

Algebra I Lesson for Order of Operations applied to the UIL Mathematics Contest

Lesson Goal: To have students learn how use the correct order of operations using questions from the UIL Mathematics Contest.

Time: 1 class period

Course: Algebra I

TEKS Addressed:

Algebra I(4)(A)(B) Foundations for functions. The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. The student is expected to: find specific function values, simplify polynomial expressions, transform and solve equations, and factor as necessary in problem situations; use the commutative, associative, and distributive properties to simplify algebraic expressions.

Overview:

The students will learn how to solve expressions using different mathematical symbols.

Materials Needed:

1. Scientific or Graphing Calculator (TI-83 or TI-84) (Optional)
2. Example problems (attached)
3. Practice problems (attached)
4. Answer Key (attached)

Procedures:

Provide students with the Example Problems paper.

After going over both examples, give the students the worksheet over practice problems.

Have students complete the practice problems the rest of the period or assign as homework. Answers need to be exact.

Assessment:

Assignment to solve expressions using different mathematical symbols

Order of Operations Example Problem 1

Evaluate:

$$(.25x\frac{1}{2} - 0.125) + (0.1666... - \frac{1}{3} \div 0.111...)$$

$$(\frac{1}{4}x\frac{1}{2} - \frac{1}{8}) + (\frac{1}{6} - \frac{1}{3} \div \frac{1}{9})$$

$$(\frac{1}{8} - \frac{1}{8}) + (\frac{1}{6} - \frac{1}{3}x9)$$

$$0 + (\frac{1}{6} - 3)$$

$$0 + (-2\frac{5}{6})$$

$$-2\frac{5}{6} = -2.8333...$$

One way to memorize the order of operations is to use the mnemonic

Please Innermost Parenthesis first.

Excuse Exponents

My Dear Multiply or Divide

Aunt Sally. Add or Subtract

This problem is a mixture of fractions and both terminating and non-terminating decimals. The easiest way to work this type of problem is to memorize the "onesies". The onesies are the decimal equivalents of the fractions from $\frac{1}{2}$ to $\frac{1}{16}$ with only one in the numerator. $\frac{1}{2} = .5$; $\frac{1}{3} = .333...$; $\frac{1}{4} = .25$; $\frac{1}{5} = .2$; $\frac{1}{6} = .1666...$; $\frac{1}{7} = .142857142857...$; $\frac{1}{8} = .125$; $\frac{1}{9} = .111...$; $\frac{1}{10} = .1$; $\frac{1}{11} = .0909...$; $\frac{1}{12} = .08333...$; $\frac{1}{13} = 0.076923076923...$; $\frac{1}{14} = 0.07142857142857...$; $\frac{1}{15} = .0666...$; and $\frac{1}{16} = .0625$.

Order of Operations Example Problem 2

Evaluate:

$$6! \div 6 - 6 \times 6 + 5$$

$$720 \div 6 - 36 + 5$$

$$120 - 31$$

$$89$$

Notice that we can collect $-36 + 5$ at the same time as the division of 720 and 6 . The $!$ math symbol does not mean that the six is excited, the $!$ symbol is called a factorial and is defined as $6 \times 5 \times 4 \times 3 \times 2 \times 1$. By definition $1! = 0! = 1$. This symbol is used in problems involving permutations and combinations as well as other problems.

Order of Operations Worksheet Problems.

Evaluate each of the following:

Show your work!

1) $10 \times 8 \div (6 + 4) - 2$

2) $9 + 1 \times 8 - 2 \div (7 - 3) \times (6 + 4) \div 5$

3) $2 + 4 \times (6 - 8) \div (7 - 5) \times 3 + 1$

4) $5! \div 5 - 5 \times 5 + 5$

5) $\frac{1}{32} \div 0.0625 + \left(\frac{1}{8} - 0.25\right) \times \frac{1}{2}$

6) $12 \times 8 \div (8 + 4) - 2$

7) $9 + 3 \times 8 - 2 \div (7 - 5) \times (6 + 4) \div 5$

8) $2 - 2 \times (6 - 8) \div (7 - 5) \times 2 + 2$

9) $7! \div 7 - 7 \times 7 + 7$

10) $\frac{1}{16} \div 0.0625 + \left(\frac{1}{8} - 0.125\right) \times \frac{1}{2}$

Order of Operations Worksheet Answers

1) 6

2) 16

3) -9

4) 4

5) 0.4375

6) 6

7) 31

8) 8

9) 678

10) 1

Algebra II Lesson for Quadratic Expressions applied to the UIL Mathematics Contest

Lesson Goal: To have students learn how to use factors to evaluate quadratic expressions using questions from the UIL Mathematics Contest.

