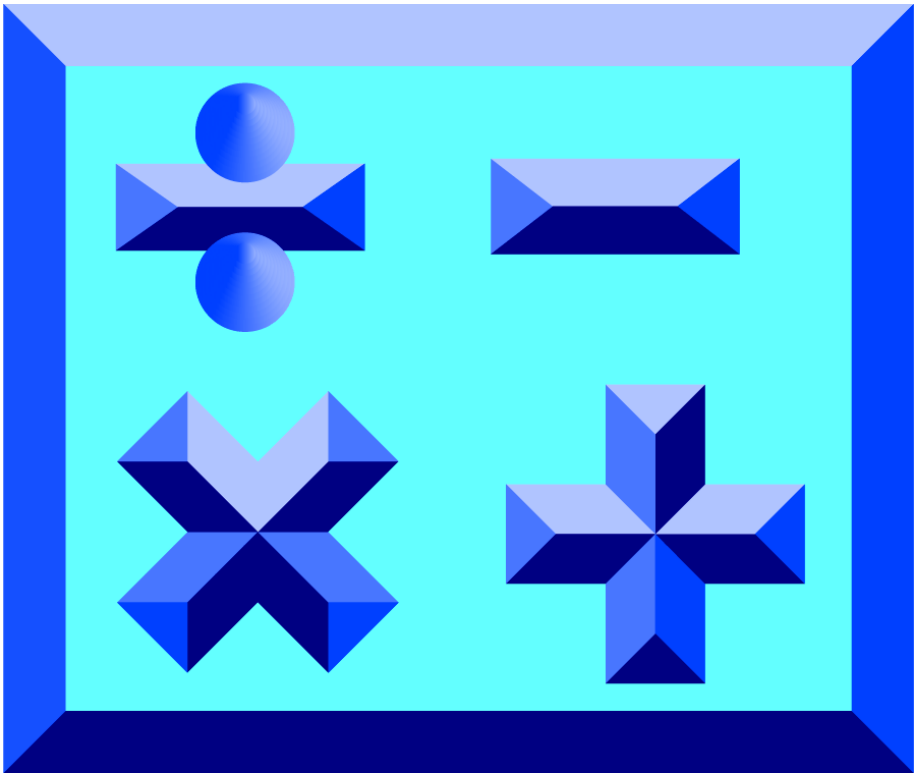
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2018



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YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $\sqrt[3]{1728} \div (16)^{\frac{1}{2}} + 8 \times (2)^{-1} - 4$

(A) 8 **(B) 3** **(C) 1.5** **(D) — 2** **(E) — 19**

2. **Two and one-fourth million is added to three hundred twenty thousand five hundred. One million one thousand one hundred is subtracted from the sum. The difference is divided by eleven. The quotient is truncated to the units place. Which digit appears the most in the final results?**

(A) 7 **(B) 6** **(C) 4** **(D) 2** **(E) 1**

3. **Find the average of the arithmetic mean, the median, and the mode of these quiz grades: 75, 95, 75, 100, 95, 80, 75, & 70. (nearest whole number)**

(A) 79 **(B) 80** **(C) 81** **(D) 83** **(E) 85**

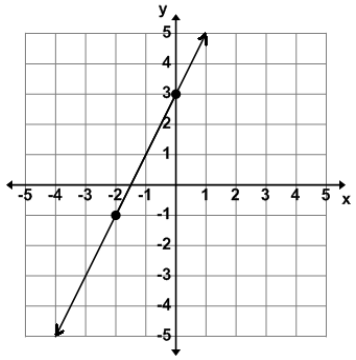
4. **Let U (universal set) = {u, i, l, m, a, t, h, b}, B = {b, u, i, l, t}, and T = {t, h, u, m, b}. Let $I = (B \cap T)^C$. Set I contains how many distinct elements?**

(A) 3 **(B) 4** **(C) 5** **(D) 6** **(E) 7**

5. **If $(3x + 1)(x - 3)(2x) = ax^3 + bx^2 + cx + d$ then $a + b + c + d =$ _____.**

(A) — 20 **(B) — 16** **(C) — 4** **(D) 3** **(E) 6**

6. **A line parallel to the line shown through the point (1, — 1) has x-intercept at point (a, b) and y-intercept at point (c, d). Find $a + b + c + d$.**



