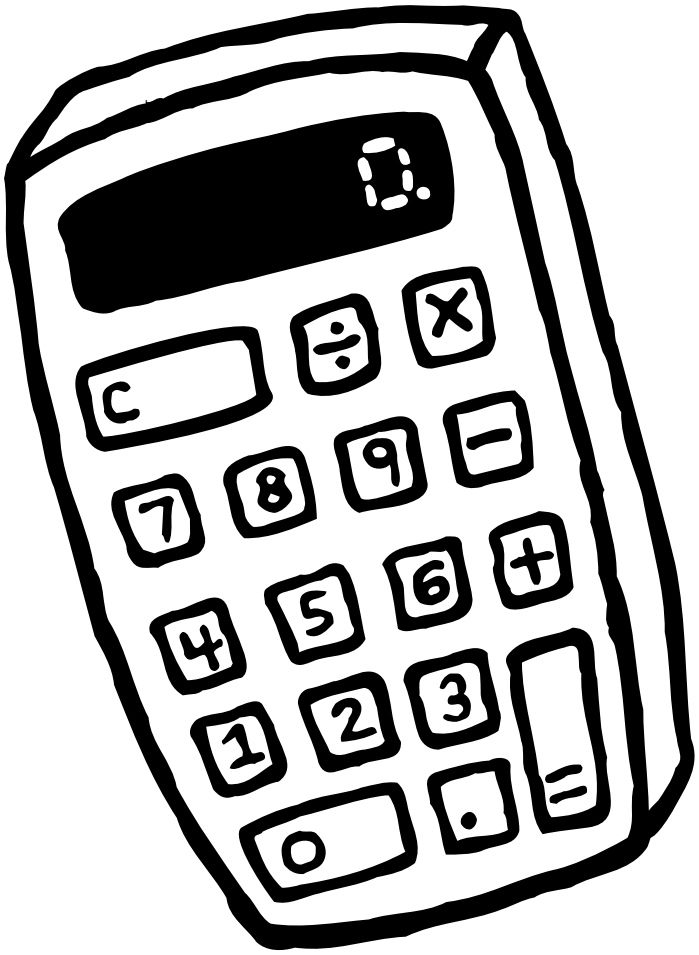
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Region • 2019



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YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate: $4 - 1 \times 3 + (2 + 0!) \div 1 \times 9$**

   (A) $1\frac{1}{3}$ **(B)** $1\frac{2}{9}$ **(C) 19** **(D) 28** **(E) 36**

2. **1 rod + 12 yards − 5.5 feet + 3 inches = _____ inches.**

   (A) 963 **(B) 825** **(C) 729** **(D) 567** **(E) 477**

3. **Travlin Tex's weekly salary consists of a base salary plus commission. He gets $500.00 base pay, 20% commission on sales up to and including $500.00, and 25% on sales over $500.00. His total sales for the last week of March was $2019.00. How much was his salary for last week?**

   (A) $979.75 **(B) $997.75** **(C) $1008.55** **(D) $1,104.75** **(E) $1,133.55**

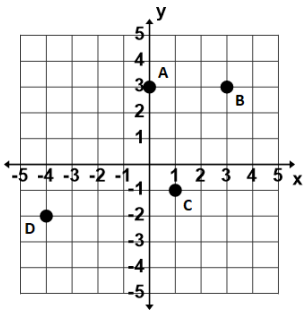
4. **The digits 4, 1, 3, 1, and 9 are each used once to form the smallest possible 5-digit odd number. What is the digit in the hundreds place?**

   (A) 9 **(B) 4** **(C) 3** **(D) 1** **(E) 0**

5. **The sequence 3, 7, 10, 17, 27, ... is a Fibonacci characteristic sequence, where 3 is the first term. Find the sum of the first ten terms.**

   (A) 759 **(B) 770** **(C) 781** **(D) 792** **(E) 797**

6. **The slope of the line containing the points A and C is m. The slope of the line containing the points B and D is n. Find the absolute value difference in the slopes.**



   (A) $\frac{3}{4}$ **(B)** $\frac{1}{5}$ **(C)** $2\frac{3}{5}$ **(D)** $3\frac{2}{7}$ **(E)** $4\frac{5}{7}$

