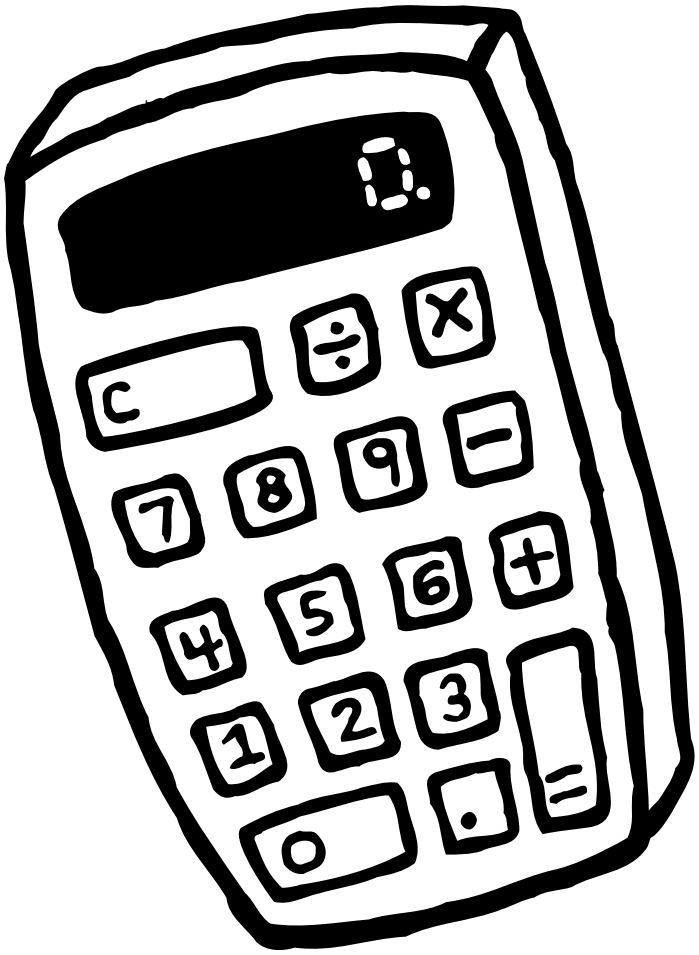
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational B • 2019



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate: $2 + 15 \div 3 - 16 \div 20 + 1 \times 9$**

   (A) -9.2 (B) -4.8 (C) 4.35 (D) 15.2 (E) 63.2

2. **Les Cash bought five radios at the local flea market at a cost of $50.00 each. He sold three of them making a 20% profit and the other two were sold at a 10% loss. How much did Les net from his sales?**

   (A) $30.00 (B) $20.00 (C) $15.00 (D) $10.00 (E) $5.00

3. **Let U = {0, 1, 2, 3, 5, 6, 9} be the universal set, J = {1, 2, 5}, and F = {1, 3, 6}. The number of elements in $\{J' \cap F'\}$, where J' and F' denote the complement sets.**

   (A) 0 (B) 1 (C) 2 (D) 3 (E) 7

4. **Let $(2x - 3)^3 = ax^3 + bx^2 + cx + d$. Find $a + b + c + d$.**

   (A) -17 (B) -1 (C) 3 (D) 125 (E) 216

5. **Let $27x^3 + 27x^2 + 9x + 1 = (3x + 1)(ax^2 + bx + c)$. Find $a + b + c$.**

   (A) 27 (B) 22 (C) 18 (D) 16 (E) 4

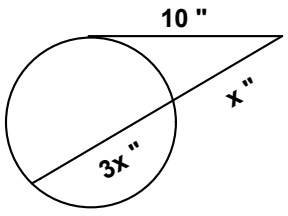
6. **Mr. Ruiz sold tickets for the local one-act play. He sold 15 more adult tickets than children tickets and he sold three times as many senior tickets as children tickets. In total, he sold 300 tickets. How many adult tickets did Mr. Ruiz sell?**

   (A) 24 (B) 57 (C) 72 (D) 129 (E) 171

7. **If two parallel lines are cut by a transversal, then each pair of consecutive interior angles is/are:**

