower number of moles or both. In Answer A the volume is lower and the moles are the same, so it can't possibly have the same pressure at a higher temperature. Answer B has a smaller volume and a higher number of moles, so that can't be right. Answer C has the same volume and the same number of moles, so that can't be right. Answers D and E both have a larger volume and a smaller number of moles, so they're both possibilities. You only have to do the calculations for one, and if it's not that one then it's the other one. Solve for pressure using the information given for Container A, then solve for pressure using the information given for container B, and if you get the same pressure, that's your answer.
Container A: $P = nRT/V = (6)(0.08206)(298)/3 = 48.9 \text{ atm}^*$
Answer D: $P = (5)(0.08206)(478)/4 = 49.0 \text{ atm}^*$
Answer E: $P = (5)(0.08206)(478)/5 = 39.2 \text{ atm}^*$
 $48.9 \approx 49.0$, so the best answer is D. *Note that when we calculate pressure we're not really calculating atmospheres unless each dot represents one mole of gas, but if that were true we couldn't use the ideal gas law. But the mole ratios of the gases in each container are the same as the dot ratios, so we can just use the number of dots as the number of moles and our pressure is in some undefined unit. (You can also do the problem leaving the constant R out of the calculation.)
- C07. (C) This is a system of two equations with two unknowns, m_{ice} and m_{water} .
Equation 1: The total mass of the ice and water is $10,000 \text{ grams}$: $m_{\text{ice}} + m_{\text{water}} = 10,000$
Equation 2: The heat gained by the ice melting and then warming up to 65°C is equal to the heat lost by the hot water cooling down: $m_{\text{ice}}\Delta H_{\text{fus}} + m_{\text{ice}}c_w\Delta T_{\text{ice}} = -m_{\text{water}}c_w\Delta T_{\text{water}}$
Plugging Equation 1 into Equation 2 yields: $m_{\text{ice}}\Delta H_{\text{fus}} + m_{\text{ice}}c_w\Delta T_{\text{ice}} = -(10,000 - m_{\text{ice}})c_w\Delta T_{\text{water}}$
Then go through some algebra to isolate m_{ice} on one side of the equation:

$$m_{\text{ice}} = \frac{-10,000c_w\Delta T_{\text{water}}}{c_w\Delta T_{\text{ice}} + \Delta H_{\text{fus}} - c_w\Delta T_{\text{water}}}$$

Plug in the numbers:

$$m_{\text{ice}} = \frac{-10,000 \times 4.184 \times (65 - 88)}{4.184 \times (65 - 0) + 334 - 4.184 \times (65 - 88)}$$

$$m_{\text{ice}} = 1370 \text{ g}, m_{\text{water}} = 10000 - 1370 = 8630 \text{ g}$$

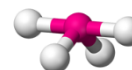


This problem might be easier to solve through trial and error by plugging in the answer choices and seeing when the heat absorbed by the water is equal to the heat lost by the water.

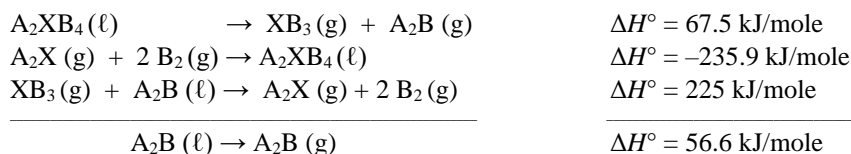
$$\begin{aligned}\text{Heat gained by the ice} &= m_{ice}\Delta H_{fus} + m_{ice}c_w\Delta T_{ice} \\ \text{Heat gained by the ice} &= (1370 \times 334) + (1370 \times 4.184 \times 25) \\ \text{Heat gained by the ice} &= (457580) + (372585) \\ \text{Heat gained by the ice} &= 830 \text{ kJ}\end{aligned}$$

$$\begin{aligned}\text{Heat lost by the water} &= -m_{water}c_w\Delta T_{water} \\ \text{Heat lost by the water} &= -8630 \times 4.184 \times -23 \\ \text{Heat lost by the water} &= 830 \text{ kJ}\end{aligned}$$

- C08. (B) Because the central S atom has a non-bonding lone pair of electrons there are five areas of electron density around the central atom, giving it trigonal bipyramidal electronic geometry. The molecular geometry of SF₄ is a non-symmetrical seesaw-shape, so it has a dipole moment and has dipole-dipole forces between molecules.

