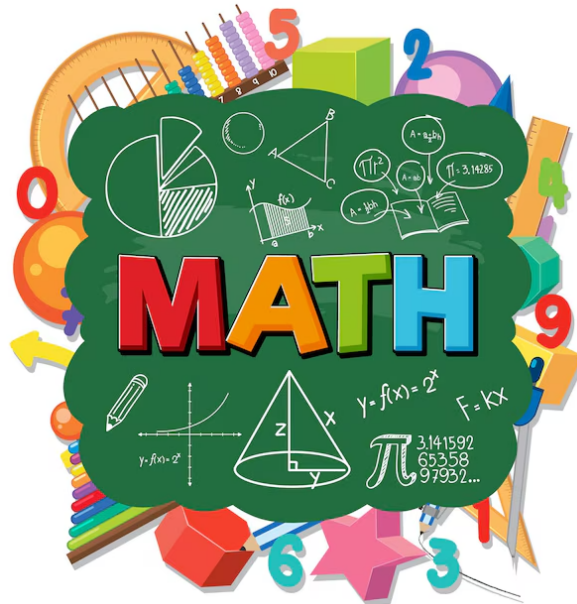UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational A • 2025

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1-2. Lathin left Rankin High on his bicycle and headed west on Hwy 67 at 24 mph for 45 minutes. Next, He turned north and cycled at 30 mph for 40 minutes on Hwy 385. Then, he turned east on Hwy 329 and cycled at 32 mph for 15 minutes to the Cross Roads Steak House where he stopped for supper.

1. Lathin ordered an 8-oz filet for $32.85, a side of broccoli for $3.20, an iced tea for $2.59, and a slice of apple pie for $5.99. The tax rate is 8.25%. He paid for his meal with three $20 bills. Since Kyina was the waitress, he told her to keep the change as a tip. How much was the tip?

   (A) $11.25   (B) $11.36   (C) $11.47   (D) $11.58   (E) $11.69

2. What is the straight-line distance from the Cross Roads Steak House to Rankin High? (nearest tenth)

   (A) 21.8 mi   (B) 22.1 mi   (C) 22.4 mi   (D) 22.7 mi   (E) 23.0 ki

3. Rohan is taking Differential Equations from Mrs. Tan. There are six tests given, with test 6 counting twice. His first five test grades are 92, 88, 82, 91 and 93. If he needs a 90.0 or higher average to earn an A, what is the minimum grade he can make on test 6 and receive an A?

   (A) 92   (B) 93   (C) 94   (D) 95   (E) 96

4. The Pasadena UIL Math team did a fundraiser in the fall. The Quebe Sisters agreed to appear and perform Mr. Cantu's favorite song, Rose of San Antone. Adult tickets cost $27.50 and student tickets cost $15.50. They sold 934 tickets and raised $21,053.00. How many adult tickets were sold?

   (A) 546   (B) 547   (C) 548   (D) 549   (E) 550

5. Nicholas has a jar full of nickels, dimes and quarters. He has a total of 141 coins with a total value of $19.40 and he has 6 more quarters than nickels. How many dimes does he have?

   (A) 46   (B) 47   (C) 48   (D) 49   (E) 50

6. Two years ago, Rob was eight times as old as Janice. In six years, Rob will be four times as old as Janice. How old is Rob?

   (A) 46   (B) 48   (C) 50   (D) 52   (E) 54

7. Madison can mow and edge 3 lawns in 10 hours. Gabriela can mow and edge 2 lawns in 7 hours. If they work together, how long would it take them to mow and edge 8 lawns? (nearest minute)

   (A) 13 hr 35 min   (B) 13 hr 40 min   (C) 13 hr 45 min   (D) 13 hr 50 min   (E) 13 hr 55 min

8-9. A large cylindrical container has a diameter of 14 feet and a height of 18 feet.