7. **Let $14x^2 + x - 4 = (ax + b)(cx + d)$. Find $a + b + c + d$.**

   (A) 14 **(B) 12** **(C) 11** **(D) 6** **(E) 5**

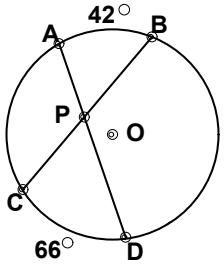
8. **The *Cheep Seets* ticket outlet sells adult tickets for $3.50 each and student tickets for $1.50 each. 250 tickets were sold totaling $505.00. How many more student tickets were sold than adult tickets?**

   (A) 120 **(B) 185** **(C) 130** **(D) $0.60** **(E) 65**

9. **$4 + (1 + 3) = (4 + 1) + 3$ is an example of the _____ property of addition.**

   (A) associative **(B) commutative** **(C) distributive** **(D) identity** **(E) inverse**

10. Given the circle with center O, find m∠BPD. (drawing is not to scale)



(A) 108° (B) 138° (C) 136° (D) 114° (E) 126°

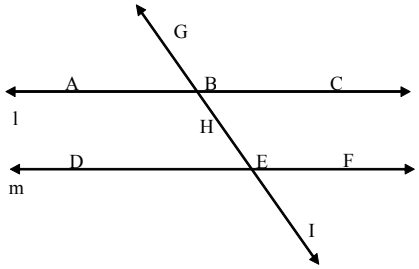
11. The Euler line is a line that cannot be determined in which of the following types of triangles?

(A) scalene (B) obtuse (C) right (D) isosceles (E) equilateral

12. A rectangle with integral sides has an area of 80 sq. inches and a perimeter of 42 inches. Two inches are cut from both the length and the width. The area of the new rectangle is how much less than the original rectangle?

(A) 30 in^2 (B) 34 in^2 (C) 38 in^2 (D) 40 in^2 (E) 42 in^2

13. The lines in the figure are coplanar with m // l. Which of the following are true statements?



1. ∠BEF & ∠BED are supplementary
2. m∠GBA = m∠IEF
3. ∠CBH ≅ ∠DEH
4. ∠ABG & ∠DEH are complementary

(A) 1 only (B) 1 & 2 (C) 2 & 3 (D) 1, 2, & 3 (E) 1, 2, 3, & 4

14. If $\frac{2x+3}{x-4} + \frac{5x-7}{2x+3} = \frac{ax^2+bx+c}{dx^2+ex+f}$, then $a-b+c-d+e-f$ equals:

(A) 12 (B) 14 (C) 33 (D) 42 (E) 66

15. If $a_1 = 4$, $a_2 = -1$, $a_3 = 3$ and $a_n = (a_{n-3} + a_{n-1})(a_{n-2})$, where $n \geq 4$, then a_6 equals:

(A) −189 (B) −24 (C) 147 (D) 216 (E) 243

16. Dee Loper and Les Spede workout on the same track. Dee runs at an average rate of 5 miles per hour. Les runs at an average rate of 3.5 miles per hour. Les starts to run at 8:00 AM and Dee starts to run at 8:30 AM. At what time will Dee catch up to Les?

(A) 9:30 AM (B) 9:40 AM (C) 10:00 AM (D) 10:10 AM (E) 10:30 AM

17. Penney Flipper tossed a penny four times and record the results, heads or tails. What are the odds that she got precisely three heads or three tails?

(A) 1:2 (B) 5:3 (C) 2:1 (D) 1:1 (E) 3:8

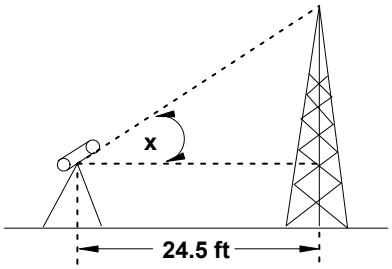
18. The expression $\frac{\sin\theta}{1-\cos\theta} - \cot\theta$ is equivalent to:

(A) $\csc\theta$ (B) $\tan\theta$ (C) $\sec\theta$ (D) $\frac{1}{2}\sin\theta$ (E) $\sin 2\theta$

19. Given: $\cos\alpha = -\frac{3}{4}$, $\frac{\pi}{2} < \alpha < \pi$ and $\cos\beta = \frac{2}{3}$, $\frac{3\pi}{2} < \beta < 2\pi$.
Evaluate: $\sin(\alpha + \beta)$ (nearest thousandth)