(A) — 1.5 **(B) 2** **(C) 0.5** **(D) — 4.5** **(E) 4**

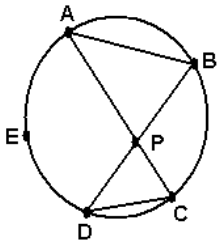
7. **Which of the following sets of numbers is closed under multiplication and addition?**
I. Primes II. Integers III. Wholes IV. Rationals

(A) I, II, & III (B) II, III, & IV (C) I, III, & IV (D) all of them (E) None of them

8. **Max Whale likes to mix his regular blend coffee with a boost blend coffee at a ratio of 3 to 1. The regular blend sells for $11.00 per pound and the boost blend sells for $8.00 per pound. Find the cost per pound of Max's special mixture of regular blend and boost blend. (nearest cent)**

(A) $14.25 **(B) $6.67** **(C) $11.67** **(D) $4.75** **(E) $10.25**

9. $\overline{AB}$, $\overline{AC}$, $\overline{BD}$, and $\overline{CD}$ are chords of circle O and point E lies on circle O. Find $m\overset{\frown}{AED}$ given $m\angle BPC = 95°$ and $m\angle BAP = 25°$.



(A) 140° **(B)** 70° **(C)** 72.5° **(D)** 120 ° **(E)** 165°

10. $\angle A$ and $\angle B$ are supplementary angles with $m\angle A = 5x - 4$ and $m\angle B = 3x + 2$. Find the absolute value difference in the measures of $\angle A$ and $\angle B$.

(A) 22.75° **(B)** 87° **(C)** 17° **(D)** 43.5° **(E)** 39.5°

11. Les Square increased the length of two opposite sides of a square by 20%, and decreased the other two opposite sides by 50%. What percent of the area of the original square is the area of the new rectangle?

(A) 30% **(B)** 40% **(C)** 60% **(D)** 70% **(E)** 80%

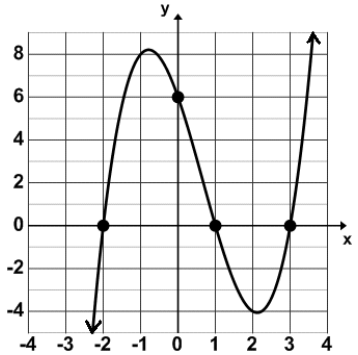
12. If $\frac{x+5}{2x-1} + \frac{Ax+B}{3x+2} = \frac{-7x^2+30x+6}{6x^2+x-2}$, where A and B are constants, then $A + B$ equals:

(A) 91 **(B)** 5 **(C)** 4 **(D)** 1 **(E)** -1

13. Let $f(x) = 2x - 1$ and $g(x) = 2 - 3x$ and $h(x) = x + 3$. Find $g\Big(h\big(f(1-x)\big)\Big)$.

(A) $6x + 2$ **(B)** $6 - 10x$ **(C)** $5 - 6x$ **(D)** 4 **(E)** $6x - 10$

14. The graph of $f(x) = Ax^3 + Bx^2 + Cx + D$ is shown here. Find $A + B + C + D$.



(A) -1 **(B)** 0 **(C)** 1 **(D)** 4 **(E)** 10

15. Les Qwik and Lotta Speed worked together to finish their research project in 12.5 hours. Lotta works 2.5 times faster than Les. How long would it have taken Lotta to do the project alone?

(A) 17.5 hrs **(B)** 15 hrs **(C)** 10 hrs **(D)** 7.5 hrs **(E)** 5 hrs

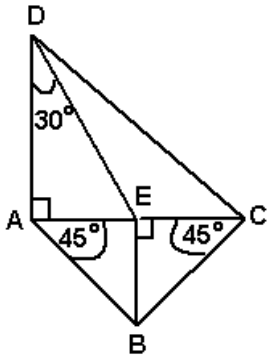
16. If you start at ($\frac{7\pi}{2}$, 0) on the x-axis and travel horizontally 15.7 radians to the left, how many times will you cross the graph of y = 2sin(3x)?