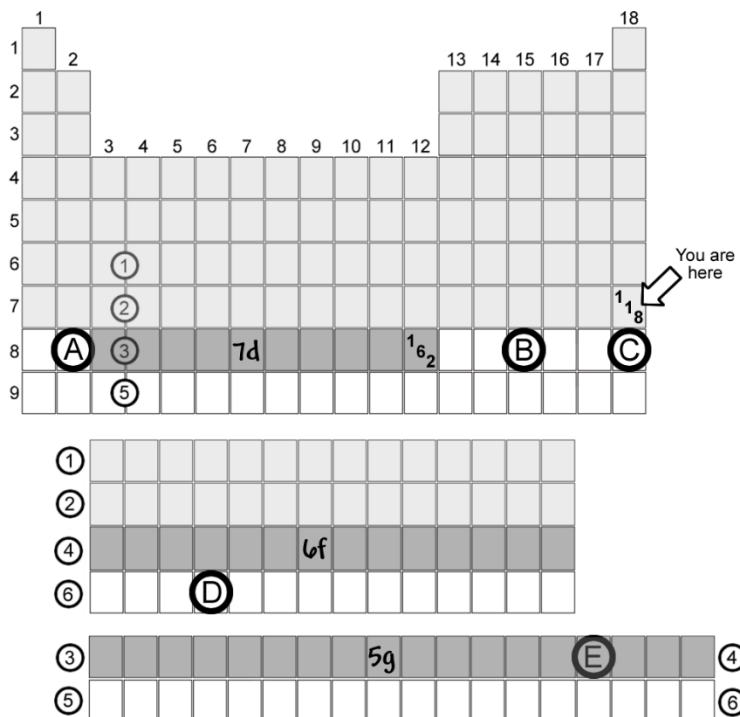


- C09. (A) What you need to get to in order to solve this problem is $\Delta G = \Delta H - T\Delta S$. Because a phase change at the boiling point or freezing point is at equilibrium, $\Delta G = 0$ and $\Delta H = T\Delta S$. Therefore $T = \Delta H / \Delta S$. The trick now is to find ΔH and ΔS . ΔS is easier, it's just $\Delta S = S^\circ_{\text{products}} - S^\circ_{\text{reactants}} = S^\circ[\text{A}_2\text{B}(\text{g})] - S^\circ[\text{A}_2\text{B}(\ell)] = 239.8 \text{ J/K} - 75.5 \text{ J/K} = 164.3 \text{ J/K}$. To find ΔH for the vaporization of A₂B we need to use Hess's Law to find ΔH for A₂B(ℓ) → A₂B(g). Reverse the second equation and change the sign on ΔH , Add the first reaction, Add the third reaction, Then add them all together.



$$T = \Delta H / \Delta S = 56,600 \text{ J} / 164.3 \text{ J/K} = 344.5 \text{ K}. \quad 344.5 \text{ K} - 273 = 71.5^\circ \text{C}$$

- C10. (B) The light shaded squares show our current periodic table, ending with 118. Element A is only two elements further, so A is Element 120. After Element A (in the 8s subshell) we go into the 5g, 6f, and 7d subshells (dark shaded), for a total of 18 + 14 + 10 = 42 elements more, which puts us at Element 162. Element 165 is only three elements further on, which is Element B.



- C11. (A) Of all the answer choices, increasing the concentration of AB_2 is the only one that will result in an increased yield of AD gas, so there is no need to calculate the costs of any of these options. Since the reaction is exothermic, it can be written like this: $\text{AB}_2(\text{aq}) + \text{C}_2\text{D}(\text{s}) \rightleftharpoons 2\text{CB}(\text{aq}) + \text{AD}(\text{g}) + \text{heat}$. LeChatelier's Principle tells us that to increase the amount of product formed, we should add more reactant. CB, AD, and heat are all products, and increasing the amounts of any of these will drive the equilibrium reaction in the backward direction and decrease the amount of product produced by the reaction. C_2D is a solid and does not appear in the equilibrium expression for this reaction, so increasing the amount of solid present will have no effect on the amount of AD produced.
- C12. (D) HF is a weak acid, so $[\text{H}^+] = \sqrt{K_a[\text{HF}]}$. If you assume only a small amount of HF dissociates and $[\text{HF}] \approx 0.0050$, you get $[\text{H}^+] = 1.76 \times 10^{-3} \text{ M}$ and $\text{pH} = 2.75$. But if you check your assumption, $(1.76 \times 10^{-3}/0.0050) \times 100 = 35.2\%$ ionization, which is way above the 5% that is considered allowable for making that simplifying assumption, so you have to use the quadratic formula to solve for $[\text{H}^+]$. $a = 1$, $b = K_a$, and $c = -K_a C_{\text{acid}}$. The result is $[\text{H}^+] = 0.00148$ and $\text{pH} = 2.83$.
- C13. (C) First you have to determine whether or not a precipitate forms by calculating Q for the reaction and comparing it with K_{sp} . $[\text{Ba}^{2+}]$ in the combined volume $= (0.500 \text{ L})(0.25 \text{ M})/(1.00 \text{ L}) = 0.125 \text{ M}$. $[\text{MoO}_4^{2-}]$ in the combined volume $= (0.500 \text{ L})(0.50 \text{ M})/(1.00 \text{ L}) = 0.250 \text{ M}$. $Q = [\text{Ba}^{2+}][\text{MoO}_4^{2-}] = (0.125)(0.250) = 3.1 \times 10^{-2}$. $Q > K$, so a precipitate forms. $[\text{Ba}^{2+}]$ and $[\text{MoO}_4^{2-}]$ react 1:1, so all 0.125 moles of Ba^{2+} will react, leaving 0.125 moles of MoO_4^{2-} in excess. This results in a common ion effect problem where solid BaMoO_4 is dissolving in a solution that is 0.125 M MoO_4^{2-} . $K_{\text{sp}} = [\text{Ba}^{2+}][\text{MoO}_4^{2-}]$, so $[\text{Ba}^{2+}] = K_{\text{sp}}/[\text{MoO}_4^{2-}] = 3.5 \times 10^{-8}/0.125 = 2.8 \times 10^{-7} \text{ M}$.
- C14. (E) First use runs 1 and 2 to determine that the reaction rate is second order with respect to B. Then use runs 1 and 3 to determine that the reaction rate is first order with respect to A. The rate law is $\text{rate} = k[\text{A}][\text{B}]^2$. Now plug in the concentrations and initial rate for any run to calculate k . $k = 0.103 \text{ M}^{-2}\text{sec}^{-1}$. Then plug in the concentrations in run #4 and the rate constant to calculate the initial reaction rate for run 4: $\text{rate} = k[\text{A}][\text{B}]^2 = (0.103)(0.200)(0.275)^2 = 1.56 \times 10^{-3} \text{ M/s}$.
- C15. (D) Pure A has a vapor pressure of 18 torr and pure B has a vapor pressure of 36 torr. The solution vapor pressure, 30 torr, is 2/3 of the way from A (18) to B (36), so the mole fraction of B is 2/3 and the mole fraction of A is 1/3. Or put another way, moles of B $= 2 \times$ moles of A. Moles of A $= (170 \text{ mL} \times 0.925 \text{ g/mL}) / 95.0 \text{ g/mol} = 1.6553$ moles. Therefore moles of B $= 3.3106$ mol. Grams of B $= 315 \text{ mL} \times 0.815 \text{ g/mL} = 256.73 \text{ g}$. $256.73 \text{ g} / 3.3106 \text{ mol} = 77.5 \text{ g/mol}$.
- C16. (C) You're passing the same total charge into each of the solutions, so as long as you're not reducing all of the metal from the solution you don't have to do the whole calculation for each one to find the answer. All you really need to know is what the mass to charge ratio is for each of these ions. The ion with the highest mass to charge ratio will produce the most mass of metal for any given charge. The mass to charge ratios for these ions are:
 A) Cu^{2+} : $63.55/2 = 31.8$
 B) Sn^{2+} : $118.71/2 = 59.4$
C) Au^{3+} : $196.97/3 = 65.7$
 D) Fe^{3+} : $55.85/3 = 18.6$
 E) Sn^{4+} : $118.71/4 = 29.7$
 F) Pb^{4+} : $207.2/4 = 51.8$
 (If you wanted to check the assumption to be sure, moles of electrons $= I \cdot t / \mathcal{F} = (1.5 \times 3 \times 60 \times 60) / 96,485 = 0.168$ mols of electrons, which isn't nearly enough to reduce out all of the metal from any of these solutions.)
- C17. (E). The balanced equation for the combustion is $\text{CH}_4 + 2\text{O}_2 \rightarrow \text{CO}_2 + 2\text{H}_2\text{O}$. Using $PV = nRT$ on just the initial CH_4 , there are $(2.0 \times 0.5) / (400 \times 0.08206) = 0.030466$ moles of CH_4 present.. Reacting away the CH_4 produces three moles of gas for every mole of CH_4 originally present, for a total of 0.091397 moles of gas. This would result in a final pressure in the total 3.0 liter volume of $P = (0.091397)(0.08206)(400) / (3.0) = 1 \text{ atm}$ pressure. But the problem says the final pressure is 1.25 atm, so there is excess O_2 in the gas bulbs. The moles of excess O_2 is calculated based on the excess pressure: $n = (0.25 \text{ atm})(3.0 \text{ L}) / (0.08206)(400 \text{ K}) = 0.022849$ moles. So the total moles of O_2 is 0.060932 moles used up in the reaction plus 0.022849 moles in excess, or 0.083781 total moles of O_2 . When all of this was in the original 1.0 L bulb, the pressure was $(0.083781)(0.08206)(400) / (1.0) = 2.75 \text{ atm}$.

