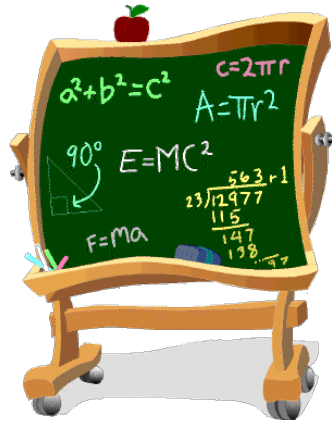




UNIVERSITY INTERSCHOLASTIC LEAGUE

# Mathematics

Invitational A • 2024



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1. Luke, Paige, Ryan and Elliot went to the Salado Pizza Palace for supper. They ordered two large pizzas which cost \$15.95 each and two pitchers of coke which cost \$5.95 each. They agreed to share the cost equally. If the tax rate is 8.125% and they added a 25% after tax tip, how much did Luke pay?

- (A) \$14.68      (B) \$14.80      (C) \$14.92      (D) \$15.04      (E) \$15.16

2. The Calhoun math team did a fundraiser in October. Stacy's favorite band, Santana, agreed to come to Port Lavaca and perform. A student ticket cost \$12.50 and an adult ticket cost \$21.50. If they sold a total of 498 tickets and they grossed \$9,213 from ticket sales, how many adult tickets were sold?

- (A) 326      (B) 328      (C) 330      (D) 332      (E) 334

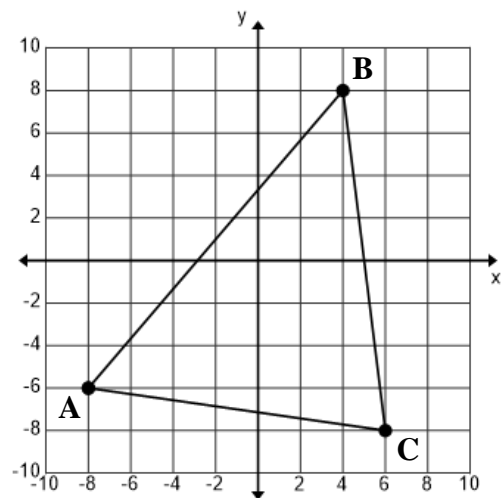
3-6. Consider  $\triangle ABC$  shown on the right.

3. The point  $P(4, b)$  lies on the perpendicular bisector of  $\overline{BC}$ .  $b = \underline{\hspace{2cm}}$ .

- (A)  $-0.125$   
 (B)  $0$   
 (C)  $0.125$   
 (D)  $0.25$   
 (E)  $0.375$

4. Find the perimeter of  $\triangle ABC$ . (nearest tenth)

- (A) 48.1  
 (B) 48.4  
 (C) 48.7  
 (D) 49.0  
 (E) 49.3



Problems 3, 4, 5, 6

5. Find the area of  $\triangle ABC$ .

- (A) 102      (B) 104      (C) 106      (D) 108      (E) 110

6. Point D, not shown, lies on  $\overline{AB}$ . If  $\overline{CD}$  is the median from point C to side  $\overline{AB}$ , then  $CD = \underline{\hspace{2cm}}$ . (nearest tenth)

- (A) 11.4      (B) 11.7      (C) 12.0      (D) 12.3      (E) 12.6

7. The roots of  $ax^3 + bx^2 + cx + d = 0$  are  $-2, 1$  and  $3$ . If  $a = 2$ , then  $c = \underline{\hspace{2cm}}$ .

- (A)  $-10$       (B)  $-8$       (C)  $-6$       (D)  $-4$       (E)  $-2$

8. The number of fleas in a house is directly related to the number of dogs and inversely related to the square of the number of ferrets. Landon's house has 240 fleas, 6 dogs and 2 ferrets. If Schafer's house has 4 ferrets and 8 dogs, how many fleas are in his house?