   (A) supplementary (B) equal (C) complementary (D) acute (E) obtuse

8. **Given the tangent and secant shown, find x. (nearest tenth)**



   (A) 2.5 " (B) 1.6" (C) 5.0 " (D) 5.8 " (E) 1.8 "

9. **Horace Troff bought a water tank for his cattle. The tank was in the shape of a rectangular prism without the top. It was 3 feet deep, 2 feet wide, and 8 feet long. How many gallons of water would it take to fill it to the top without spilling over?**

   (A) 279 gal (B) 299 gal (C) 311 gal (D) 359 gal (E) 478 gal

10. Find the 5^{th} term of row 10.

				1						row 0
			1		1					row 1
		1		2		1				row 2
	1		3		3		1			row 3
1		4		6		4		1		row 4
				...						

(A) 120 **(B) 126** **(C) 210** **(D) 252** **(E) 330**

11. The *Lick'em Slow* lollipop company package 5 lollipops per pack. The company has chocolate, raspberry, coconut, grape, lime, and licorice lollipops. How many different packs of 5 lollipops can they package?

(A) 252 **(B) 720** **(C) 42** **(D) 720** **(E) 210**

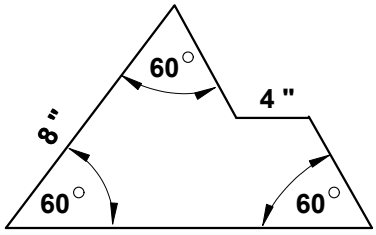
12. Nicole Taas is going to flip a coin three times and record the results. What is the probability she gets at least one head? (nearest whole percent)

(A) 38% **(B) 50%** **(C) 67%** **(D) 75%** **(E) 88%**

13. Which of the following mathematicians is noted for work on conic sections and the construction of astrolabes used for navigation?

(A) Aryabhata **(B) Noether** **(C) Agnesi** **(D) Theano** **(E) Hypatia**

14. Find the perimeter this pentagon?



(A) 24" **(B) 28"** **(C) 32"** **(D) 36"** **(E) not enough data**

15. 4 bushels + 2 pecks + 1 gallon + 3 quarts = _____ pints.

(A) 128 **(B) 256** **(C) 286** **(D) 302** **(E) 512**

16. Find the range of the function $f(x) = 2 - 3|x + 4|$.

(A) $-4 \le y \le 2$ **(B) $y \ge 2$** **(C) $-4 \ge y \ge 4$** **(D) $y \le 2$** **(E) $y \le -3$**

17. Which of the following are the side lengths of an obtuse triangle?

(A) 6, 8, 9 **(B) 5, 6, 7** **(C) $4, 4, 4\sqrt{2}$** **(D) $3, 3\sqrt{3}, 6$** **(E) 8, 8, 12**

18. Let $2x - y = 5$ and $3x + y = 6$. Find $20x + 19y$.

(A) 32.6 (B) 5 (C) 19.5 (D) 63 (E) 1.6

19. The expression $(\sin\theta + \cos\theta)^2 - 1$ is equivalent to:

(A) $\sin 2\theta$ (B) $\sin\theta\cos\theta$ (C) 0 (D) $\cos^2\theta - 1$ (E) $\sin^2\theta + 1$

20. Let $f(x) = |x^2 - 7x + 10|$. Find the sum of the local maximum and minimum values.

(A) $5\frac{3}{4}$ (B) $5\frac{1}{2}$ (C) $4\frac{1}{4}$ (D) $3\frac{1}{2}$ (E) $2\frac{1}{4}$

21. Let $f''(x) = 6x + 12$, $f'(-1) = 0$, and $f(1) = 12$. Find $f(-2)$.

(A) 10 (B) 2 (C) 0 (D) -2 (E) -6

22. $(613_8 - 316_8) \times 4_8 =$ _____ $_8$.