- C18. (A) The deBroglie equation tells you the wavelength of a moving particle based on its mass and velocity.

$$\lambda = \frac{h}{mv}$$

m in this equation is the mass of the individual particle (one helium atom) in kg.

First calculate the root mean square velocity of a helium atom at 298 K by combining

$KE = \frac{3}{2}RT$ and $KE = \frac{1}{2}mv^2$ into $v = \sqrt{\frac{3RT}{m}}$. Note that since this equation uses the gas law constant R , which always refers to *one mole* of a gas, in this equation m is the molar mass of helium in kg/mol:

$$m = \frac{4.00\text{g}}{\text{mol}} \times \frac{1\text{ kg}}{1000\text{ g}} = 0.00400\text{ kg/mol}$$

$$v = \sqrt{\frac{3RT}{m}} = \sqrt{\frac{(3)(8.314\text{ J/mol} \cdot \text{K})(298\text{ K})}{0.00400\text{ kg/mol}}} = 1363\text{ m/s}$$

In the deBroglie equation we need the mass of one helium atom in kg:

$$0.00400\frac{\text{kg}}{\text{mol}} \times \frac{1\text{ mol}}{6.022 \times 10^{23}\text{ atoms}} = 6.642 \times 10^{-27}\text{ kg/atom}$$

$$\lambda = \frac{h}{mv} = \frac{6.626 \times 10^{-34}\text{ J} \cdot \text{s}}{(6.642 \times 10^{-27}\text{ kg})(1363\text{ m/s})} = 7.32 \times 10^{-11}\text{ m} = 0.0732\text{ nm}$$

- C19. (D) $500\text{ mL} \times 1.22\text{ g/mL} = 610\text{ g}$ of solution. $2.70\text{ mol/L} \times 0.500\text{ L} = 1.85\text{ mol}$ of CaCl_2 . The mass of the CaCl_2 is $1.35\text{ mol} \times 110.98\text{ g/mol} = 149.82\text{ g}$. The mass of water is therefore $610 - 149.82 = 460.18\text{ g}$. $460.18\text{ g} / 18.02\text{ g/mol} = 25.54\text{ mol H}_2\text{O}$. The mole fraction of water is the moles of water divided by the total moles of everything in the sample. Because CaCl_2 breaks up into Ca^{2+} and 2 Cl^- , the total moles in the sample = moles of H_2O + moles of Ca^{2+} + moles of $\text{Cl}^- = 25.54 + 1.35 + 2.70 = 29.59$ moles. $\chi_{\text{H}_2\text{O}} = 25.54/29.59 = 0.863$.
- C20. (B) $K_{\text{sp}} = [\text{M}^+][\text{X}^-]$, so when the MX just begins to precipitate out, the $[\text{M}^+]$ concentration = $K_{\text{sp}}/[\text{X}^-] = 8.5 \times 10^{-9}/0.0050 = 1.70 \times 10^{-6}\text{ M}$. When $[\text{M}^+] = 1.70 \times 10^{-6}\text{ M}$, $[\text{Z}^-] = K_{\text{sp}}/[\text{M}^+] = 1.7 \times 10^{-10}/1.70 \times 10^{-6}\text{ M} = 1 \times 10^{-4}\text{ M}$. The percent of Z^- remaining is $(1 \times 10^{-4}\text{ M} / 0.0025) \times 100\% = 4.00\%$