8. Find the total area of the cylinder. (nearest whole number)

   (A) 1088 ft^2 (B) 1092 ft^2 (C) 1096 ft^2 (D) 1100 ft^2 (E) 1104 ft^2

9. The cylindrical container stores _____ gallons of water when it is completely filled. (nearest whole number)

   (A) 20,716   (B) 20,720   (C) 20,724   (D) 20,728   (E) 20,732

10. Three times the measure of the complement of ∠T is 16 greater than the supplement of ∠T.
    m∠T = _____.

    (A) 36°   (B) 37°   (C) 38°   (D) 39°   (E) 40°

11. Find the perimeter of △ABC. (nearest tenth)

    (A) 41.0   (B) 41.2   (C) 41.4
    (D) 41.6   (E) 41.8
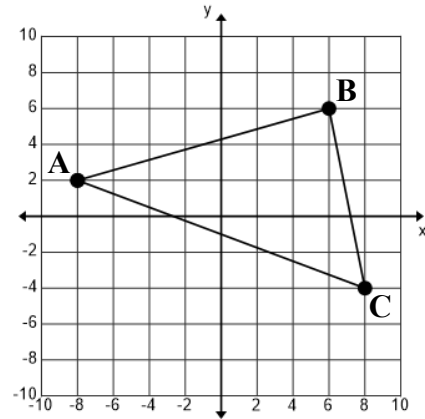
12. Find the area of △ABC. (nearest whole number)

    (A) 74   (B) 75   (C) 76
    (D) 77   (E) 78

13. Point D is the midpoint of $\overleftrightarrow{AC}$. BD = _____.
    (nearest tenth)

    (A) 8.6   (B) 8.8   (C) 9.0
    (D) 9.2   (E) 9.4

Problems 11, 12, 13, 14

14. △ABC is classified as a/an _____ triangle.

    (A) acute   (B) right   (C) obtuse   (D) isosceles   (E) equilateral

15-18. Consider a circle with center O. Chord $\overline{AC}$ is
       parallel to chord $\overline{DF}$. Diameter $\overline{BE}$ is
       perpendicular to chord $\overline{DF}$ and BE = 26.
       $\overline{AC}$ intersects $\overline{BO}$ at point G and BG = 8.
       $\overline{DF}$ intersects $\overline{OE}$ at point H and OH = 8.
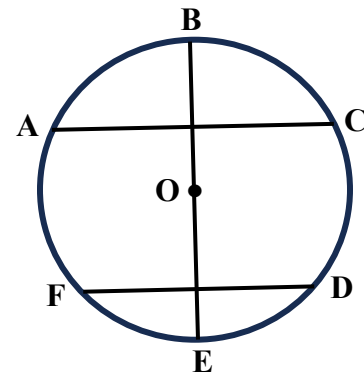
15. The area of △OGC = _____. (nearest tenth)

    (A) 29.6   (B) 29.8   (C) 30.0   (D) 30.2   (E) 30.4

16. FD = _____. (nearest tenth)

    (A) 19.7   (B) 19.9   (C) 20.1   (D) 20.3   (E) 20.5

17. m∠COD = _____°. (nearest tenth)

    (A) 60.0   (B) 60.3   (C) 60.6   (D) 60.9   (E) 61.2

18. The area of sector AOC is _____. (nearest whole number)

    (A) 193   (B) 195   (C) 197   (D) 199   (E) 201

UIL Mathematics

19-20. Consider △DEF with EF = 35, DF = 37 and m∠DEF = 90°. Point G lies on $\overline{DF}$ with DG = 12.

19. The area of △DEG = _____. (nearest whole number)

   (A) 68   (B) 70   (C) 72   (D) 74   (E) 76

20. The perimeter of △EGF = _____. (nearest whole number)

   (A) 68   (B) 70   (C) 72   (D) 74   (E) 76

21. Find the domain of $f(x) = \dfrac{\sqrt{3x-8}}{x-5}$.

   (A) $x \in R \left| x \geq \dfrac{8}{3} \right.$ (B) $x \in R \left| x \neq 5 \right.$ (C) $x \in R \left| x > \dfrac{8}{3}, x \neq 5 \right.$

   (D) $x \in R \left| x \leq \dfrac{8}{3} \right.$ (E) $x \in R \left| x \geq \dfrac{8}{3}, x \neq 5 \right.$

22. Consider the circle $x^2 + y^2 + 6x - 14y - 6 = 0$. The area of the circle is _____.
   (nearest whole number)

   (A) 192   (B) 195   (C) 198   (D) 201   (E) 204

23. Consider the sequence 13, 20, 27, 34, 41, 48, … Find the sum of the first 24 terms.

   (A) 2240   (B) 2242   (C) 2244   (D) 2246   (E) 2248

24. On July 4, 2020, Payton placed $18,000 into an account that earns 5.76% annual interest compounded monthly. On July 4, 2021, Jacob placed $18,000 into an account that earns 6.12% annual interest compounded quarterly. On July 4, 2030, Payton will have _____ more in his account than Jacob.