- (A) 72      (B) 74      (C) 76      (D) 78      (E) 80

9. When  $2x^3 + 6x^2 + 4x + 8$  is divided by  $x + 2$ , the remainder is \_\_\_\_\_.
- (A) 0                      (B) 2                      (C) 4                      (D) 6                      (E) 8
10. Matthew is taking Linear Algebra this year at Southwestern University during 7<sup>th</sup> period. Each of the 4 test grades count once and the final exam grade counts twice when averaging his grade for the course. His test grades are 86, 92, 91 and 87. He needs to have an average of at least 90.0 to earn an A for the course. What is the minimum score he needs to make on the final exam to earn an A?
- (A) 90                      (B) 91                      (C) 92                      (D) 93                      (E) 94
11. Points A, B, C and D lie on a circle such that  $\overleftrightarrow{BD}$  intersects  $\overleftrightarrow{AC}$  at point E. If  $BE = 4$ ,  $AE = 6$  and  $DE = 9$ , then  $CE =$  \_\_\_\_\_. (nearest tenth)
- (A) 5.8                      (B) 6.0                      (C) 6.2                      (D) 6.4                      (E) 6.6
12. Points F, G, H and I lie on a circle such that  $\overleftrightarrow{HF}$  intersects  $\overleftrightarrow{IG}$  at point J.  
Given: minor arcs GF and IH with  $m\widehat{GF} = 88^\circ$  and  $m\widehat{IH} = 108^\circ$ . Find  $m\angle HJG$ .
- (A)  $80^\circ$                       (B)  $82^\circ$                       (C)  $84^\circ$                       (D)  $86^\circ$                       (E)  $88^\circ$
- 13-14.  $\triangle ABC$  is similar to  $\triangle DEF$ . Point G lies on  $\overline{AC}$  and  $\overline{BG}$  bisects  $\angle ABC$ .  
Given:  $AB = 22$ ,  $BC = 20$ ,  $AC = 14$  and  $EF = 15$ .
13. Find the perimeter of  $\triangle DEF$ . (nearest tenth)
- (A) 42.0                      (B) 42.5                      (C) 43.0                      (D) 43.5                      (E) 44.0
14.  $CG =$  \_\_\_\_\_. (nearest tenth)
- (A) 6.7                      (B) 6.9                      (C) 7.1                      (D) 7.3                      (E) 7.5
15. Consider rhombus PQRS with  $PR = 12.274$  and  $QS = 25.166$ . The area of the rhombus is \_\_\_\_\_. (nearest whole number)
- (A) 150                      (B) 152                      (C) 154                      (D) 156                      (E) 158
- 16-17. A large cylindrical container has a diameter of 4 feet and a height of 8 feet.
16. Find the total area of the cylinder. (nearest whole number)
- (A)  $117 \text{ ft}^2$                       (B)  $120 \text{ ft}^2$                       (C)  $123 \text{ ft}^2$                       (D)  $126 \text{ ft}^2$                       (E)  $129 \text{ ft}^2$
17. How many gallons of water are required to completely fill the container? (nearest whole number)
- (A) 740                      (B) 743                      (C) 746                      (D) 749                      (E) 752

18-19. Consider a circle with center  $O$  and diameter  $\overline{BD}$ . Chord  $\overline{AC}$  is perpendicular to  $\overline{BD}$ .  $\overline{AC}$  intersects  $\overline{BD}$  at point  $E$ . Given:  $AC = 80$  and  $BE = 32$ .

18. Find the area of  $\triangle OEC$ . (nearest whole number)

- (A) 180                      (B) 182                      (C) 184                      (D) 186                      (E) 188

19. Find the length of minor arc  $CD$ . (nearest tenth)

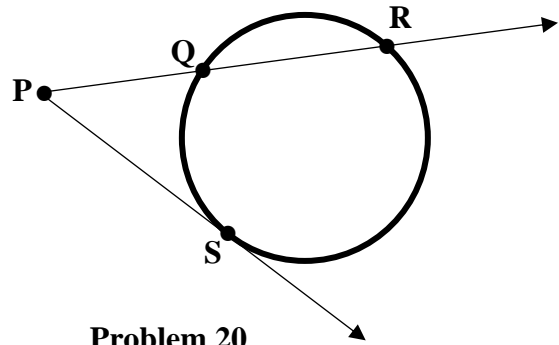
- (A) 72.6                      (B) 72.9                      (C) 73.2                      (D) 73.5                      (E) 73.8

20.  $\overline{PS}$  is tangent to the circle at point  $S$ .

$PQ = 16$  and  $QR = 22$ .