(A) — 0.118 (B) — 0.401 (C) 0.450 (D) 0.599 (E) 1.000

20. Sir Vayor used his theodolite to measure the height of the tower to be 20 ft 3 inches tall. His theodolite was 5 ft 6 inches from the level ground. What angle x did he use to compute the height? (nearest minute)



(A) 15° 20' (B) 31° 3' (C) 37° 0' (D) 39° 34' (E) 46° 25'

21. The harmonic mean of the roots of $x^5 - 15x^4 + 85x^3 - 225x^2 + 274x - 120 = 0$ is? (nearest tenth)

(A) 1.6 (B) 2.2 (C) 2.7 (D) 3.0 (E) 3.5

22. Find the sum of the 3rd term and the 11th term of row 12.

			1			row 0
		1		1		row 1
	1		2		1	row 2
1		3		3		1 row 3
1		4		6	4	1 row 4
			...			

(A) 132 (B) 136 (C) 144 (D) 168 (E) 180

23. $(214_7 \times 5_7 - 413_7) \div 6_7$ has a remainder of _____.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 5

24. Find the area bounded by the curves y = sin(x) and y = cos(x), where $0 \le x \le 2\pi$.

(A) $2\sqrt{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\sqrt{2}$ (D) 1 (E) 0

25. Let $f(x) = \frac{3}{\sqrt{2x+1}}$. Find $f'(12)$.

(A) — 0.024 (B) — 0.012 (C) — 0.006 (D) 0.6 (E) 1.666...

26. What is the slope of the secant line to the graph of $f(x) = 2 - 3x - 4x^2$ passing through the points $(-2, p)$ and $(2, q)$?

(A) 7 (B) 4 (C) — 1 (D) — 2 (E) — 3

27. How many distinct 4-letter code words can be made from the letters in the words "ODIOUS"?

(A) 360 (B) 216 (C) 192 (D) 180 (E) 120

28. Find the 30^{th} term given the sequence: 3, 8, 15, 24, 35, 48, ...

(A) 899 (B) 915 (C) 930 (D) 960 (E) 1,030

29. Let U = {r, e, g, i, o, n, m, a, t, h} be the universal set, G = {g, r, e, a, t}, and T = {t, i, m,e}. The number of elements in $\{G \cup T\}'$, where the symbol, $'$, denotes the complement set.

(A) 1 (B) 3 (C) 5 (D) 6 (E) 7

30. How many teaspoons will it take to get $\frac{3}{4}$ cup?

(A) 16 tsp (B) 18 tsp (C) 32 tsp (D) 36 tsp (E) 48 tsp

31. Let $(ax + d)^3 = 8x^3 + bx^2 + cx + 1$. Find $a + b + c + d$.

(A) 17 (B) 18 (C) 21 (D) 27 (E) 81

32. Given: $x^2y - xy^2 - x + y = 56$ and $xy = 8$. Find $x^2 + y^2$

(A) 48 (B) 64 (C) 65 (D) 80 (E) 81

33. Which of the following's most notable accomplishment was conceiving the ideas of differential and integral calculus?

(A) Franciscus Vieta (B) Christian Goldbach (C) Leonardo Bigollo
(D) Claudius Ptolemy (E) Gottfried Leibniz

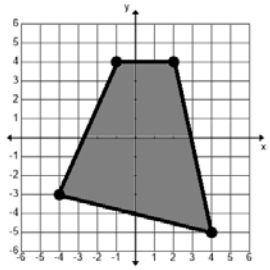
34. Let $x + 2y = 3$, $3y - z = 2$, and $2x + z = 3$. Find z.