(A) 8 (B) 11 (C) 14 (D) 15 (E) 16

17. Given: f(x) = 3cos[4π(x + 1)] — 2. Find the sum of the numeric values of the period and the vertical displacement.

(A) — 1.5 (B) — 1 (C) 0 (D) 2 (E) 3.5

18. Find DC if AE = 3".



(A) $3\sqrt{7}$ in (B) $2\sqrt{10}$ in (C) $3\sqrt{8}$ in (D) $4\sqrt{3}$ in (E) $3\sqrt{13}$ in

19. How many negative real roots will $x^5 + x^4 - 2x^3 + x^2 - 1 = 0$ have?

(A) 3 or 1 (B) 4, 2, or 0 (C) 1 (D) 0 (E) 2 or 0

20. Which of the following is true about the function f(x) = $\frac{x^2+4}{x^3-3}$?

I. f(x) is odd II. f(x) is even III. f(x) has 3 asymptotes.

(A) I & III (B) II & III (C) I only (D) III only (E) none of these

21. Meagan Money invested some money in the stock market. Her investment increased 8% by the end of the first year, decreased 2% by the end of the second year, and increased 12% by the end of the third year. What was Meagan's average rate of return over the three year period? (nearest tenth)

(A) 6.2% (B) 6.0% (C) 5.8% (D) 5.6% (E) 5.5%

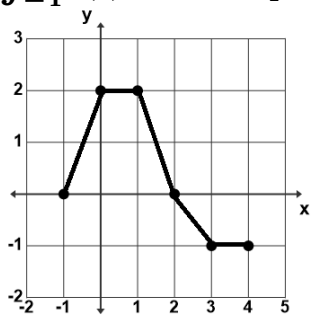
22. The vertex of a parabola is located at (3, 1) and the focus is located at (3, 3). Find the directrix of the parabola.

(A) y = 5 (B) y = 1 (C) y = 0 (D) y = $-\frac{1}{3}$ (E) y = — 1

23. Let f(x) = $\frac{5x-2}{4+3x}$. Find f′(— 2).

(A) 6.5 (B) 6 (C) 6 (D) 14 (E) 13

24. Find the value of $\int_{-1}^{4} f(x)\,dx$ for the piecewise-linear function f, $-1 \le x \le 4$, shown below?



(A) 1 **(B) 2.5** **(C) 4** **(D) 5.5** **(E) 8**

25. In a triple play game, Willie When performs three tasks. He flips a quarter, and success would be heads. He rolls a single die, and success would be a six. He picks a card from a standard deck of cards, and success would be picking a heart. If any of these task are successful, He will win the game. What is the probability he will win? (nearest whole percent)

(A) 2% **(B) 31%** **(C) 92%** **(D) 69%** **(E) 48%**

26. If two dice are tossed, what is the probability that the sum of the faces is a prime number?

(A) $\frac{5}{36}$ **(B)** $\frac{13}{36}$ **(C)** $\frac{7}{9}$ **(D)** $\frac{7}{12}$ **(E)** $\frac{5}{12}$

27. The *Blow Upp* balloon company package 6 balloons per pack. The company has red, blue, white, pink, yellow, green, and magenta colored balloons. How many different packs of 6 balloons can they package?

(A) 12,012 **(B) 5,040** **(C) 924** **(D) 720** **(E) 42**

28. Leonardo Pisano Bigollo was an Italian mathematician who referenced and made known which of the following special sequences of numbers to Western mathematics?

(A) {1,1,2,3,5,8,...} **(B) {1,3,6,10,15,21,...}** **(C) {0,2,4,6,8,10,...}**
(D) {2,3,5,7,11,13,...} **(E) {1,5,25,125,625,3125,...}**

29. Which of the following numbers is an abundant, happy, and lucky number?

(A) 28 **(B) 31** **(C) 44** **(D) all of these** **(E) none of these**

30. Find k if GCF(48, k) = 8 and LCM(48, k) = 336.

(A) 56 **(B) 64** **(C) 72** **(D) 80** **(E) 88**

31. $\{(x, y) \mid x,y \in \{\text{Integers}\}, -10 \le x \le 10, \text{ and } -10 \le y \le 10\}$ is the solution set of $2x + 5y = 10$. How many such ordered pairs exist?