(A) 1100 (B) 1110 (C) 1210 (D) 1332 (E) 1364

23. Find the greatest common divisor of 270, 504, and 882.

(A) 21 (B) 18 (C) 9 (D) 6 (E) 2

24. 2153A16B19 ÷ 11 has a remainder of 6. Find A − B.

(A) 2 (B) 3 (C) 5 (D) 7 (E) 9

25. Which of the following points of concurrency is on a side of a right triangle but not a vertex point, on the interior of an acute triangle, and on the exterior of an obtuse triangle?

(A) centroid (B) circumcenter (C) incenter (D) orthocenter (E) none of these

26. The roots of $x^4 + x^3 - 7x^2 - x + 6 = 0$ are p, q, r, and s.
Find $(p + q + r + s) + (pqr + pqs + prs + qrs) - (pqrs)$.

(A) 7 (B) -1 (C) -6 (D) -8 (E) -14

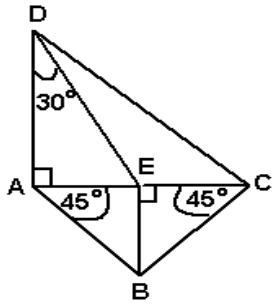
27. Andy Foundette knows of an odd integer between 600 and 800 that is divisible by both 7 and 9. He added the digits of this number. What sum did Andy get?

(A) 7 (B) 9 (C) 14 (D) 16 (E) 18

28. Which point is the reflection of the point $(-7, 5)$ over $y = -x$?

(A) $(-5, 7)$ (B) $(-7, 5)$ (C) $(5, -7)$ (D) $(7, -5)$ (E) $(7, 5)$

29. Find DC if CE = 5".



(A) $5\sqrt{7}$ in (B) 10 in (C) $7\sqrt{5}$ in (D) $5\sqrt{3}$ in (E) $3\sqrt{7}$ in

30. Points P and R are on a circle with center C such that m∠PCR = 94°. Point Q lies outside of the circle such that QP and QR are tangent to the circle. Find m∠PQR.

(A) 94° (B) 90° (C) 88° (D) 86° (E) 84°

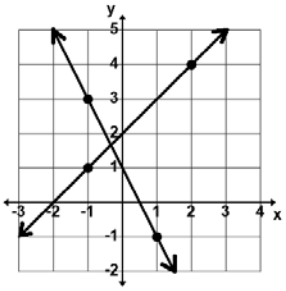
31. Nicole Taas is going to flip a coin three times and record the results. What are the odds against her getting exactly two heads?

(A) 5:3 (B) 4:4 (C) 5:8 (D) 3:8 (E) 3:5

32. The number 215 is a member of which of the following sets of special types of numbers:
(E)vil (H)appy (O)dious (U)nhappy

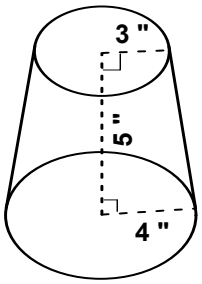
(A) H & O (B) E & U (C) O & U (D) E & H (E) none of these

33. The point of intersection of the two lines shown is (h, k). Find h + k.



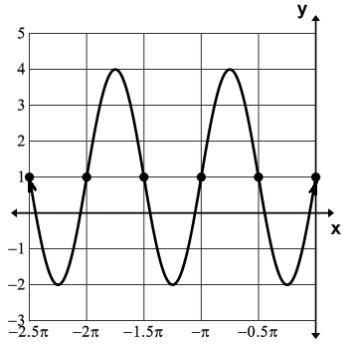
(A) $1\frac{1}{3}$ (B) $1\frac{5}{12}$ (C) $1\frac{1}{2}$ (D) $1\frac{7}{12}$ (E) 2

34. Find the volume of the figure shown. (nearest tenth)



(A) 754.0 in^3 (B) 36.7 in^3 (C) 185.0 in^3 (D) 581.2 in^3 (E) 193.7 in^3

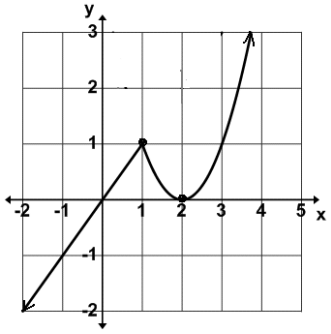
35. Which of the following equations is represented by the graph shown?