(A) 1 (B) $2\frac{1}{3}$ (C) 3 (D) $5\frac{2}{3}$ (E) 7

35. Let $f(x) = x + 1$ and $g(x) = 2x - 1$ and $h(x) = -x - 2$. Find $h\Big(f\big(g(x-1)+1\big) - x\Big)$

(A) $2x - 1$ (B) $3x$ (C) $3x - 1$ (D) $-2x + 1$ (E) $-x - 1$

36. Find the area of the shaded quadrilateral with integral vertices.



(A) 44.5 units^2 (B) 43 units^2 (C) 42.5 units^2 (D) 41.5 units^2 (E) 40 units^2

37. Brad flew his new plane 87 miles from Dumas to Perryton on a bearing of 70°. Then he flew 98 miles from Perryton to Shamrock on a bearing of 115°. What distance would he fly to return directly to Dumas? (nearest mile)

(A) 171 miles (B) 152 miles (C) 138 miles (D) 131 miles (E) 125 miles

38. Find the sum of the x-values of the points where y = cos 2x intersects with y = 2cos x and $0 \leq x \leq 2\pi$. (nearest hundredth)

(A) 2.39 (B) 4.33 (C) 6.28 (D) 7.48 (E) 8.29

39. Let $f(x) = |2x - 1| + |x - 2|$. Find the minimum value of f(x).

(A) -2.5 (B) -2 (C) 0.5 (D) 1.5 (E) 2

40. Determine the value of $5 + 5(\frac{2}{3}) + 5(\frac{2}{3})^2 + 5(\frac{2}{3})^3 + ... + 5(\frac{2}{3})^{15}$ to the nearest thousandths.

(A) 14.977 (B) 14.965 (C) 14.949 (D) 14.923 (E) 14.917

41. Roland Bones tosses a pair of dice. What is the probability the sum of the top faces is greater than 7 and the number of dots (pips) on each die is a Fibonacci number ?(nearest whole percent)

(A) 6% (B) 8% (C) 13% (D) 15% (E) 31%

42. $(413_8 - 312_4) \times 11_2 =$ _____ $_{10}$.

(A) 639 (B) 303 (C) 23 (D) 248 (E) 1,111

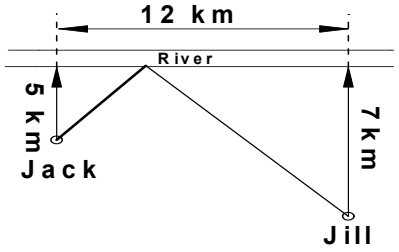
43. Les Arria inherited 3 "leagues" of land. He had to sell 2 "labors" of land to pay the inheritance tax. How many total acres did he have left? (nearest integer)

(A) 13,639 acres (B) 12,931 acres (C) 12,754 acres (D) 12,657 acres (E) 8,502 acres

44. Find the range of the function $f(x) = 4 - 3|2x - 4|$, where $0 \leq x \leq 4$.

(A) $2 \geq y \geq -2$ (B) $y \geq -4$ (C) $-8 \leq y \leq 4$ (D) $y \leq 4$ (E) $y \geq -8$

45. Jack flew his drone to the river to fill a pail of water and delivered it to Jill. The drone flew the minimum distance possible. How much shorter would the trip be if he had water at his house and the drone took it directly to Jill? (nearest hundredth)



(A) 4.24 km (B) 4.81 km (C) 5.92 km (D) 6.00 km (E) 6.72 km

46. A baseball pitcher's earned-run average varies directly with the number of earned runs and inversely as the number of innings pitched. Willy Tossett had an earned-run average 1.8, giving up 18 runs in 90 innings. Based on his average, how many earned runs would he give up having pitched 240 inning?

(A) 36 (B) 42 (C) 45 (D) 48 (E) 54

47. Point P (0, 3) lies in the x-y plane. Point P is rotated 90° clockwise about the point (— 2, 1) to Point Q. Point Q is reflected across the line x = 3 to point R. Point R is translated vertically 4 units up to point S (x, y). Find x + y.

(A) 10 units (B) 9 units (C) 7 units (D) 5 units (E) 3 units

48. How many of the following are the side lengths of an acute triangle?
I. $3, 3\sqrt{3}, 6$ II. 7, 9, 11 III. 8, 9, 20 IV. 12, 13, 14

(A) 1 (B) 2 (C) 3 (D) all of them (E) none of them

49. The geometric mean of 4, 13, and N is 9.96 (rounded off to the nearest hundredth). The harmonic mean of 4, 13, and N is? (rounded off to the nearest hundredth)

(A) 12.00 (B) 9.04 (C) 8.72 (D) 7.96 (E) 7.90

50. Given: f(x) = 4 — 13sin(20x + 19). If the amplitude is decreased by 5, the phase shift is doubled, the displacement is increased by 2, and the frequency is divided by 2 then the sum of the amplitude, phase shift, displacement and frequency is? (nearest integer)

