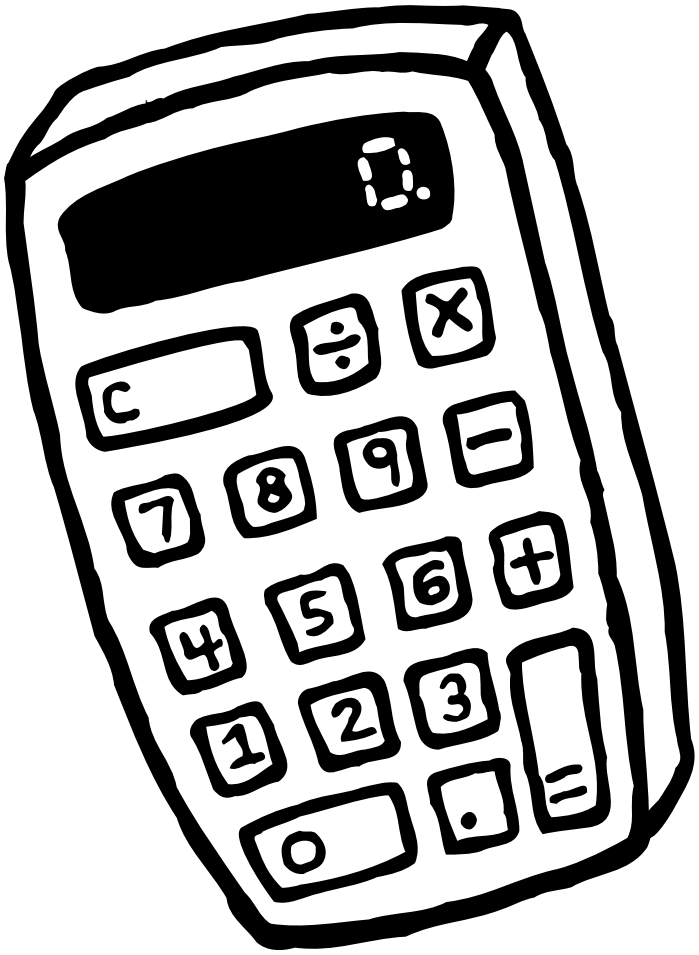
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2019



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $5 - 4 \times (2! + 1) + 9 \div 3 - 5$

   (A) -9 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 9

2. **Nine million five hundred two thousand three hundred five is subtracted from five hundred four million two thousand nineteen. Which of the following digits appears the most in the difference?**

   (A) 9 **(B)** 7 **(C)** 4 **(D)** 1 **(E)** 0

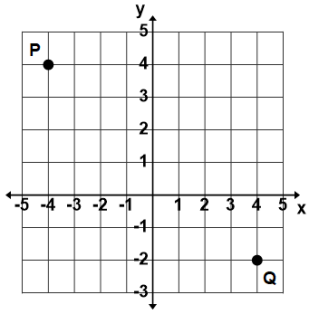
3. **Lotta Dowe bought four calculators at a cost of $35.00 each while shopping at a garage sale. She sold them at her garage sale for $60.00, $50.00, $45.00, and $30.00. What percent profit did she net from these transactions? (nearest whole percent)**

   (A) 30% **(B)** 32% **(C)** 45% **(D)** 54% **(E)** 68%

4. **If the sales tax on $34.75 is $2.52, what would it cost, including tax for an item that sells for $504.00? (nearest cent)**

   (A) $540.55 **(B)** $536.55 **(C)** $517.80 **(D)** $511.25 **(E)** $506.52

5. **The line containing points P and Q crosses the y-axis at (0, y) and the x-axis at (x, 0). Find $x + y$.**



   (A) $-1\frac{1}{3}$ **(B)** $-\frac{2}{3}$ **(C)** $\frac{1}{3}$ **(D)** $1\frac{2}{3}$ **(E)** $2\frac{1}{3}$

6. $\frac{3-5}{4} = \frac{3}{4} - \frac{5}{4}$ **is an example of the _____ property of addition.**

   (A) associative (B) commutative (C) distributive (D) identity (E) inverse

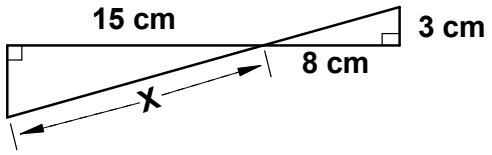
7. **Let** $(5x + 3)(4x - 1)(2x) = ax^3 + bx^2 + cx + d$. **Find** $a + b + c + d$.