(A) $\cos(2x + \frac{\pi}{2}) + 3$ **(B)** $3\sin(\frac{\pi}{2}x - \pi) + 1$ **(C)** $3\cos(2x - \frac{\pi}{2}) + 1$

(D) $3\sin(2x - \pi) - 1$ **(E)** $\cos(2x - \pi) - 3$

36. The graph of f(x) is shown. For what values of x is f(x) differentiable?



(A) $1 < x < 2$ **(B)** $x < 1$ and $x > 1$ **(C)** $x < 2$ and $x > 2$

(D) all values of x **(E)** no values of x

37. Given that the set of natural numbers continue in the triangular pattern shown below, find the sum of the 7^{th} number in row 8 and the 8^{th} number in row 9.

			1				(row 1)
		2	3	4			(row 2)
	5	6	7	8	9		(row 3)
10	11	12	13	14	15	16	(row 4)
			...				(...)

(A) 128 **(B)** 130 **(C)** 132 **(D)** 134 **(E)** 136

38. Find $f(4) - f(0) + f(1)$ **if** $f(x) = \begin{cases} x - 1 & \text{if } x < 1 \\ x + 4 & \text{if } 1 \le x \le 4 \\ 1 - x & \text{if } x > 4 \end{cases}$

(A) 8 **(B)** 9 **(C)** 12 **(D)** 14 **(E)** 17

39. Let $\begin{bmatrix} a & 2 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & 1 \\ 6 & b \end{bmatrix} = \begin{bmatrix} 15 & 19 \\ 33 & 46 \end{bmatrix}$. Find $a + b$.

(A) 61 (B) 52 (C) 18 (D) 10 (E) 11

40. Find the sum of the x-values in $\left\{ x \middle| 2\cos^4(x) - 3\cos^2(x) + 1 = 0,\ x \in [-\pi, \frac{3\pi}{2}] \right\}$. (nearest hundredth)

(A) 3.14 (B) 3.93 (C) 4.71 (D) 5.50 (E) 8.64

41. Which of these trapezoidal means are used for finding the volume of a frustrum of a cone?

(A) Geometric (B) Heronian (C) Centroidal (D) Arithmetic (E) Harmonic

42. A parabola has a vertical axis of symmetry with a vertex at (1, 4) and focus at (1, 2). Find the equation of the directrix of this parabola.

(A) $y = 3$ (B) $y = \frac{1}{4}$ (C) $y = 6$ (D) $y = \frac{1}{2}$ (E) $y = 8$

43. N. A. Hurry stops at a convenience store. The probability that she buys a loaf of bread is 60%, the probability she buys a gallon of milk is 50%, and the probability she buys both bread and milk is 30%. What is the probability she will buy either bread or milk or both?

(A) 20% (B) 40% (C) 70% (D) 80% (E) 100%

44. Find the digit in the hundred-thousandth place of the sum of $1 + 3 + \frac{9}{2} + \frac{27}{6} + \frac{81}{24} + \ldots$.

(A) 8 (B) 6 (C) 5 (D) 3 (E) 0

45. Nicole Taas is going to flip a coin three times and record the results. What is the probability she gets at least two tails given that the first flip was a tail. (nearest whole percent)

(A) 38% (B) 50% (C) 67% (D) 75% (E) 88%

46. Les Stickum has $60.00 to buy some 48-cent stamps and some 1-dollar stamps. He has to buy twice as many 48-cent stamps as 1-dollar stamps. What is the greatest number of 48-cent stamps can he buy?