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- P01. (D) pages 132-133: “In fact, these satellite photographs are now so precise that it is possible to detect tiny, minuscule ripples in the background radiation due to the quantum uncertainty principle. At the instant of creation, there should have been quantum fluctuations that caused these ripples. A perfectly smooth Big Bang would have violated the uncertainty principle.”
- P02. (C) page 155: “...another astonishing discovery was made by Juan Maldacena in 1997. He jolted the entire physics community by showing something that was once considered impossible: that a supersymmetric Yang-Mills theory, which describes the behavior of subatomic particles in four dimensions, was dual, or mathematically equivalent, to a certain string theory in ten dimensions.”
- P03. (A) page 165: “At present, one leading candidate for dark matter is called the weakly interacting massive particles (WIMPs). Among them, one likely possibility is the photino, the supersymmetric partner of the photon.”
- P04. (E) Right after its formation, the Sun’s heat and solar wind began to vaporize and push outward the gas and dust surrounding it. Only heavy minerals and those that could withstand higher heat remained close to the newly formed star. Closest to the Sun were metal oxides and silicates, with sulfates and hydrated minerals a little farther out. Ices of water, ammonia, and methane only survived in the outer layers of the protoplanetary disk. As planets began to form, the presence of those ices provided a vast reservoir of raw materials that quickly created large, gravitationally strong protoplanets. These larger protoplanets had enough gravity to sweep up the surrounding gases and create the gas giants we see today in the outer solar system. The lack of ices in the inner solar system left little material for planetary formation, resulting in small worlds composed almost entirely of metals and silicates.
- P05. (A) Following the correct order of operations, we first calculate the squared term. This term has three significant digits, so the squared value will also have three significant digits: $(8.94)^2 = 79.9236$. Here, we list all of the digits, but underline those that are significant. Next is the subtraction. For this, we look at the decimal places of the two numbers. The first number is significant to the tenths place, while the second number is significant to the hundredths place. We must go with the least number of decimal places, so the answer will be significant only to the tenths place. Completing this calculation: $79.9236 - 73.42 = 6.5036$. Notice that this result has only two significant digits. Lastly, we divide by pi. Pi is considered an exact number, so every digit of pi that we include is significant. We must round the answer to the least number of significant digits of the values being divided, which will be two (based on the numerator). Thus, we get: $\frac{6.5036}{3.14159} = 2.07016 \approx 2.1$.
- P06. (D) First, we need to examine the motion of the basket (and, thus, ourselves): starting from rest, the basket accelerates directly upward at 2.55m/s^2 . After 3.00 seconds, the position and velocity of the basket are as follows: $y = y_i + v_{iy}t + \frac{1}{2}a_yt^2 = 0 + 0 + (0.5)(2.55)(3.00)^2 = 11.475\text{m}$, and $v_y = v_{iy} + a_yt = 0 + (2.55)(3.00) = 7.65\text{ m/s}$. This is our position and velocity at the moment that we throw the ball. We throw the ball horizontally at 8.00m/s relative to us, which means that the initial velocity of the ball, relative to the ground, is 8.00m/s horizontally plus 7.65m/s vertically. The ball also has an initial vertical position of 11.475m above the ground. After it is thrown, the ball is in free-fall, with a downward acceleration of 9.80m/s^2 . Starting with the vertical motion, we have: $y = y_i + v_{iy}t - \frac{1}{2}gt^2 \rightarrow 0 = 11.475 + 7.65t - 4.9t^2$. Rearranging terms and simplifying, we get a quadratic equation: $t^2 - 1.561t - 2.342 = 0$. Solving this gives times of $t = 2.50\text{s}$, -0.938s . Ignoring the negative time option, we conclude that the time for the ball to reach the ground is $t = 2.50\text{s}$. Now, turning to the horizontal motion, the ball starts directly above the launch point ($x_i = 0$), and with a horizontal velocity of 8.00m/s . There is no horizontal acceleration. Thus, we have $x = x_i + v_{ix}t + \frac{1}{2}a_xt^2 = 0 + (8.00)(2.50) + 0 = 20.0\text{m}$. This is the distance from the launch point where the ball lands.

- P07. (B) The force diagram is shown below. There are four forces: gravity (mg , downward), the applied force (F , up and right), the normal force (F_N , up and left, perpendicular to the plane), and the frictional force (F_f , down and left, parallel to the plane).

For inclined plane problems, it is customary to tilt the coordinate system so that the x-axis is parallel to the plane, and the y-axis is perpendicular to the plane. In this tilted coordinate system, the frictional force is in the negative x-direction, and the normal force is in the positive y-direction. Both the gravitational force and the applied force must be split into components. For the gravitational force: in the negative x-direction is the component $mg \sin \theta_1$ and in the negative y-direction is the component $mg \cos \theta_1$, where $\theta_1 = 18^\circ$. For the applied force, we must first determine its angle relative to the tilted coordinate system: $\phi = \theta_2 - \theta_1$ which gives

$\phi = 32 - 18 = 14^\circ$. Then the component

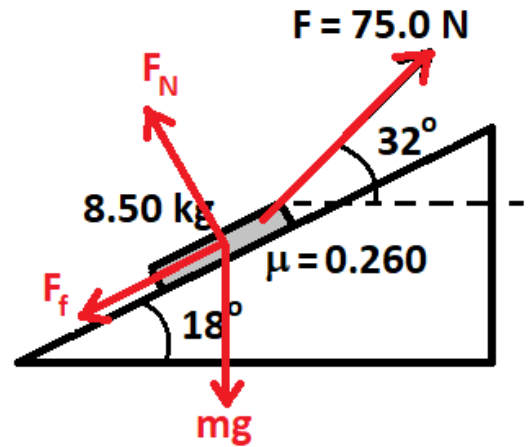
$F \cos \phi$ is in the positive x-direction and the component $F \sin \phi$ is in the positive y-direction. The motion of the sled is entirely in the x-direction, so the forces in the y-direction sum to zero:

$$\sum F_y = F_N + F \sin \phi - mg \cos \theta = 0 \rightarrow F_N + (75) \sin 14 - (8.50)(9.8) \cos 18 \rightarrow F_N = 61.08 \text{ N.}$$

Now we can determine the magnitude of the frictional force: $F_f = \mu F_N = (0.26)(61.08) = 15.88 \text{ N}$.