Time: 1 class period

Course: Algebra II

TEKS Addressed:

Algebra II (8)(D)(10)(D) Quadratic and square root functions. The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to solve quadratic equations using the quadratic formula and solve quadratic equations and inequalities using graphs, tables, and algebraic methods. Rational functions. The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. The student is expected to determine the solutions of rational equations using graphs, tables, and algebraic methods.

Overview:

The students will learn how to solve quadratic expressions using basic factoring techniques.

Materials Needed:

1. Example problem (attached)
2. Practice problems (attached)
3. Answer Key (attached)

Procedures:

Provide students with the Example Problems paper.

After going over the example, give the students the worksheet over practice problems.

Have students complete the practice problems the rest of the period or assign as homework. Answers need to be exact.

Assessment:

Practice problems to use factors to evaluate quadratic expressions

Quadratic Expressions Example Problem

Simplify:

$$\left(\frac{2x - 2y}{x^2 + 2xy + y^2} \right) \div \left(\frac{x - y}{x + y} \right) \times \left(\frac{x^2 - y^2}{2} \right)$$

$$\left(\frac{2(x - y)}{(x + y)^2} \right) \times \left(\frac{x + y}{x - y} \right) \times \left(\frac{(x + y)(x - y)}{2} \right)$$

$$x - y$$

The first step is to factor each quadratic and to change division to multiplication by inverting the fraction. It is very helpful to memorize the following basic factoring definitions (Even the cubic's).

$$(A + B)^2 = A^2 + 2AB + B^2$$

$$(A - B)^2 = A^2 - 2AB + B^2$$

$$A^2 - B^2 = (A + B)(A - B)$$

$$A^3 - B^3 = (A - B)(A^2 + AB + B^2)$$

$$A^3 + B^3 = (A + B)(A^2 - AB + B^2)$$

In order to factor other quadratic expressions, use the factors of the product of the constant term and the quadratic term to find the coefficient of the linear term. (A-C Method).

Factor: $4x^2 + 3x - 22$; $4(-22) = -88$ Factors of -88: $(-1, -2, -4, -8, 11, 22, 44, 88)$; Of these pairs, $(-8$ and $11)$ give us the proper product and sum.

$$4x^2 - 8x + 11x - 22$$

$$4x(x - 2) + 11(x - 2)$$

$$(4x + 11)(x - 2)$$

Quadratic Expressions Worksheet Problems.

Evaluate each of the following:

Show your work!

$$1) \quad \left(\frac{x-2}{x^2+2x+1} \right) \div \left(\frac{x^2-4}{x+1} \right)$$

$$2) \quad \left(\frac{x-3}{x^2+4x+4} \right) \div \left(\frac{x^2-9}{x+2} \right)$$

$$3) \quad \left(\frac{x-4}{x^2+6x+9} \right) \div \left(\frac{x^2-16}{x+3} \right)$$

$$4) \quad \left(\frac{x^2+5x+6}{x^2-x-20} \right) \div \left(\frac{x^2+x-2}{x^2+3x-4} \right)$$

$$5) \quad \left(\frac{x^2+5x-6}{x^2-x-30} \right) \div \left(\frac{x^2+x-2}{x^2+4x-5} \right)$$

$$6) \quad \left(\frac{4x^2+3x-22}{x^2+x-6} \right) \div \left(\frac{4x+11}{x+3} \right)$$

$$7) \quad \left(\frac{3x-3y}{x^2+2xy+y^2} \right) \div \left(\frac{2(x-y)}{x+y} \right) \times \left(\frac{x^2-y^2}{6} \right)$$