(A) 2 **(B) 3** **(C) 4** **(D) 5** **(E) 6**

32. Which of the following points of concurrency are always on the exterior of an obtuse triangle?
(1) circumcenter (2) centroid (3) incenter (4) orthocenter

(A) 1 only (B) 1, 3, & 4 (C) 1 & 4 (D) 2 & 3 (E) 4 only

33. An elongated square pyramid is a nonahedron. It has 9 faces and 9 vertices. How many edges does it have?

(A) 18 (B) 16 (C) 20 (D) 9 (E) 11

34. Find C if the remainder when $(3x^3 + 2x^2 - x + C) \div (x + 1)$ is 4.

(A) -1 (B) 0 (C) 1 (D) 2 (E) 4

35. Find $a + b + c + d$ given the Fibonacci characteristic sequence: a, 2, b, c, 20, d, 51, ...

(A) 58 (B) 80 (C) 52 (D) 47 (E) 73

36. Given the function f(x) = sin x, find the slope of the secant line between x = 0 and x = $\frac{\pi}{2}$.

(A) 0 (B) $-\frac{\pi}{2}$ (C) $\frac{2}{\pi}$ (D) π (E) no slope

37. Ester Bunnee had a box of chocolate eggs. She hid half of them in the yard for the big hunt. Then she put two of the remaining eggs in her room for a late night snack. The remaining six eggs were put in the refrigerator for a later day. How many chocolate eggs were in the original box?

(A) 32 (B) 28 (C) 24 (D) 20 (E) 16

38. Sir Benjamin Hall was looking at the circular face of the famous *Big Ben* clock. He noted that the time was 5:43 pm. What was the measure of the acute angle formed by the big hand and the little hand at that time?

(A) 65° (B) 75.5° (C) 85° (D) 86.5° (E) 89.5°

39. Chip Shought hit his golf ball over a pond onto the edge of the green. He had to walk around the pond to his ball. He walked 70 yards on a bearing of 250° from the tee. Then he walked 90 yards on a bearing of 50° to his ball. What was the straight line distance from the tee to his ball? (nearest yard)

(A) 29 yds (B) 34 yds (C) 30 yds (D) 35 yds (E) 44 yds

40. I. C. Itt spotted a plane flying over his house. He noted that the angle of elevation from him to the plane was 32°40' and he was 1,530 meters from his house. Using this information I. C. was able to determine the altitude of the plane. What was the altitude of the plane? (nearest meter)

(A) 2,386 meters (B) 1,287 meters (C) 981 meters (D) 971 meters (E) 826 meters

41. Mei Chado is 5' 4" tall. She is walking at a rate of 3 ft/sec toward a street light that is 16 feet tall. At what rate is the tip of her shadow moving? (nearest tenth)

(A) 4.5 ft/sec (B) 4.0 ft/sec (C) 3.5 ft/sec (D) 3.0 ft/sec (E) 1.5 ft/sec

42. 14 out of 17 Millersviewites have spouses. 4 out of 6 Millerviewites own at least 3 acres and a travel trailer. What is the probability that a Millersviewite has a travel trailer given that a Millersviewite has a spouse? (nearest whole percent)

(A) 83% (B) 81% (C) 78% (D) 67% (E) 24%

43. Anthony and Chuck take three number sense tests. Anthony is twice as likely to score higher than Chuck. What are the odds that Anthony scores higher on all three tests? Due to an unknown tiebreaker, there are no ties.

(A) $\frac{8}{11}$ (B) $\frac{11}{27}$ (C) $\frac{8}{19}$ (D) $\frac{1}{3}$ (E) $\frac{1}{9}$

44. If $12x^2 + ax - 5 = (bx - 5)(2x + c)$ then abc = _____.

(A) 11 (B) 3 (C) 1 (D) — 15 (E) — 24

45. Let $e^{(2x-3)} = 4e^{(5x+6)}$. Find $e^{(x)}$. (nearest hundredth)

(A) .03 (B) 0.22 (C) 0.42 (D) — 2.23 (E) — 3.46

46. The set of Lucas numbers is {1,3,4,7,11, ...}, where $L_1 = 1$. The set of Fibonacci numbers is {1,1,2,3,5, ...}, where $F_1 = 1 = F_2$. If $L_{10} = F_x + F_y$, where $y > x$, then y is _____.

(A) 8 (B) 9 (C) 10 (D) 11 (E) 12

47. Let f(x) = ax + 4 and g(x) = bx — 1, where a and b are positive integers. Find a + b if f(g(x)) = g(f(x)).

(A) 5 (B) 4 (C) 3 (D) 2 (E) 1

48. Let $f(x) = 4x^2 - 4x + 1$. The tangent to f(x) at (x, y) is parallel to y = 4x — 2. Find x + y.

(A) 4 (B) 2 (C) 1 (D) 0 (E) — 1

49. In honor of Valentines day, let $x = 2 + \cfrac{14}{2 + \cfrac{14}{2 + \cfrac{14}{2 + \cfrac{14}{2 + \ldots}}}}$. Find x. (nearest tenth)

(A) 4.9 (B) 4.7 (C) 3.9 (D) 2.7 (E) 2.1

50. If the following patten continues, determine which of the following numbers will be in row 10.

1	row 0
1 1	row 1
1 2 1	row 2
1 3 3 1	row 3
1 4 6 4 1	row 4
1 5 10 10 5 1	row 5
...	...