The sum of the forces in the x-direction gives the acceleration of the sled:

$$\sum F_x = F \cos \phi - F_f - mg \sin \theta = ma \rightarrow (75) \cos 14 - 15.88 - (8.50)(9.8) \sin 18 = (8.50)a, \text{ giving } 31.15 = (8.50)a \rightarrow a = 3.66 \text{ m/s}^2.$$



- P08. (C) We solve this problem by using conservation of momentum. We must consider the momenta in both the x- and y-directions, before and after the collision. Before the collision, the first ball has non-zero momentum exclusively in the x-direction: $p_{ix1} = m_1 v_{ix1} = (4.50)(12.0) = 54.0 \text{ kgm/s}$. The momentum in the y-direction for this ball is zero: $p_{iy1} = 0$. Also, since the second ball is initially stationary, both its x- and y-direction momenta are initially zero: $p_{ix2} = 0$, $p_{iy2} = 0$. Thus, the total initial momentum in the x-direction is $p_{ix} = p_{ix1} + p_{ix2} = 54.0 + 0 = 54.0 \text{ kgm/s}$. And the total initial momentum in the y-direction is $p_{iy} = p_{iy1} + p_{iy2} = 0 + 0 = 0$. By conservation of momentum, the total final momentum in each direction must equal the total initial momentum in each direction:

$$p_{fx} = p_{ix} = 54.0 \text{ kgm/s, and } p_{fy} = p_{iy} = 0.$$

Now, let's consider the components of the final momenta: for the x-direction, we have components from both the first ball and the second ball:

$$p_{fx} = p_{fx1} + p_{fx2} = m_1 v_{fx1} + m_2 v_{fx2} = m_1 v_{f1} \cos \theta_1 + m_2 v_{f2} \cos \theta_2. \text{ This leads to}$$

$$p_{fx} = (4.50)(6.00) \cos 58.0 + (6.50)v \cos \theta = 54.0. \text{ Simplifying, this gives:}$$

$$14.308 + (6.50)v \cos \theta = 54 \rightarrow (6.50)v \cos \theta = 39.692 \rightarrow v \cos \theta = 6.106.$$

For the y-direction we also have components from both the first ball and the second ball:

$$p_{fy} = p_{fy1} + p_{fy2} = m_1 v_{fy1} + m_2 v_{fy2} = m_1 v_{f1} \sin \theta_1 + m_2 v_{f2} \sin \theta_2. \text{ Keeping in mind that } \theta_2 \text{ is an angle below the x-axis, we get } p_{fy} = (4.50)(6.00) \sin 58.0 - (6.50)v \sin \theta = 0 \rightarrow$$

$$22.897 - (6.50)v \sin \theta = 0 \text{ which leads to } (6.50)v \sin \theta = 22.897 \rightarrow v \sin \theta = 3.523.$$

$$\text{Dividing the two resulting equations gives us: } \frac{v \sin \theta}{v \cos \theta} = \tan \theta = \frac{3.523}{6.106} = 0.577 \rightarrow$$

$$\theta = \tan^{-1}(0.577) = 30.0^\circ. \text{ The direction for this angle is "down and right" or "South of East."}$$

Finally, since we know the final angle of the second ball, we can determine the final speed of the second ball: $v \sin \theta = v \sin 30.0 = 3.523 \rightarrow v = 7.05 \text{ m/s}$. Thus, the correct answer is a velocity of 7.05 m/s at an angle of 30.0° South of East.

- P09. (C) Since the system is in static equilibrium, the forces in the x-direction, and those in the y-direction, must sum to zero. Similarly, the torques must also sum to zero. Let's start by identifying the forces: at the base we have both a horizontal component force (F_h , right) and a vertical component force (F_v , upward). At the midpoint of the beam is the weight of the beam ($m_b g$, down) and at the end of the beam is the weight of the crate ($M_c g$, down). Lastly, there is the horizontal tension of the cable attached to the wall (T , left).

In most problems like this, it is best to start with the torques. To calculate the torques, we must first choose a pivot point. The best place for a pivot point in this problem is at the base of the beam. The definition of torque is $\tau = Fr \sin \theta$ where F is the force, r is the torque arm (the distance from the pivot point to the force location) and θ is the angle between the force and the torque arm. Torques that would cause clockwise rotation are considered negative, while those that would cause counterclockwise rotation are positive. So, we have the following torques: $\tau_h = F_h(0) = 0$, $\tau_v = F_v(0) = 0$. We chose the pivot point to be at the base of the beam so that these would be zero. Now for the non-zero torques, being careful to correctly determine the angles: first, the beam weight torque:

$\tau_1 = m_b g \left(\frac{1}{2}L\right) \sin(90 - 57) = (60.0)(9.8)(10) \sin 33 = 3202\text{Nm}$ clockwise; now the crate weight torque: $\tau_2 = M_c g(L) \sin(90 - 57) = (93.0)(9.8)(20) \sin 33 = 9928\text{Nm}$ clockwise; and, finally, the cable torque: $\tau_3 = T(15.0) \sin(57) = 12.58T$ counterclockwise. Adding up all of the torques and setting the sum to zero: $\sum \tau = -3202 - 9928 + 12.58T = 0 \rightarrow 12.58T = 13130 \rightarrow T = 1044\text{N}$. This is the tension in the horizontal cable.