$PS = \underline{\hspace{2cm}}$ . (nearest tenth)

- (A) 24.1  
(B) 24.4  
(C) 24.7  
(D) 25.0  
(E) 25.3



Problem 20

21. Leonardo has a collection of 24 marbles that are identical in size, but vary in color. Twenty of them are green, 2 are red and 2 are blue. How many different ways can he arrange them in a row?

- (A) 7,695                      (B) 15,939                      (C) 31,878                      (D) 63,756                      (E) 127,512

22. Consider the sequence 7, 11, 15, 19, 23, 27,... Find the sum of the first 19 terms.

- (A) 813                      (B) 814                      (C) 815                      (D) 816                      (E) 817

23. Consider the sequence 4, 6, 9, 13.5, 20.25, 30.375,... Find the sum of the first 11 terms. (nearest tenth)

- (A) 681.8                      (B) 682.9                      (C) 684.0                      (D) 685.1                      (E) 682.2

24-25. The measure of an interior angle of a regular polygon is  $135^\circ$ . The length of each side is 12.

24. Find the area of the polygon. (nearest whole number)

- (A) 686                      (B) 689                      (C) 692                      (D) 695                      (E) 698

25. Find the area of the circle that is inscribed in the polygon. (nearest whole number)

- (A) 656                      (B) 659                      (C) 662                      (D) 665                      (E) 668

26.  $f(x) = \frac{2x+7}{3x-2}$  and  $g(x) = \frac{4x-1}{5x+2}$ .  $f^{-1}(2) - g^{-1}(2) = \underline{\hspace{2cm}}$ .

- (A)  $3\frac{5}{12}$       (B)  $3\frac{1}{2}$       (C)  $3\frac{7}{12}$       (D)  $3\frac{2}{3}$       (E)  $3\frac{3}{4}$

27. Consider the function  $g(x) = \sqrt{\frac{5x+4}{3x}}$ . Find the range of  $g(x)$ .

- (A)  $(-\infty, \infty)$       (B)  $(-0.8, \infty)$       (C)  $\left(\sqrt{\frac{5}{3}}, \infty\right)$       (D)  $[0, \infty)$       (E)  $(0, \infty)$

28-29. Consider the polynomial function  $f(x) = 3x^5 + x^4 - 5x^3 + dx^2 - 8x + 4$ .

28. If  $f(-1) = 20$ , then  $d = \underline{\hspace{2cm}}$ .

- (A) 1      (B) 3      (C) 5      (D) 7      (E) 9

29. The polynomial function  $f(x)$  has            real zeros.

- (A) 1      (B) 2      (C) 3      (D) 4      (E) 5

30. On June 1<sup>st</sup> of 2018, Ian's grandpa placed \$25,000 into an account for Ian that earns interest at a rate of 7.25% compounded monthly. Ian plans to withdraw all of the money in the account on June 1<sup>st</sup> of 2024 and use it toward the purchase of a new RAV4 Hybrid from Georgetown Toyota. If the total cost including tax, title and license is \$38,773.56, how much money will Ian have to come up with?

- (A) \$100      (B) \$200      (C) \$300      (D) \$400      (E) \$500

31. Wyatt is playing 5 card poker with his friends on the Georgetown High School track team. They are using a standard deck of 52 cards. What is the probability that he will be dealt a hand (5 cards) with at least one queen? (nearest thousandth)

- (A) 0.329      (B) 0.335      (C) 0.341      (D) 0.347      (E) 0.353

32. Find the period of the graph of  $y = 3 + 2\cos\left(\frac{\pi x}{3} - \frac{\pi}{6}\right)$ .

- (A) 2      (B) 3      (C) 6      (D)  $\pi$       (E)  $2\pi$

33. Six chairs are arranged in a row. Six students from Calhoun are to sit in the chairs. If Seth insists on sitting next to Doreen, how many distinct seating arrangements are possible?