(A) 84 (B) 110 (C) 126 (D) 215 (E) 252

51. The fraction $\frac{30}{\sqrt{3}+\sqrt{5}+\sqrt{8}}$ can be written as $a\sqrt{30}+b\sqrt{3}+c\sqrt{5}+d\sqrt{8}$. Find $a+b+c+d$.

(A) 15 (B) 10 (C) 8 (D) 6 (E) 3

52. Let f(x) = $\sqrt{6-\sqrt{2x+7}}$. The domain of f(x) is $\{x \mid p \le x \le q\}$. Find $\frac{P+Q}{2}$.

(A) -3.5 (B) -3 (C) 5.5 (D) 6 (E) 9

53. Points P (— 1, 1), Q(3, 5), R(17, 1), and S(x, y) are the coordinates of the vertices of a parallelogram. How many possible coordinates of S exist for the fourth vertex?

(A) 1 (B) 2 (C) 3 (D) 4 (E) infinitely many

54. Given: $9x - 6y = 21$ and $6x - 4y = k$. Find the value of k such that this system of equations has an infinite number of solutions.

(A) 31.5 (B) 24 (C) 21 (D) 14 (E) 13.5

55. If x is in QIII then $\frac{1-\cos(2x)}{\sin(2x)}$ = tan kx and k equals:

(A) $1\frac{1}{2}$ (B) 1 (C) $\frac{2}{3}$ (D) $\frac{1}{2}$ (E) $\frac{1}{3}$

56. An "*emirp*" number is a prime number that becomes a new prime number when the digits are reversed. Single digit primes and palindromic primes cannot be *emirp* numbers. How many prime numbers less than 20 are considered to be *emirp* numbers?

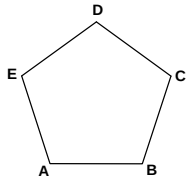
(A) 8 (B) 4 (C) 3 (D) 2 (E) 0

57. Let f(x) = $\begin{cases} -x+5 & x < -2 \\ x^2+1 & -2 \le x \text{ and } x \le 1 \\ 2x^3-1 & 1 \le x \end{cases}$. Which of the following is/are true?

1. f is continuous at — 2 2. f is differentiable at x = 1 3. f has a local minimum at x = 0

(A) 1 & 3 (B) 2 & 3 (C) 2 only (D) 3 only (E) 1, 2, & 3

58. Given the regular pentagon shown, find BC with AC + AD + BE + BD + CE = 44.5". (nearest tenth)



(A) 8.9" **(B) 5.2"** **(C) 8.1"** **(D) 4.5"** **(E) 5.5"**

59. Let $(131_b) \times 3_b = k_b$, where k_b is a 3-digit number. Find b if $k_b = 1323_4$.

(A) 4 **(B) 5** **(C) 6** **(D) 7** **(E) 8**

60. If P, Q, and R are different digits, then the largest possible three-digit sum for PPP + QP + P = ? has which of the following forms?

(A) PPQ **(B) PQR** **(C) QQP** **(D) QQR** **(E) RRQ**

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation B ● 2018
Answer Key

1. B	**21. C**	**41. A**
2. D	**22. E**	**42. B**
3. A	**23. A**	**43. C**
4. C	**24. B**	**44. E**
5. B	**25. D**	**45. A**
6. A	**26. E**	**46. D**
7. B	**27. C**	**47. D**
8. E	**28. A**	**48. B**
9. A	**29. E**	**49. A**
10. E	**30. A**	**50. E**
11. C	**31. D**	**51. D**
12. E	**32. C**	**52. C**
13. E	**33. B**	**53. C**
14. B	**34. E**	**54. D**
15. A	**35. A**	**55. B**
16. D	**36. C**	**56. D**
17. A	**37. E**	**57. D**
18. A	**38. D**	**58. E**
19. E	**39. B**	**59. B**
20. E	**40. C**	**60. D**