   (A) 20 **(B)** 48 **(C)** 54 **(D)** 60 **(E) not enough information**

8. **Simplify:** $\left(\frac{x^2 - 2x - 15}{x^2 + 2x - 3}\right) \div \left(\frac{x^2 - 2x - 8}{x^2 + x - 2}\right)$.

   (A) $\frac{x-5}{x-4}$ **(B)** $\frac{x+5}{x-4}$ **(C)** $x + 4$ **(D)** $x - 5$ **(E)** $\frac{x+5}{x+4}$

9. Find x to the nearest tenth. (drawing is not to scale)



(A) 15.6 cm (B) 16.0 cm (C) 16.7 cm (D) 17.0 cm (E) cannot be determined

10. If two chords of a circle are unequal in length and neither chord is the diameter, then the longer chord is _____ the center of the circle.

(A) nearer to (B) equal distance to (C) through (D) below (E) farther from

11. Line *m* contains the points A (— 5, 3) and B (4, — 5). Which of the following points would be the midpoint of segment AB?

(A) (— 1, 0) (B) (— 0.5, — 0.5) (C) (— 1, — 1) (D) (— 0.5, — 1) (E) (0, — 1.5)

12. Horace Tank bought a rectangular prism shaped trough to water his sheep. It was 2 feet deep, 3 feet wide, and 12 feet long. He used a water hose to put 350 gallons into the trough. What percent of the trough's capacity was filled with water? (nearest %)

(A) 36% (B) 49% (C) 53% (D) 60% (E) 65%

13. Which of the following mathematicians replaced the operation of multiplication by the word "and" and addition by the word "or"?

(A) Boole, George (B) Cantor, Georg (C) Descartes, Rene
(D) Euler, Leonard (E) Goldbach, Christian

14. Given the table of values : find $p + q$, where $p, q \geq 0$.

x	— 1	2	1	3	4	7	q
f(x)	— 2	7	— 2	22	43	p	70

(A) 124 (B) 134 (C) 137 (D) 147 (E) 152

15. Given the triangular pattern shown, find the sum of the terms in row 8 minus the first and last terms.

1 (row 1)
2 1
4 4 1
8 12 6 1
16 32 24 8 1
32 A B C D 1
...

(A) 2,058 (B) 664 (C) 1,930 (D) 2,187 (E) 6,304

16. If the following pattern continues, find the sum of the terms on the left side of the equality sign of row 20.

$$1 + 2 = 3 \qquad \text{row 1}$$
$$4 + 5 + 6 = 7 + 8 \qquad \text{row 2}$$
$$9 + 10 + 11 + 12 = 13 + 14 + 15 \qquad \text{row 3}$$

(A) 8,610 (B) 8,810 (C) 8,400 (D) 8,390 (E) 8,000

17. The sequence 2, 4, 6, 10, 16, ... is a Fibonacci characteristic sequence. Find the ratio of the 22nd term to the 20th term given that 2 is the first term. (nearest tenth)

(A) 1.62 (B) 1.89 (C) 2.62 (D) 3.43 (E) 5.24

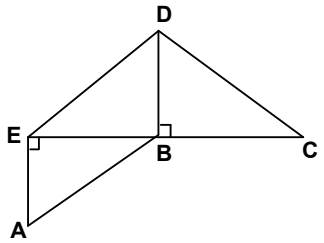
18. $101111101_2 + 10100100_2 - 1111001_2 =$ _____ $_8$.

(A) 1232 (B) 1212 (C) 1210 (D) 650 (E) 522

19. $(503_8 \times 11_8 - 504_8) \div 7_8$ has a remainder of _____.

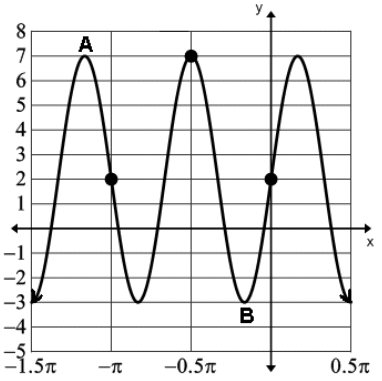
(A) 0 (B) 2 (C) 4 (D) 5 (E) 6

20. Given: $m\angle BAE = 60°$, $m\angle BCD = 45°$, BD = 5", and $\triangle ABE \cong \triangle DBE$. Find the perimeter of the pentagon ABCDE. (nearest inch)