Quadratic Expressions Answers

$$1) \quad \frac{1}{(x+2)(x+1)} \text{ or } \frac{1}{x^2 + 3x + 2}$$

$$2) \quad \frac{1}{(x+2)(x+3)} \text{ or } \frac{1}{x^2 + 5x + 6}$$

$$3) \quad \frac{1}{(x+3)(x+4)} \text{ or } \frac{1}{x^2 + 7x + 12}$$

$$4) \quad \frac{x+3}{x-5}$$

$$5) \quad \frac{(x+6)(x-1)}{(x-6)(x+2)} \text{ or } \frac{x^2 + 5x - 6}{x^2 - 4x - 12}$$

$$6) \quad 1$$

$$7) \quad x - y$$

Pre-Calculus Lesson for Trigonometric Functions applied to the UIL Mathematics Contest

Lesson Goal: To have students learn how to transform and develop equations using questions from the UIL Mathematics Contest.

Time: 1 class period

Course: Pre-Calculus

TEKS Addressed:

Pre-Calculus (1)(A)(2)(A)The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions. The student is expected to describe parent functions symbolically and graphically, including $f(x) = x^n$, $f(x) = \frac{1}{x}$, $f(x) = \log_a x$, $f(x) = \frac{1}{x}$, $f(x) = e^x$, $f(x) = |x|$, $f(x) = ax$, $f(x) = \sin x$, $f(x) = \arcsin x$, etc. The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems. The student is expected to apply basic transformations, including $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(b \cdot x)$, and compositions with absolute value functions, including $|f(x)|$, and $f(|x|)$, to the parent functions.

Overview:

The students will learn how to solve trigonometric function problems on the UIL Mathematics test.

Materials Needed:

1. Example problems (attached)
2. Practice problems (attached)
3. Answer Key (attached)

Procedures:

Provide students with the Example Problems paper.

After going over the examples, give the students the worksheet over practice problems.

Have students complete the practice problems the rest of the period or assign as homework.

Assessment:

Practice assignment over how to transform and develop equations

Trigonometric Functions applied to the UIL Mathematics Contest Examples

$f(x) = A\sin(Bx - C) + D$ where

A is the amplitude of the function where the amplitude is $\frac{1}{2}$ the distance between the maximum and the minimum value.

Since the period of the parent function $y = \sin(x)$ is 2π , the period of the standard function is $\left| \frac{2\pi}{B} \right|$

C is the horizontal displacement.

D is the vertical displacement.

Example 1

A sine function, $f(x)$ has a maximum value of 7, a minimum value of 3, a period of $\frac{\pi}{3}$, a horizontal phase shift of $\frac{\pi}{2}$. Determine the value of $f\left(\frac{\pi}{3}\right)$. (nearest tenth)

$$A = \frac{1}{2}(7-3) = 2; \quad B = \left| \frac{2\pi}{\frac{\pi}{3}} \right| = 6; \quad C = \frac{\pi}{2}; \quad D = \frac{1}{2}(7+3) = 5;$$

$$f\left(\frac{\pi}{3}\right) = 2\sin\left(6\left(\frac{\pi}{3}\right) - \frac{\pi}{2}\right) + 5 = 3.$$

Example 2

The general form of the sine function can be written as $y = A\sin[B(x-C)] + D$. The graph of the parent sine function, $y = \sin(x)$, where $0 < x < 2\pi$, will be totally in quadrant I by setting which of the following:

- A) $A > 1$ B) $A < 1$ C) $C < -2\pi$ D) $D < 1$ E) $D > 1$

Since x is positive and there is no horizontal shift, the function can only be in quadrant I or IV. The only way the function will remain in quadrant I is if it is shifted vertically up by more than 1. D is the vertical shift so E) is the answer.