(A) 96 (B) 62 (C) 60 (D) 48 (E) 30

47. The *Wind Gust* electric company erected a wind turbine 1500 yards from the main station on a bearing of 110°. They erected a second wind turbine 800 yards from the main station on a at a bearing of 300°. How far apart were the two turbines? (nearest yard)

(A) 2,105 yds (B) 2,229 yds (C) 2,292 yds (D) 2,300 yds (E) 2,490 yds

48. Kanyu Emahjun changed the rectangular point (— 1, 4) to the polar point (r, θ). Kanyu found the sum of r and θ to be: (nearest whole number)

(A) 6 (B) 5 (C) 76 (D) 3 (E) 80

49. Bill Defense is fencing in a non-square rectangular area of 3,200 square feet. The cost of the fencing for two sides of the rectangle will cost $1.00 per foot and the other two sides will cost $2.00 per foot. What is the lowest possible cost for the fence?

(A) $80.00 (B) $160.00 (C) $320.00 (D) $356.00 (E) $400.00

50. Let $e^{(3x+2)} = 4e^{(x-5)}$. Find $e^{(x)}$. (nearest hundredth)

(A) 0.02 (B) 0.06 (C) 0.07 (D) 0.22 (E) 0.24

51. If $A + B = 14$ and $A \times B = 26$, then $|B - A|$ = _____.

(A) $7 + \sqrt{23}$ (B) $4\sqrt{23}$ (C) $7 - \sqrt{23}$ (D) $\sqrt{23}$ (E) $2\sqrt{23}$

52. How many points of intersection are there for the curves $r = 1 + 3\cos\theta$ and $\theta = \frac{1}{3}$?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

53. $(2x^2 + kx + 1) \div (x + 3)$ has a remainder of 43. Find k.

(A) — 8 (B) — 3 (C) 5 (D) $14\frac{1}{3}$ (E) $21\frac{1}{3}$

54. Let $f(x) = \frac{2x^3 + x^2 + 11x + 5}{x^2 + 5}$ and s(x) be the slant asymptote of f. Find the value of s(— 3).

(A) — 163 (B) $-9\frac{5}{6}$ (C) $-5\frac{3}{14}$ (D) — 5 (E) — 3

55. Given: f is a continuous function on the interval [0, 2] such that $\int_0^2 f(x)\,dx = 5$. Find $\int_0^1 f(2y)\,dy$.

(A) 10 (B) 5 (C) 2.5 (D) 1.25 (E) 0.625

56. How many distinct 4-letter code words can be made from the letters in the words "PIZZA PIE" if the first letter must be a vowel and the second letter must be a consonant?

(A) 54 (B) 88 (C) 98 (D) 120 (E) 354

57. Expand $10^B \div (10^{(2B)} - 10^B - 1)$ for B = 3. What is the 21st digit after the decimal place?

(A) 0 (B) 1 (C) 3 (D) 5 (E) 8

58. Given: $x^2y + xy^2 + x + y = 63$ and $xy = 6$. Find $x^2 + y^2$

(A) 99 (B) 69 (C) 57 (D) 54 (E) 7

59. Three integers, p, q, and r exist such that they form an arithmetic progression and their product is a prime number. Find the absolute value difference of the smallest and largest of the three integers.

(A) 0 (B) 2 (C) 3 (D) 4 (E) 5

60. Given: $25! \div 5^k$ is an integer. What is the greatest value of k?

(A) 2 (B) 3 (C) 4 (D) 5 (E) 6

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University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation B • 2019
Answer Key

1. D	**21. E**	**41. B**
2. B	**22. E**	**42. C**
3. C	**23. B**	**43. D**
4. B	**24. C**	**44. D**
5. D	**25. B**	**45. D**
6. C	**26. C**	**46. C**
7. A	**27. E**	**47. C**
8. C	**28. A**	**48. A**
9. D	**29. A**	**49. C**
10. C	**30. D**	**50. B**
11. A	**31. A**	**51. E**
12. E	**32. B**	**52. D**
13. E	**33. A**	**53. A**
14. C	**34. E**	**54. D**
15. D	**35. C**	**55. C**
16. D	**36. B**	**56. C**
17. E	**37. A**	**57. C**
18. A	**38. D**	**58. B**
19. A	**39. D**	**59. D**
20. E	**40. B**	**60. E**

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____