Now we sum up the forces. Let's start with the horizontal: $\sum F_x = F_h - T = 0 \rightarrow F_h = T = 1044\text{N}$. And for the vertical forces: $\sum F_y = F_v - m_b g - m_c g = 0 \rightarrow F_v = (60)(9.8) + (93)(9.8) \rightarrow F_v = 1499\text{N}$. Lastly, we determine the magnitude of the net force on the base of the beam:

$$F_{base} = \sqrt{F_h^2 + F_v^2} = \sqrt{(1044)^2 + (1499)^2} = \sqrt{3336937} = 1827\text{N}.$$

- P10. (E) Though we are dealing with an oscillatory system, we solve this problem by considering conservation of energy. Initially, the energy stored in the stretched spring is $U_0 = \frac{1}{2}kx_0^2 = (0.5)(380)(0.20)^2 = 7.60\text{J}$.

After one period, the energy remaining in the system is $U_1 = \frac{1}{2}kx_1^2 = (0.5)(380)(0.16)^2 = 4.864\text{J}$. The difference is $\Delta U = 7.60 - 4.864 = 2.736\text{J}$. Thus, the work done by friction caused the conversion of 2.736J of energy to heat. Now we consider the force of friction: the vertical forces acting on the mass are the weight (mg , down) and the normal force (F_N , upward). Since there is no motion in the vertical direction, these two forces must sum to zero: $\sum F_y = F_N - mg = 0 \rightarrow F_N = mg = (1.20)(9.8) = 11.76\text{N}$. From this we can write an expression for the frictional force: $F_f = \mu F_N = (11.76)\mu$. The frictional force multiplied by the distance that the mass slides equals the work done by friction. That is,

$W = F_f d = \Delta U = 2.736\text{J}$. So, how far did the box slide? Initially it slides 20cm to return to equilibrium, but at that point it is moving quickly, and slides past equilibrium. On the other side, the mass will have lost some energy and will only slide back about 18cm. Then it stops, turns around, and slides forward, back towards equilibrium. Moving forward, the mass passes through equilibrium again. The mass slides to a stop 16.0cm from equilibrium, completing one period. The total distance that the mass slides is

$20\text{cm} + 18\text{cm} + 18\text{cm} + 16\text{cm} = 72\text{cm}$. This is an approximation of the motion of the mass, but it is close enough for us to determine the correct answer choice. Putting this into the work expression, we get $F_f d = (11.76)\mu(0.72) = 2.736 \rightarrow \mu = 0.32$.

- P11. (E) Starting at the bottom left vertex, this engine cycle consists of an isovolumetric process, an isothermal process, and an isobaric process. The work done by an isovolumetric process is zero, thus $W_{1 \rightarrow 2} = 0$. To determine the work done by the isothermal process, we must use the ideal gas law to find the temperature of the isotherm. Using the vertex at the top, we have: $PV = nRT \rightarrow (350\text{kPa})(20\text{L}) = (1.50)(8.314)T_2$. Converting the pressure and volume, we get: $(350,000\text{Pa})(0.020\text{m}^3) = 7000 = 12.47T_2 \rightarrow T_2 = 561\text{K}$. This is the isotherm temperature, with which we can find the work done during the isothermal process: $W_{2 \rightarrow 3} = nRT \ln\left(\frac{V_f}{V_i}\right) = (1.50)(8.314)(561) \ln\left(\frac{70}{20}\right) = 8770\text{J}$. Finally, we need to find the work done during the isobaric process:

$W_{3 \rightarrow 1} = P\Delta V = P(V_1 - V_3) = (100,000\text{Pa})(0.020 - 0.070) = -5000\text{J}$. Thus, the total work done by the gas in one complete cycle is: $W = W_{1 \rightarrow 2} + W_{2 \rightarrow 3} + W_{3 \rightarrow 1} = 0 + 8770 - 5000 = 3770\text{J}$.