(A) 9 (B) 12 (C) 17 (D) 20 (E) 26

51. Three bags labeled 1, 2, and 3 each contain four ping pong balls labeled 1, 2, 3, and 4. I. M. Kahnfuzed randomly selects one ball from each of the bags. What is the probability of getting three different numbered balls given that the first ball chosen is an even number? (nearest whole percent)

(A) 50% (B) 38% (C) $33\frac{1}{3}\%$ (D) 22% (E) 11%

52. Let $A = \begin{bmatrix} 1 & -3 \\ 5 & -7 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 6 \\ -4 & k \end{bmatrix}$. Find k if the determinant of A — B is 33.

(A) — 23 **(B)** — 22 **(C)** 8 **(D)** 9 **(E)** 45

53. Let P and Q be two prime numbers such that $P + Q = 150$ and $PQ = 5{,}561$. Find $|P - Q|$.

(A) 8 **(B)** 12 **(C)** 16 **(D)** 24 **(E)** 28

54. Which of the following functions have a horizontal, a vertical, and an oblique asymptote?

I. $f(x) = \frac{x^2 - 1}{x^2 + 3x + 2}$ II. $f(x) = \frac{x^2 - 3x - 4}{x + 3}$ III. $f(x) = \frac{x^3 + 2x^2 - x - 3}{x^2 - 2x + 1}$

(A) I & III **(B)** III only **(C)** II & III **(D)** I, II, & III **(E)** none of these

55. Willie Skor had 6-wk test grades of 88, 91, 65, 73, and 84. He wants to make a high enough grade on his last 6-wk test to average 80. What will the median of his six 6-wk grades be if his average is exactly 80?

(A) 79 **(B)** 80 **(C)** 81.5 **(D)** 83.5 **(E)** 85

56. The function $g(x) = x^3 + x^2 - 8x - 2$ has a minimum at (x, y) and a maximum at (x, z) on the interval [— 2, 2]. Find $y + z$. (nearest tenth)

(A) 8.0 **(B)** 2.5 **(C)** 0 **(D)** 1.5 **(E)** 6.0

57. For what values of x does the series $x^n(n^2 + 3)^{-(\frac{1}{2})}$ converge?

(A) $-1 \leq x < 3$ **(B)** $0 \leq x < 3$ **(C)** $-.5 \leq x < 3$ **(D)** $1 \leq x < 3$ **(E)** $-1 \leq x < 1$

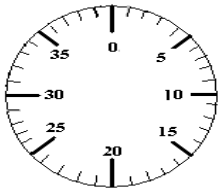
58. Forty students took the regional math test last year. Twelve of them were 9th graders. All of them had an equal chance to go to state as one of the three medal winners or as one of the two alternates. What was the probability that at least one 9th grader made it to state? (nearest whole percent)

(A) 1% **(B)** 13% **(C)** 30% **(D)** 67% **(E)** 85%

59. What is the sum of the digits of the least emirp prime that is greater than 100?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

60. T-S-P is the combination needed to open the lock box with the combination dial shown below. How many distinct combinations exist if T is a triangular number, S is a square number, and P is a pentagonal number?



(A) 384 **(B)** 240 **(C)** 117 **(D)** 21 **(E)** 19

DO NOT DISTRIBUTE TO STUDENTS BEFORE OR DURING THE CONTEST

University Interscholastic League
MATHEMATICS CONTEST
HS • Regional • 2019
Answer Key

1. D	**21. B**	**41. B**
2. D	**22. A**	**42. A**
3. A	**23. D**	**43. B**
4. C	**24. A**	**44. C**
5. C	**25. A**	**45. B**
6. E	**26. E**	**46. D**
7. B	**27. C**	**47. B**
8. A	**28. D**	**48. B**
9. A	**29. B**	**49. E**
10. E	**30. D**	**50. C**
11. E	**31. C**	**51. B**
12. C	**32. D**	**52. D**
13. D	**33. E**	**53. C**
14. E	**34. A**	**54. E**
15. C	**35. E**	**55. C**
16. B	**36. A**	**56. D**
17. D	**37. A**	**57. E**
18. A	**38. C**	**58. E**
19. E	**39. D**	**59. B**
20. B	**40. A**	**60. B**

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____