   (A) $881.65   (B) $882.70   (C) $883.75   (D) $883.80   (E) $883.85

25. A hungry hawk is perched on the edge of the roof of the Tuscola State Bank. The hawk spots a tasty mouse on the ground. The angle of depression from the hawk to the mouse is 22°. The mouse begins moving toward the bank and when the mouse has moved 140 feet closer to the bank, the angle of depression is 37°. How tall is the bank? (nearest foot)

   (A) 118 ft   (B) 120 ft   (C) 122 ft   (D) 124 ft   (E) 126 ft

26. Consider the function $f(x) = x^4 + x^3 + cx^2 + 11x + 30$. If $f(1) = 24$, then $f(4) =$ _____.

   (A) 88   (B) 90   (C) 92   (D) 94   (E) 96

27. Find the shortest distance from the graph of $x^2 + y^2 = 25$ to the point $(8, -2)$.
    (nearest tenth)

    (A) 3.0   (B) 3.2   (C) 3.4   (D) 3.6   (E) 3.8

28. Consider the function $f(x) = 2 - 3\cos\left(\dfrac{5\pi x}{3} + \dfrac{\pi}{6}\right)$. The period of $f(x)$ is _____.

    (A) $\dfrac{5}{6}$ (B) $\dfrac{6}{5}$ (C) $\dfrac{3}{2}$ (D) π (E) 2π

29. Given: $f(x) = 2x + 1$, $g(x) = 3\sqrt{x-5}$, $h(x) = (g \circ f)(x)$. Find the domain of $h(x)$.

    (A) $(-\infty, \infty)$ (B) $[5, \infty)$ (C) $\left[-\dfrac{1}{2}, 5\right]$ (D) $[2, \infty)$ (E) $(5, \infty)$

30. Point $A\left(-7\sqrt{6}, -7\sqrt{2}\right)$ is rotated 150° counterclockwise about the origin to point $B(a, b)$.
    $a + b =$ _____. (nearest tenth)

    (A) 19.2   (B) 19.5   (C) 19.8   (D) 20.1   (E) 20.4

31-32. A regular hexagon has a perimeter of 72 cm².

31. The area of the hexagon is _____ cm². (nearest whole number)

    (A) 362   (B) 365   (C) 368   (D) 371   (E) 374

32. The area of a circle inscribed in the hexagon is _____ cm². (nearest whole number)

    (A) 327   (B) 330   (C) 333   (D) 336   (E) 339

33. Find the shortest distance from the point $(9, 6)$ and the graph of the parametric equations
    $x = 4 - t$ and $y = 7 + t$. (nearest tenth)

    (A) 2.6   (B) 2.8   (C) 3.0   (D) 3.2   (E) 3.4

34. In his physics class, Anthony attached a long spring to the ceiling. He attached a 500-g mass to the
    bottom of the spring and waited for the spring to come to rest at its equilibrium position. Then he
    pulled the mass down 20 cm and released it at $t = 0$. The mass oscillated vertically between 20 cm
    below equilibrium and 20 cm above equilibrium. The position of the mass varied sinusoidally with
    time with a period of two seconds. How far above its equilibrium position was the mass at $t = 2.75$ s?
    (nearest centimeter)

    (A) 11 cm   (B) 12 cm   (C) 13 cm   (D) 14 cm   (E) 15 cm

35. If vector $\mathbf{u} = \langle 8, -6 \rangle$ is orthogonal to vector $\mathbf{v} = \langle -3, b \rangle$, then b = _____ .

(A) −6   (B) −4   (C) −2   (D) 2.25   (E) 4.5

36. In Alexandra's favorite movie, Doc was playing 5-card poker using a standard 52-card deck. He was dealt 4 queens, which surprised the other players because the probability of being dealt 4 of a kind is _____ . (not just 4 queens, but 4 of any kind) (nearest hundred-thousandth)

(A) 0.00024   (B) 0.00036   (C) 0.00048   (D) 0.00060   (E) 0.00072

37-38 . Consider $\triangle ABC$ with vertices $A(2, -3, 5)$, $B(3, -2, -1)$ and $C(-5, 4, -3)$.

37. Find the perimeter of $\triangle ABC$. (nearest tenth)

(A) 28.5   (B) 28.7   (C) 28.9   (D) 29.1   (E) 29.3

38. $m\angle ABC =$ _____ °. (nearest whole number)

(A) 97   (B) 99   (C) 101   (D) 103   (E) 105

39. Consider the sequence $\dfrac{2}{1}, \dfrac{3}{3}, \dfrac{5}{6}, \dfrac{7}{10}, \dfrac{11}{15}, \dfrac{13}{21}, ...$ The sum of the 10th term and 11th term is _____ .