- P12. (E) This circuit contains batteries in multiple different branches, so it is best solved by using Kirchhoff's Laws. Let the current in the left branch be I_1 , directed upward. Let the current in the middle branch be I_2 , directed downward. Finally, let the known current in the right branch be $I_3 = 22.0\text{mA} = 0.022\text{A}$, directed upward. Using the Kirchhoff node rule at the top node, we determine the relationship between the currents: $I_1 + I_3 = I_2 \rightarrow I_1 + 0.022 = I_2$. Now, using the Kirchhoff loop rule on the left loop, going clockwise around the loop, we get: $19 - (510)I_1 - (620)I_2 = 0$. Combining this with the node rule relationship: $19 - 510I_1 - 620(I_1 + 0.022) = 5.36 - 1130I_1 = 0 \rightarrow I_1 = 0.00474\text{A}$. Then we get $I_2 = 0.00474 + 0.022 = 0.0267\text{A}$. Lastly, we use Kirchhoff's loop rule and go counterclockwise around the right-side loop: $V - (620)I_2 - (380)I_3 = 0 \rightarrow V - (620)(0.0267) - (380)(0.022) = V - 24.9 = 0 \rightarrow V = 24.9\text{V}$.
- P13. (C) To find the electric field in this situation, we must rely on Gauss' Law: $\oint E \cdot dA = \frac{Q_{\text{inside}}}{\epsilon_0}$. The system, fortunately, has spherical symmetry, which allows us to reduce the surface integral to a simple multiplication: $\oint E \cdot dA = EA$ where A is the surface area of the Gaussian surface (the dashed circle). Since the Gaussian surface is a sphere, we get $A = 4\pi R^2 = 4\pi(1.20)^2 = 18.1\text{cm}^2 = 0.00181\text{m}^2$. The charge contained inside the Gaussian surface is then given by $Q_{\text{inside}} = \iiint \rho dV$. Again, we take advantage of the spherical symmetry of the system, reducing the three-dimensional integral to a single integration over the radial variable: $Q_{\text{inside}} = \int_0^R \rho(r)4\pi r^2 dr$. Putting in the expression for the charge density, $Q_{\text{inside}} = \int_0^R (0.0300)(4 - r^2)4\pi r^2 dr = \int_0^R 0.377(4r^2 - r^4)dr$. Completing the integration gives: $Q_{\text{inside}} = (0.377)\left(\frac{4}{3}R^3 - \frac{1}{5}R^5\right)$. Plugging in the radius of the Gaussian surface, $R = 1.20\text{cm}$, we get $Q_{\text{inside}} = (0.377)[(1.33)(1.20)^3 - (0.200)(1.20)^5] = (0.377)(1.806) = 0.681\text{nC}$. Now we put it all together: $EA = \frac{Q_{\text{inside}}}{\epsilon_0} = E(0.00181\text{m}^2) = \frac{0.681\text{nC}}{8.854 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}} = \frac{6.81 \times 10^{-10} \text{C}}{8.854 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}} \rightarrow E(0.00181\text{m}^2) = 76.9 \frac{\text{Nm}^2}{\text{C}} \rightarrow |E| = 4.25 \times 10^4 \text{N/C}$.
- P14. (B) The magnitude of the magnetic field produced by a long straight wire is given by $|B| = \frac{\mu_0 I}{2\pi r}$. The distance from the lower wire to the point P is simply $r_2 = 30.0\text{cm} = 0.300\text{m}$, but the distance from the upper wire to the point P must be found by using the Pythagorean theorem: $r_1 = \sqrt{(30.0)^2 + (40.0)^2} = 50.0\text{cm} = 0.500\text{m}$. Now we can find the magnitudes of the magnetic fields produced by each wire: $|B_2| = \frac{\mu_0 I_2}{2\pi r_2} = \frac{(4\pi \times 10^{-7})(32.0)}{2\pi(0.300)} = 2.133 \times 10^{-5}\text{T} = 21.33\mu\text{T}$. Likewise, $|B_1| = \frac{\mu_0 I_1}{2\pi r_1} = \frac{(4\pi \times 10^{-7})(42.0)}{2\pi(0.500)} = 1.68 \times 10^{-5}\text{T} = 16.8\mu\text{T}$. At this point, we have to consider the directions of these magnetic fields. The magnetic fields form in circles around the wires, and since the currents are going in the +z direction, the magnetic fields are oriented counterclockwise in circles around the wires. This means that the magnetic field B_2 will be directed entirely in the positive y-direction. Thus $B_{2x} = 0$ and $B_{2y} = 21.33\mu\text{T}$. The magnetic field B_1 , however, has components in both the x- and y-directions. These can be determined using the right triangle formed by the point P, the origin, and the upper wire. We must be careful, though, because the direction of the magnetic field is perpendicular to the line connecting the upper wire to the point P. This means that the horizontal component of B_1 is $B_{1x} = \frac{40}{50}|B_1| = (0.8)(16.8) = 13.44\mu\text{T}$ (positive x-direction), and the vertical component of B_1 is $B_{1y} = \frac{30}{50}|B_1| = (0.6)(16.8) = 10.08\mu\text{T}$ (positive y-direction). Now we combine the fields from the two wires to get the total magnetic field at the point P. For the horizontal: $B_x = B_{1x} + B_{2x} = 13.44 + 0 = 13.44\mu\text{T}$, and for the vertical: $B_y = B_{1y} + B_{2y} = 10.08 + 21.33 \rightarrow B_y = 31.41\mu\text{T}$. Finally, we can use the components to find the magnitude of the total magnetic field at the point P due to the currents in the two wires: $|B| = \sqrt{B_x^2 + B_y^2} = \sqrt{(13.44)^2 + (31.41)^2} = 34.2\mu\text{T}$.