Trigonometric Functions applied to the UIL Mathematics Contest Problems

- 1) A sine function, $f(x)$ has a maximum value of 3, a minimum value of -5, a period of $\frac{\pi}{2}$, a horizontal phase shift of $\frac{\pi}{4}$. Determine the value of $f\left(\frac{\pi}{6}\right)$. (nearest tenth)

$$A = \frac{1}{2} (3 - (-5)) = 4; B = \left| \frac{2\pi}{\frac{\pi}{2}} \right| = 4; C = \frac{\pi}{4}; D = \frac{1}{2} (3 + (-5)) = -1;$$

$$f\left(\frac{\pi}{6}\right) = 4\sin\left(4\left(\frac{\pi}{6}\right) - \frac{\pi}{4}\right) - 1 = ?$$

- 2) A sine function, $f(x)$ has a maximum value of 11, a minimum value of 3, a period of $\frac{\pi}{2}$, a horizontal phase shift of $\frac{\pi}{6}$. Determine the value of $f\left(\frac{\pi}{3}\right)$. (nearest tenth)

- 3) A **cos** function, $f(x)$ has a maximum value of 11, a minimum value of 3, a period of $\frac{\pi}{2}$, a horizontal phase shift of $\frac{\pi}{6}$. Determine the value of $f\left(\frac{\pi}{3}\right)$. (nearest tenth)

- 4) The general form of the sine function can be written as $y = A\sin[B(x-C)] + D$. The graph of the parent sine function, $y = \sin(x)$, where $0 < x < 2\pi$, will be totally in quadrant IV by setting which of the following: (Unless defined $A=B=1$ and $C=D=0$ and more than one answer may be necessary).

A) $A > 1$ B) $A < 1$ C) $C < -2\pi$ D) $D < 1$ E) $D > 1$

- 5) The general form of the sine function can be written as $y = A\sin[B(x-C)] + D$. The graph of the parent sine function, $y = \sin(x)$, where $0 < x < 2\pi$, will be totally in quadrant II by setting which of the following: (Unless defined $A=B=1$ and $C=D=0$ and more than one answer may be necessary).

A) $A > 1$ B) $A < 1$ C) $C < -2\pi$ D) $D < 1$ E) $D > 1$

Trigonometric Functions applied to the UIL Mathematics Contest Answers

- 1) 2.9
- 2) 5.0
- 3) 3.5
- 4) D
- 5) C & E (Shift the function both left and up)

Pre-Calculus Lesson for Logarithmic Expressions applied to the UIL Mathematics Contest

Lesson Goal: To have students learn how to transform and develop Expressions using questions from the UIL Mathematics Contest.

Time: 1 class period

Course: Pre-Calculus

TEKS Addressed:

Pre-Calculus (1)(A)(2)(A)The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, power (including radical), exponential, logarithmic, trigonometric, and piecewise-defined functions. The student is expected to describe parent functions symbolically and graphically, including $f(x) = x^n$, $f(x) = \frac{1}{x}$, $f(x) = \log_a x$, $f(x) = \frac{1}{x}$, $f(x) = e^x$, $f(x) = |x|$, $f(x) = ax$, $f(x) = \sin x$, $f(x) = \arcsin x$, etc. The student interprets the meaning of the symbolic representations of functions and operations on functions to solve meaningful problems. The student is expected to apply basic transformations, including $a \cdot f(x)$, $f(x) + d$, $f(x - c)$, $f(b \cdot x)$, and compositions with absolute value functions, including $|f(x)|$, and $f(|x|)$, to the parent functions.

Overview:

The students will learn how to solve Logarithmic Expression problems on the UIL Mathematics test.

Materials Needed:

1. Example problems (attached)
2. Practice problems (attached)
3. Answer Key (attached)

Procedures:

Provide students with the Example Problems paper.

After going over the examples, give the students the worksheet over practice problems.

Have students complete the practice problems the rest of the period or assign as homework.