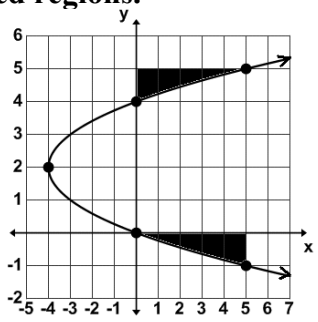
(A) 42 in (B) 38 in (C) 37 in (D) 35 in (E) 34 in

21. The sinusoidal graph below reaches a maximum at point $A(x_1, y_1)$ and a minimum at point $B(x_2, y_2)$. Find $x_1 + x_2$. (nearest tenth)



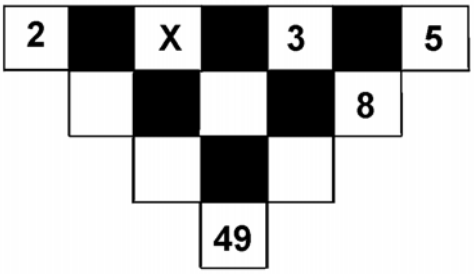
(A) —1.6 (B) — 2.1 (C) — 3.1 (D) — 4.2 (E) — 4.7

22. Find the area of the shaded regions.



(A) $4\frac{5}{6}$ **sq. units** **(B) 5 sq. units** **(C)** $5\frac{1}{6}$ **sq. units** **(D)** $5\frac{1}{3}$ **sq. units** **(E)** $5\frac{2}{3}$ **sq. unit**

23. The number in the unshaded box is found by adding the numbers connected with it from the row above it. (ex. 8 is found using 3 and 5. What is the value of X?



(A) 7 **(B) 8** **(C) 9** **(D) 11** **(E) 12**

24. 504P05Q102 ÷ 9 has a remainder of 7. Find the least value of P + Q, where P,Q > 0.

(A) 9 **(B) 8** **(C) 6** **(D) 3** **(E) 1**

25. How many integers, n, satisfy the inequality $-\frac{5}{6} < \frac{3n}{8} < \frac{7}{10}$ **?**

(A) 7 **(B) 6** **(C) 5** **(D) 4** **(E) 3**

26. If $a_1 = -5$, $a_2 = 0$, $a_3 = 4$ **and** $a_n = (a_{n-2})(a_{n-1}) + (a_{n-3})$, **where** $n \geq 4$, **then** a_6 **equals:**

(A) 104 **(B) 20** **(C) — 4** **(D) — 20** **(E) — 24**

27. Let $4x - 3y = 7$ **and** $2x + 5y = 7$. **Find** $6x + 2y$.

(A) $1\frac{1}{13}$ **(B)** $6\frac{6}{13}$ **(C)** $11\frac{11}{13}$ **(D) 14** **(E) 34**

28. Let $f(x) = x + 2$ **and** $g(x) = 3x - 4$ **and** $h(x) = 5x + 6$. **Find** $f(g(x)) + g(h(x)) + h(f(x))$.

(A) $9x + 4$ **(B)** $20x + 32$ **(C)** $23x - 24$ **(D)** $20x - 4$ **(E)** $23x + 28$

29. Determine the range of $f(x) = -3\cos[5(x + \pi)] - 2$.

(A) $[3, -2]$ **(B)** $[-5, 1]$ **(C)** $[-7, -2]$ **(D)** $[-5, 2]$ **(E)** $[5, -1]$

30. If $\sqrt{x^5\sqrt{x^4\sqrt{x^3}}} = \sqrt[n]{x^k}$, where k and n are relatively prime, then k = ?

(A) 60 (B) 39 (C) 31 (D) 12 (E) 8

31. The expression $\frac{\cos\theta}{1+\sin\theta} + \tan\theta$ is equivalent to:

(A) $\csc\theta$ (B) $\tan\theta$ (C) $\sec\theta$ (D) $\cot\theta$ (E) $\sin\theta\cos\theta$

32. Evaluate: $\prod_{n=1}^{4} [(-n)^{(n-1)} + (n-1)]$

(A) 737 (B) 671 (C) 600 (D) −50 (E) −74

33. The first Germain prime of the form $4k + 3$ is G. Find k.

(A) 11 (B) 7 (C) 5 (D) 4 (E) 2

34. If $\frac{d}{dx}\left(\frac{5x-4}{2x+3}\right) = \frac{a(bx+c)-b(ax-d)}{(bx+c)^2}$, then $a + b + c + d$ = ?

(A) 5 (B) 6 (C) 14 (D) 18 (E) 23

35. Let $\begin{bmatrix} -2 & -3 \\ 1 & 4 \end{bmatrix}^{-1} = \begin{bmatrix} d & e \\ f & g \end{bmatrix}$ then $f + g$ = ?