(A) $\dfrac{109}{110}$ (B) $\dfrac{164}{165}$ (C) $\dfrac{329}{330}$ (D) 1   (E) $\dfrac{331}{330}$

40. Find the shortest distance from the point $(-8, -4)$ and the graph of the polar equation

$r = \dfrac{5}{4\cos\theta + 3\sin\theta}$. (nearest tenth)

(A) 9.4   (B) 9.6   (C) 9.8   (D) 10.0   (E) 10.2

41. The y-intercept of the line tangent to the graph of $y = 4 - 2\tan(.25x)$ at $x = \dfrac{\pi}{6}$.

(nearest tenth)

(A) 4.0   (B) 4.2   (C) 4.4   (D) 4.6   (E) 4.8

42. A 13-ft-long ladder is leaning against the wall of Haneli's house. The base of the ladder is being pulled away from the wall at 18 inches per second. Consider the triangle formed by the ground, the wall of the house and the ladder. The rate at which the area of the triangle is changing when the base of the ladder is 5 ft from the wall is _____ ft^2/s. (nearest tenth)

(A) 7.2   (B) 7.4   (C) 7.6   (D) 7.8   (E) 8.0

43. At the instant the traffic light turned green, Carlos' car, which had been waiting at the intersection, began to accelerate at a constant rate of 7.2 ft/s². At the same instant, Napolean's car passed Carlos in the next lane, traveling at a constant speed of 30 mph. How fast will Carlos be going when he catches Napolean? (nearest whole number)

(A) 45 mph (B) 50 mph (C) 55 mph (D) 60 mph (E) 65 mph

44. Julian used the Trapezoidal Rule with four subintervals of equal width to approximate the value of $\int_0^\pi 2\sin(x)dx$. This value is _____ less than the exact value. (nearest thousandth)

(A) 0.199 (B) 0.202 (C) 0.205 (D) 0.208 (E) 0.211

45. Given: $f(x)=-\dfrac{1}{2}x^3-x-1$. Find the value of $\left(f^{-1}\right)'(x)$ when $x=5$.

(A) $-\dfrac{1}{7}$ (B) $-\dfrac{1}{6}$ (C) $-\dfrac{1}{5}$ (D) $-\dfrac{1}{4}$ (E) $-\dfrac{1}{3}$

46. Given: $2y\ln(x)+y^2=21$. Find $\dfrac{dy}{dx}$ when $y=3$. (nearest hundredth)

(A) −0.12 (B) −0.10 (C) −0.08 (D) −0.06 (E) −0.04

47. Consider the differential equation $\dfrac{dy}{dx}=2x-3y$ with the initial condition $y(0)=2$. Use Euler's Method with three steps of equal size to approximate the value of $y(1.5)$. (nearest tenth)

(A) 0.1 (B) 0.2 (C) 0.3 (D) 0.4 (E) 0.5

48. Mr. Cantu has a small herd of elk on his estate. One of the elk mommies gave birth to a calf that weighed 35 pounds. He named the calf Daniel. Daniel gains weight at the rate $\dfrac{dw}{dt}=k(900-w)$ with w in pounds and t in years. Daniel weighed 511 pounds on his first birthday. Solve the differential equation and predict Daniel's weight on his fourth birthday. (nearest pound)

(A) 855 pounds (B) 860 pounds (C) 865 pounds (D) 870 pounds (E) 875 pound

49. Find the fourth degree Maclaurin polynomial for $f(x)=e^{2x}$ and use it to approximate $f(0.5)$ The exact value of $f(0.5)$ is _____ greater than the approximation. (nearest hundred-thousandth)

(A) 0.00993 (B) 0.00995 (C) 0.00997 (D) 0.00999 (E) 0.0101

50. Consider the curve defined by the parametric equations $x(\theta) = 2\cos(\theta)$, $y(\theta) = 2\sin(\theta)$. The x-intercept of the line tangent to the curve when $\theta = \dfrac{\pi}{6}$ is the point (a, b). $a = $ ____. (nearest tenth)

    (A) 2.3   (B) 2.5   (C) 2.7   (D) 2.9   (E) 3.1

51-52. Consider the region bounded above by the graph of $y = f(x)$, which is part of a circle, below by the graph of $y = g(x)$, which is part of a parabola, and on the right by the y-axis.
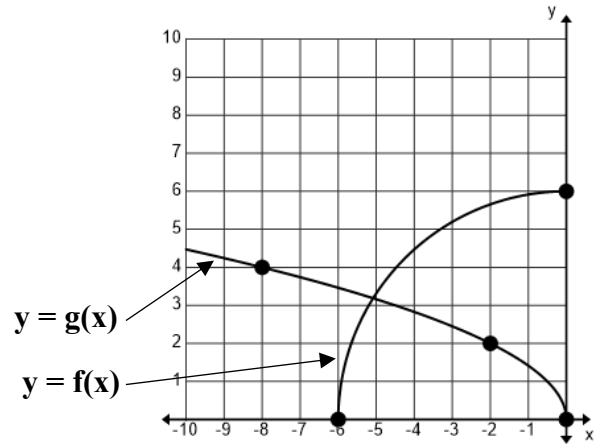
51. Find the area of the bounded region. (nearest tenth)