Assessment:

Practice problems to transform and develop Expressions

Logarithmic Expressions applied to the UIL Mathematics Contest Examples

$\log_b a = x$ may be rewritten as $b^x = a$.

$$\log a = \log_{10} a$$

$$\ln a = \log_e a$$

$$\log_b 1 = 0$$

$$\log_b a = \frac{\log a}{\log b}$$

$$\log xy = \log x + \log y$$

$$\log \frac{x}{y} = \log x - \log y$$

$$\log x^y = y \log x$$

Example 1

If $\log_a(4) = P$, $\log_a(3) = Q$, and $\log_a(2) = R$, then $\log_a(1.5)$ can be written as:

- A) PQR B) P+R+Q C) $\frac{RQ}{P}$ D) R+Q-P E) $\frac{Q+R}{P}$

Answer D) R+Q-P

$$\log_a(2) + \log_a(3) - \log_a(4) = \log_a \left[\frac{(2)(3)}{4} \right] = \log_a(1.5)$$

Example 2

Simplify: $[\log_a(6k) - \log_a(4)] + [\log_a(3) - \log_a(5k)]$

- A) $\log_a(9) - \log_a(10)$ B) $2\log_a(22.5k)$ C) $\log_a(1.6)$
D) $\log_a(k) - \log_a(1)$ E) $\log_a(11k)$

Answer A)

$$\log_a \left[\frac{6k}{4} \right] + \log_a \left[\frac{3}{5k} \right] = \log_a \left[\frac{6k(3)}{4(5)k} \right] = \log_a \left[\frac{9}{10} \right] = \log_a(9) - \log_a(10)$$

Logarithmic Expressions applied to the UIL Mathematics Contest Problems

1) Simplify: $\log_b(2xy) + \log_b\left(\frac{x}{4y}\right) - \log_b(4x^2)$

2) Simplify: $\log_b(3xy) - \log_b\left(\frac{x}{4y}\right) + \log_b(3x)$

3) Simplify: $\log_b(8wt) - 2\log_b\left(\frac{w}{4t}\right) + \log_b(4w)$

4) Simplify: $-3\log_b(2fg) + \log_b\left(\frac{12g}{4f}\right) - \log_b(4f^2)$

5) Simplify: $[\log_a(7w) - \log_a(5)] - [\log_a(14) - \log_a(10w)]$

6) Simplify: $[\log_a(9t) - \log_a(4u)] - [\log_a(3u) - \log_a(8t)]$

7) Simplify: $[\log_a(7h^2) - \log_a(3g)] + [\log_a(15g) - \log_a(5h)]$

8) If $\log_a(20) = P, \log_a(8) = Q,$ and $\log_a(10) = R,$ then $\log_a(16)$ can be written as:

A) PQR B) P - R+Q C) $\frac{RQ}{P}$ D) R+Q-P E) $\frac{Q+P}{R}$

9) If $\log_a(14) = P, \log_a(7) = Q,$ and $\log_a(21) = R,$ then $\log_a(42)$ can be written as:

A) PQR B) P+R - Q C) $\frac{RP}{Q}$ D) R+Q-P E) $\frac{P+R}{Q}$

10) If $\log_a\left(\frac{4}{5}\right) = P, \log_a\left(\frac{3}{4}\right) = Q,$ and $\log_a\left(\frac{2}{3}\right) = R,$ then $\log_a\left(\frac{2}{5}\right)$ can be written as:

A) PQR B) P+R- Q C) $\frac{RP}{Q}$ D) R+Q+P E) $\frac{P+R}{Q}$

Logarithmic Expressions applied to the UIL Mathematics Contest Answers

1) $-\log_b(8)$

2) $\log_b(36xy^2)$

3) $\log_b(512t^3)$

4) $\log_b\left(\frac{3}{64f^6g^2}\right)$ or $\log_b(3) - \log_b(64f^6g^2)$

5) $\log_b(w^2)$ or $2\log_b(w)$

6) $\log_b\left(\frac{6t^2}{u^2}\right)$

7) $\log_b(7h)$

8) B)

9) B)

10) D)

Pre-Calculus Lesson for Vectors applied to the UIL Mathematics Contest

Lesson Goal: To have students learn to use vectors as models using questions from the UIL Mathematics Contest.

Time: 1 class period

Course: Pre-Calculus

TEKS Addressed:

Pre-Calculus (6)(A)(B) The student uses vectors to model physical situations. The student is expected to use the concept of vectors to model situations defined by magnitude and direction; and analyze and solve vector problems generated by real-life situations.