(A) −1 (B) −0.2 (C) 0.6 (D) 0.8 (E) 1.5

36. Let $(1-2i)^5 = (a + bi)$. Find $a + b$. (nearest whole number)

(A) 35 (B) 41 (C) 57 (D) 78 (E) 79

37. Expand $10^B \div (10^{(2B)} - 10^B - 1)$ for B = 3. What is the 35^{th} digit after the decimal place?

(A) 1 (B) 3 (C) 4 (D) 5 (E) 8

38. The points A, B, C, D, and E are collinear and arranged alphabetically from left to right. Find BD given the following: AE = 24 cm, AC = 8 cm, DE = 6 cm, and $AB = \frac{2}{3}DE$.

(A) 4 cm (B) 8 cm (C) 10 cm (D) 12cm (E) 14 cm

39. One hundred twenty-five students attended the invitational math/science/robotics competition. Eighty took the math test, seventy took the science test, and fifty took both math and science. How many did not take the math or science test?

(A) 5 (B) 15 (C) 20 (D) 25 (E) 50

40. Currently, Les Cash has $75.00 and his sister, Lotta Cash, has $105.00. Starting next week, Les will save $10.00 a week and Lotta will save $6.00 a week. How many weeks will it take until Les has more money than Lotta?

(A) 5 (B) 7 (C) 8 (D) 9 (E) 11

41. Point P (4, — 4) lies in the x-y plane. Point P is reflected across the y-axis to point Q. Point Q is rotated 90° clockwise about the origin to point R. Point R is translated horizontally + 3 units and vertically — 5 units to point S. Point S is reflected across the x-axis to point T (x, y). Find $x + y$.

(A) 8 (B) — 2 (C) — 5 (D) 2 (E) 0

42. The *PeaKann* company sells shelled pecans for $7.95 a pound. The *PeaNutt* company sells shelled peanuts for $3.75 a pound. The two companies come together to create a mixture of pecans and peanuts that will sell for $5.50 a pound. What percent of the mixture would be pecans if the bag of mixture held 3 pounds? (nearest percent)

(A) 87% (B) 69% (C) 48% (D) 42% (E) 25%

43. Saul T. Water sailed his yacht from his Port Aransas dock 30 miles on a bearing of 280°. Then he turned and sailed on a bearing of 75°. After two hours he turned to head to his home dock on a bearing of 150°. How far was he from his dock when he turned for home? (nearest tenth)

(A) 12.8 miles (B) 13.1 miles (C) 15.0 miles (D) 23.8 miles (E) 26.1 miles

44. Nicole Koin flips a nickel five times and records the results, heads or tails. What are the odds that she got at least three tails?

(A) 1:2 (B) 3:2 (C) 1:1 (D) 1:3 (E) 2:1

45. Ken Dahl has 6 different romance novels, 4 different historical novels, and 5 different mystery novels. How many different sets of 7 novels can he form if 3 are mystery novels, 3 are romance novels, and 1 is a historical novel?

(A) 29 (B) 800 (C) 6,435 (D) 400 (E) 14,400

46. The hour hand and the minute hand on a circular clock form a 160° angle at which of the following times?

(A) 3:20 (B) 11:44 (C) 7:20 (D) 5:32 (E) 2:40

47. The roots of $x^4 - 13x^3 + 56x^2 - 92x + 48 = 0$ are p, q, r, and s.
Find $(pqrs) + (pqr + pqs + prs + qrs) - (pq + pr + ps + qr + qs + rs)$.

(A) 196 (B) 84 (C) 12 (D) — 12 (E) — 100

48. How many x-values in $\left\{x \middle| 3\sin(x) - 2\cos^2(x) = 0,\ x \in [-\frac{10\pi}{3}, \frac{8\pi}{3}]\right\}$ exist?