    (A) 14.7
    (B) 14.9
    (C) 15.1
    (D) 15.3
    (E) 15.5

$y = g(x)$

$y = f(x)$

Problems 51, 52

52. Find the volume of the solid generated when the bounded region is revolved about the line $x = 4$. (nearest whole number)

    (A) 567   (B) 570   (C) 573   (D) 576   (E) 579

Test #	1	2	3	4	5	6
Score	262	258	186	308	258	242

53-54. Raymundo took six number sense tests this week. His scores appear in the table above.

53. The difference between the median score and the mean score is _____. (nearest tenth)

    (A) 4.9   (B) 5.1   (C) 5.3   (D) 5.5   (E) 5.7

54. A modified box plot showed that _____ of the test scores are outliers.

    (A) 0   (B) 1   (C) 2   (D) 3   (E) 4

55. Assume that the Norwegian Muskox has a mean weight of 628 pounds with a standard deviation of 42 pounds. Edgar owns a 702-pound muskox. This weight places him at the _____ percentile.

    (A) 88th   (B) 90th   (C) 92nd   (D) 94th   (E) 96th

56. Luka enjoys playing horse against Russell. They have played hundreds of times over the years and Luka wins 25% of the time. Luka watched Argyle play Grapevine last night and after the game he challenged Russell to 12 games of horse. Find the probability that Luka won at least 6 of the games. (nearest thousandth)

    (A) 0.054   (B) 0.057   (C) 0.060   (D) 0.063   (E) 0.066

57. Consider the difference D of two independent random variables X and Y. $D = X - Y$.
Given that $\sigma_x = 5.20$ and $\sigma_Y = 4.98$, what is σ_D, the standard deviation of D?

(A) 0.22 (B) 1.50 (C) 7.20 (D) 8.88 (E) 10.18

Week #	1	2	3	4	5	6
Minutes	45	55	60	70	75	90

58-59. Rachel is planning to enter the Ray Roberts 10-mile Trail Run in ten weeks. During the work week, she runs 3 miles every morning at 5:00 AM. She goes on a long run every Saturday morning. These long runs are on an unmarked trail, so she goes by time. The times of her first six long runs appear above. She calculated a LSRL to fit the data.

58. Find the value of the residual for the week 5 long run. (nearest tenth)

(A) −3.5 (B) −3.3 (C) −3.1 (D) −2.9 (E) −2.7

59. Use the LSRL to predict the time of her last training run (week 9) before the race. (nearest minute)

(A) 109 mi (B) 112 mi (C) 115 mi (D) 118 mi (E) 121 mi

60. Assume that a study by the Montana High School Association randomly selected 128 juniors and 112 seniors who participate in cross country to be part of a study. One of questions asked students if they plan to participate in cross country at the college level. Results of the study showed that 32 juniors and 22 seniors are planning to run cross country in college. An appropriate test was performed at the $\alpha = 0.05$ level to consider whether there is a significant difference in the responses of the juniors and the seniors to this question. The MHSA concluded, "Based on a p-value of _____, we fail to reject H_0 at the $\alpha = 0.05$ level. We conclude that there is insufficient evidence to conclude that the proportion of juniors who plan to run cross country at the college level differs from the proportion of seniors who plan to run cross country at the college level." (nearest hundredth)

(A) 0.321 (B) 0.324 (C) 0.327 (D) 0.330 (E) 0.333

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational A ● 2025
Answer Key

1. E	21. E	41. A
2. C	22. D	42. B
3. A	23. C	43. D
4. C	24. B	44. D
5. B	25. C	45. A
6. C	26. B	46. C
7. B	27. B	47. E
8. D	28. B	48. C
9. D	29. D	49. B
10. B	30. C	50. A
11. E	31. E	51. E
12. A	32. E	52. A
13. D	33. B	53. E
14. A	34. D	54. C
15. C	35. B	55. E
16. E	36. A	56. A
17. C	37. D	57. C
18. D	38. B	58. A
19. A	39. C	59. B
20. D	40. C	60. A