- P15. (C) It is possible that you already know the answer if you happen to have memorized the rms conversion for a sawtooth wave. However, let us show the mathematics of how we obtain the correct result. The definition of the root-mean-square for any waveform is $V_{rms} = \sqrt{\frac{1}{T} \int_0^T (V(t))^2 dt}$. We already are given the voltage waveform function, so we can go ahead and put that into the formula. Keep in mind that T is the period of the waveform, and it should cancel out before we get to the final answer. Putting it together gives: $V_{rms} = \sqrt{\frac{1}{T} \int_0^T \left(\frac{V_0}{T} t\right)^2 dt} = \sqrt{\frac{1}{T} \int_0^T \frac{V_0^2}{T^2} t^2 dt} = \sqrt{\frac{V_0^2}{T^3} \int_0^T t^2 dt} = \sqrt{\frac{V_0^2}{T^3} \frac{t^3}{3} \Big|_0^T} = \sqrt{\frac{V_0^2}{T^3} \frac{T^3}{3}} = \sqrt{\frac{V_0^2}{3}} = \frac{V_0}{\sqrt{3}}$.
- P16. (A) We cannot use the thin lens equation for this thick of a lens. Instead, we will have to treat each surface of the lens as an independent refracting surface. The equation for image formation at a refracting surface is $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$, where n_1 is the index of refraction of the material where the light rays are located before refracting, and n_2 is the index of refraction of the material where the light rays are located after refracting. R is the radius of curvature of the refracting surface, and is positive for a convex entry side and negative for a concave entry side.
- For our first surface (surface a), we have a convex entry side; thus, $R_a = +25.0\text{cm}$. Our light starts at the object, so $n_{1a} = 1.00$ and the light ends up in the glass, so $n_{2a} = 1.62$. Thankfully, we can work the entire problem in centimeters. Thus, the first image, formed by the first refracting surface, can be found: $\frac{n_{1a}}{p_1} + \frac{n_{2a}}{q_1} = \frac{n_{2a} - n_{1a}}{R_{1a}} \rightarrow \frac{1.00}{21.0} + \frac{1.62}{q_1} = \frac{1.62 - 1.00}{25} \rightarrow \frac{1.62}{q_1} = \frac{0.62}{25} - \frac{1}{21} \rightarrow q_1 = -71.0\text{cm}$. So, the image from the first surface is located 71.0cm to the left of the curved surface of the lens. This first image becomes the object for the second surface. Moving over to the location of the second surface (surface b), we can find the object distance: $p_2 = D - q_1 = 30\text{cm} - (-71\text{cm}) = 101\text{cm}$. That is, the object for the second surface (which is the image formed by the first surface) is located 101cm to the left of the second surface. We have to be careful with the indices of refraction: the image may appear to be outside the lens, but the light rays are currently in the glass travelling toward the right surface of the lens. For this side of the lens, the light starts in the glass, so $n_{1b} = 1.62$, and the light ends in the air, so $n_{2b} = 1.00$. Finally, this second surface is flat, which means that it has an infinite radius of curvature. That is, $R_b = \infty$.
- Putting it all together, we can find the location of the final image: $\frac{n_{1b}}{p_2} + \frac{n_{2b}}{q_2} = \frac{n_{2b} - n_{1b}}{R_{1b}} \rightarrow \frac{1.62}{101} + \frac{1.00}{q_2} = \frac{1.00 - 1.62}{\infty} \rightarrow \frac{1.62}{101} + \frac{1}{q_2} = 0 \rightarrow q_2 = -62.3\text{cm}$. Therefore, the final image is located 62.3cm to the left of the flat surface of the lens. Moving back to the curved surface, we find that this image is located $62.3\text{cm} - 30.0\text{cm} = 32.3\text{cm}$ to the left of the curved surface of the lens.
- P17. (D) Since the wavefunction is already normalized, this question is pretty easy. The mathematical definition for the expectation value of position is $\langle x \rangle = \int \Psi^* x \Psi dx$. Putting in the given wavefunction and limits, we calculate: $\langle x \rangle = \int_1^{\infty} \frac{\sqrt{3}}{x^2} x \frac{\sqrt{3}}{x^2} dx = \int_1^{\infty} \frac{3}{x^3} dx = \frac{3}{-2} \frac{1}{x^2} \Big|_1^{\infty} = \frac{3}{-2} \left[\frac{1}{\infty} - \frac{1}{1} \right] = \frac{3}{2}$.
- Thus, $\langle x \rangle = 1.50$ units.
- P18. (B) From the decay products, and knowing some conservation laws, we can deduce the properties of the unknown particle. First, we know that the Sigma particle is a baryon (made up of three quarks). Baryon number is conserved, so it must be the same on the left and on the right of the decay equation. We have a baryon on the right; thus, the unknown particle on the left must also be a baryon. This means that our unknown particle is made up of three quarks, eliminating choices D and E. Second, we know that the particle lifetime is fairly long, which indicates that the decay is mediated by the weak force. Weak force decays occur when a quark changes flavor. The Sigma baryon has a quark structure of uus , so the unknown particle must have a different quark structure, which eliminates choice A. Finally, charge is conserved. The total charge on the left of the decay equation must equal the total charge on the right. On the right, we have $Q = +1 + 1 - 1 + 1 = +2$. Therefore, the unknown particle must have a charge of +2, which eliminates choice C (usc has a charge of only +1).
- Thus, only uuc matches all of the criteria – it is a baryon, has a charge of +2, and requires a flavor change to decay into a Sigma particle. This is the most likely quark structure of the unknown particle.

- P19. (A) From the plots, we can clearly see the moment of collision. We can also estimate the cart velocities before and after the collision. For cart 1, we can see that the velocity prior to the collision is about 23cm/s, and the velocity after the collision is about 6cm/s. For cart 2, we can see that it is stationary prior to the collision, and that it has a velocity of about 13cm/s after the collision. Momentum is conserved in a collision, so we can set the initial momentum of the system equal to the final momentum of the system: $p_i = p_f$. These consist of the initial and final momenta of the two carts: $p_{1i} + p_{2i} = p_{1f} + p_{2f}$. Recalling that momentum is mass multiplied by velocity, and noting that all of the velocities are positive (or zero) and in the same direction, we have: $M_1 v_{1i} + M_2 v_{2i} = M_1 v_{1f} + M_2 v_{2f} \rightarrow M_1(23) + M_2(0) = M_1(6) + M_2(13)$. This leads to $23M_1 = 6M_1 + 13M_2 \rightarrow 17M_1 = 13M_2 \rightarrow \frac{M_2}{M_1} = \frac{17}{13} = 1.3$.
- P20. (D) In an AC circuit, the power factor is defined as the cosine of the phase angle between the voltage and the current. In a simple series circuit such as this one, the phase angle between the voltage and the current is the same as the phase angle between the resistance and the impedance. One can build a right triangle in which the resistance, R , is the horizontal leg, the reactance, X , is the vertical leg and the impedance, Z , is the hypotenuse. Then, the power factor is given by $\cos \phi = \frac{R}{Z}$. To calculate the reactance, we need to read a point off of the power factor curve given. I chose the point $(750\Omega, 0.6)$. From the point, we can determine the total impedance: $\frac{R}{Z} = 0.6 = \frac{750}{Z} \rightarrow Z = 1250\Omega$. Keep in mind that the total impedance will depend on the exact point chosen. After that, we can use the Pythagorean Theorem to find the reactance: $Z^2 = R^2 + X^2 \rightarrow (1250)^2 = (750)^2 + X^2 \rightarrow X = 1000\Omega$. Although the total impedance will vary, the result for the reactance will be about the same for any point on the curve.