(A) 3 (B) 5 (C) 6 (D) 7 (E) 8

49. Find the sum of the coefficients of the x^5y^3 term and the x^4y^4 term in the expansion of $(2x - y)^8$.

(A) -1792 (B) -672 (C) 2,912 (D) 1,334 (E) 672

50. The function $f(x) = x^3 - 5x + 3$ has a minimum at (m, n) and a maximum at (M, N) on the interval [0, 3]. Find $(m + M) - (n + N)$. (nearest tenth)

(A) -9.4 (B) -12.0 (C) -13.7 (D) -14.6 (E) -16.3

51. Roland Decubes tosses a pair of dice twice. What is the probability the sum of the dots on the top faces of the first toss is the same as the sum on the second toss? (nearest whole percent)

(A) 11% (B) 14% (C) 22% (D) 36% (E) 65%

52. Given that the number of minutes needed to solve a set of problems varies directly as the number of problems and inversely as the number of people working on the solutions. A problem solving committee consisting of 4 people took 24 minutes to solve 12 problems. How many minutes will it take to solve 30 problems if 2 more people are added to the committee?

(A) 32 min (B) 40 min (C) 48 min (D) 56 min (E) 60 min

53. Given: A= 5 and B = 6. Which of the following trapezoidal means of A and B have the least numerical value?

(A) Geometric (B) Heronian (C) Centroidal (D) Arithmetic (E) Harmonic

54. Points A and B are on a circle with center O and point C lies outside of the circle such that segments CA and CB are tangent to the circle. Find $m\angle BOA$ if the $m\angle ACB = 77°$.

(A) 113° (B) 38.5° (C) 123° (D) 83.5° (E) 103°

55. The *Get The Lead Out* pencil shop packages four map pencils in each pack. The shop has red leaded ones, black leaded ones, blue leaded ones, green leaded ones, and purple leaded ones. How many different packs of four pencils can they package?

(A) 20 (B) 70 (C) 105 (D) 120 (E) 280

56. For what values of x is the expression $\sqrt{2x - \sqrt{6 - 5x}}$ defined over the real numbers?

(A) $\frac{1}{2} \le x \le \frac{5}{6}$ (B) $-2 \le x \le 1$ (C) $\frac{1}{2} \le x \le 1\frac{1}{5}$ (D) $\frac{3}{4} \le x \le 1\frac{1}{5}$ (E) $x \le \frac{3}{4}$

57. Let $f(x) = \begin{cases} ax + b, & \text{if } x < 0 \\ 2\sin(x) + 3\cos(x) & \text{if } x \ge 0 \end{cases}$.
Find the value of $a + b$ where the values of a and b exist so that f(x) is differentiable at x = 0.

(A) 0 (B) 2 (C) 3 (D) 5 (E) 6

58. Which of the following sequences converge to 1?

1. $a_n = \frac{\ln(n+1)}{n}$ **2.** $a_n = (8)^{\frac{1}{n}}$ **3.** $a_n = \frac{\ln(n)}{(n)^{\frac{1}{n}}}$ **4.** $a_n = \sqrt[n]{10n}$

(A) 2, 3, & 4 (B) 2 & 4 (C) 1 & 3 (D) 1, 2, & 3 (E) none of these

59. A package of Mathlink cubes contains 5 green ones, 4 yellow ones, and 3 orange ones. Three cubes are randomly drawn from the package, without replacement. What is the probability that all are green given that the first one is not orange? (nearest whole percent)

(A) 22% (B) 11% (C) 10% (D) 6% (E) 5%

60. How many distinct 6-letter code words can be made from the letters in the word TEENAGER ?

(A) 20,188 (B) 20,160 (C) 36 (D) 3,720 (E) 12,120

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University Interscholastic League
MATHEMATICS CONTEST
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Answer Key

1. A	**21. D**	**41. E**
2. C	**22. B**	**42. D**
3. B	**23. D**	**43. B**
4. A	**24. B**	**44. C**
5. E	**25. D**	**45. B**
6. C	**26. A**	**46. E**
7. B	**27. D**	**47. B**
8. A	**28. E**	**48. C**
9. B	**29. B**	**49. B**
10. A	**30. C**	**50. A**
11. D	**31. C**	**51. A**
12. E	**32. B**	**52. B**
13. A	**33. E**	**53. E**
14. D	**34. C**	**54. E**
15. A	**35. C**	**55. B**
16. A	**36. E**	**56. D**
17. C	**37. C**	**57. D**
18. D	**38. E**	**58. B**
19. A	**39. D**	**59. D**
20. C	**40. C**	**60. E**