Overview:

The students will learn how to solve Vector problems on the UIL Mathematics test.

Materials Needed:

1. Example problems (attached)
2. Practice problems (attached)
3. Answer Key (attached)

Procedures:

Provide students with the Example Problems paper.

After going over the examples, give the students the worksheet over practice problems.

Have students complete the practice problems the rest of the period or assign as homework.

Assessment:

Assignment to use vectors as models

Vector Problems applied to the UIL Mathematics Contest Examples

Note: All of the problems presented are from recent UIL mathematics tests. There may exist alternate methods for solving these problems without vectors. Every problem may be solved by using vectors.

Example 1

Ima Hyker is camped out in the forest. After breakfast she walks 16 km on a bearing of 327° then she walks 12 km on a bearing of 237° . What bearing will Ima have to walk to head straight back to her camp? (nearest degree)

Use component vectors to solve:

$r\cos\theta$ = horizontal component

$r\sin\theta$ = vertical component

$$16\cos 327^\circ + 12\cos 237^\circ = 6.88$$

$$16\sin 327^\circ + 12\sin 237^\circ = -18.77$$

$$\langle 6.88, -18.77 \rangle$$

$$\tan^{-1}\left(\frac{-18.77\dots}{6.88\dots}\right) = -69.86\dots^\circ$$

So Ima is 20 km from camp at a bearing of -69.86° .

$$\sqrt{(6.88\dots)^2 + (-18.77\dots)^2} = 20$$

To get back to camp Ima must head in a direction of $-69.86^\circ + 180^\circ = 110.14^\circ \approx 110^\circ$

Example 2

An eagle looks down from his nest on a cliff and spots a rodent for dinner. The rodent is 120 feet away from the base of the cliff and the angle of depression from the eagle to the rodent is $14^\circ 45'$. How far is the rodent from the eagle? (nearest inch)

The horizontal component of the problem is 120 feet.

$$14^\circ 45' = 14 \frac{45}{60}^\circ = 14.75^\circ$$

$$r\cos(14.75) = 120$$

$$r = 1489 \text{ in} = 41 \text{ yds } 1' 1''$$

Vector Problems applied to the UIL Mathematics Contest Problems

- 1) Saul D. Wood is building a wheelchair ramp that is 18 feet long from the ground to the top of a porch. The height of the porch is 18 inches. The approximate angle of inclination of the ramp is:
- 2) On the map legend, the map scale shows inches being approximately 125 miles. The distance from Perryton, Texas to Del Rio Texas on the map is $8\frac{1}{2}$ inches. Approximately how many miles is it from Perryton to Del Rio?
- 3) The measure of the acute angle between two lines with slopes of 1 and -3 is: (nearest degree).
- 4) Willie Landett looks down from the cockpit of his jet at an angle of depression of $52^{\circ}20'$ and sees an aircraft carrier. The jet is 700 feet above level of the carrier's deck. How far away is the carrier? (nearest foot)
- 5) On the map legend, $\frac{5}{8}$ of an inch represents 125 miles. The distance from El Paso to Tyler is approximately 750 miles. How far apart are they on the map?
- 6) Bea Safe installs a security sensor light over her back door. The sensor is 8 feet above the driveway. What will the angle of depression need to be in order for the security light to cover all 40 feet of the driveway? (nearest minute)
- 7) Mark Despot is staking out a triangular garden. After placing his first stake, he steps off 50 feet on a bearing of 75° and places a second stake. Then he walks 75 feet on a bearing of 100° and places the third stake. What is the distance from the third stake to the first stake? (nearest foot)
- 8) Captains Chip Ahoy and Rusty Nale leave port in their respective ships at 2p.m. Chip's ship sails on a bearing of 40° traveling at a speed of 10 mph. Rusty's ship sails on a bearing of 140° traveling at a speed of 15 mph. How far apart will the ships be at 5 p.m.? (nearest mile)

Vector Problems applied to the UIL Mathematics Contest Answers

1) 4.8°

2) 586 miles

3) 63°

4) 540 ft

5) $3\frac{3}{4}in.$

6) $11^\circ 10'$

7) 122 ft

8) 58 miles