- (A) 36      (B) 120      (C) 240      (D) 480      (E) 720

34. Ariana leaves Georgetown and flies on a bearing of  $210^\circ$  for 120 miles. She lands at an airport and picks up her mom and then she flies on a bearing of  $300^\circ$  for 180 miles and lands at a different airport near her lake home. After 4 days, Ariana flies directly back to Georgetown. If her Bell 407 helicopter travels at an average speed of 112 mph, how much time will it take her to fly directly from her lake house to Georgetown? (nearest minute)
- (A) 1 hr 44 min    (B) 1 hr 47 min    (C) 1 hr 50 min    (D) 1 hr 53 min    (E) 1 hr 56 min
35. Consider a hard drive disk with a diameter of 3.5 inches. If it is operating at 7200 rpm, what is the linear velocity of a point on the outer edge of the disk? (nearest whole number)
- (A) 69 mph    (B) 71 mph    (C) 73 mph    (D) 75 mph    (E) 77 mph
36. The equation of an ellipse that is centered at  $(4, 2)$ , with vertices  $(0, 2)$  and  $(8, 2)$ , and with eccentricity  $= \frac{\sqrt{7}}{4}$  is  $ax^2 + by^2 + cx + dy + e = 0$ .  $e =$  \_\_\_\_\_.
- (A) 16    (B) 32    (C) 64    (D) 80    (E) 96
37. The y-intercept of the graph of the polar equation  $r = \frac{-6}{2\sin\theta - \cos\theta}$  is  $(0, b)$ .  $b =$  \_\_\_\_\_.
- (A) -6    (B) -5    (C) -4    (D) -3    (E) -2
38. Find the angle between the vectors  $u = 3i + 5j$  and  $v = 4i + 7j$ . (nearest tenth)
- (A)  $1.2^\circ$     (B)  $2.3^\circ$     (C)  $3.4^\circ$     (D)  $4.5^\circ$     (E)  $5.6^\circ$
39. Consider the curve represented by the parametric equations  $x = 2\sec(\theta)$  and  $y = 4\tan(\theta)$ . Find the distance between the foci. (nearest tenth)
- (A) 8.7    (B) 8.9    (C) 9.1    (D) 9.3    (E) 9.5
40. Find the distance between the point  $P(1, 2, 3)$  and the plane  $2x - 3y + 5z = 4$ . (nearest tenth)
- (A) 1.1    (B) 1.3    (C) 1.5    (D) 1.7    (E) 1.9
41. Given:  $f(0.5) = 0.5$ ,  $f(6.5) = 0.5$ , and  $f'(x) = \frac{2\pi \cos^2\left(\frac{\pi x}{6}\right) - \pi}{3}$ . Find the sum of the values of  $x$  in the open interval  $(0.5, 6.5)$  that satisfy the Mean Value Theorem for the function  $f$  on the closed interval  $[0.5, 6.5]$ ? (rad) (nearest tenth)
- (A) 5.6    (B) 5.8    (C) 6.0    (D) 6.2    (E) 6.4

42. Consider the function  $h(x) = .5e^x$ . At what value of  $x$  is the slope of the line tangent to the graph of  $y = h(x)$  equal to 2.2? (nearest hundredth)

- (A) 1.46            (B) 1.48            (C) 1.58            (D) 1.60            (E) 1.62

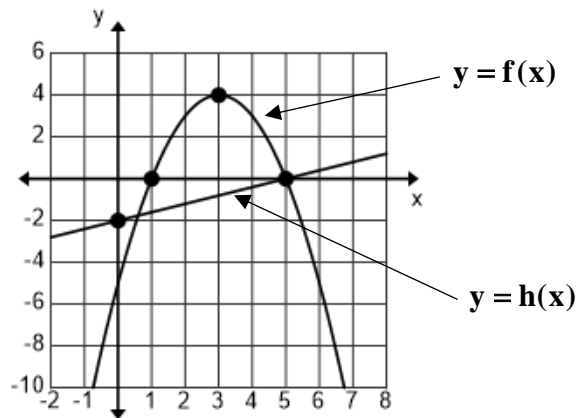
43-44. Consider the graphs of  $y = f(x)$  and  $y = h(x)$ .

43. Find the area bounded by the graphs of  $y = f(x)$  and  $y = h(x)$ . (nearest tenth)

- (A) 14.2            (B) 14.4            (C) 14.6  
(D) 14.8            (E) 15.0

44. Find the volume generated by revolving the region bounded by the graphs of  $y = f(x)$  and  $y = h(x)$  about the line  $x = -4$ . (nearest tenth)

- (A) 602.2            (B) 603.3            (C) 604.4            (D) 605.5            (E) 606.6

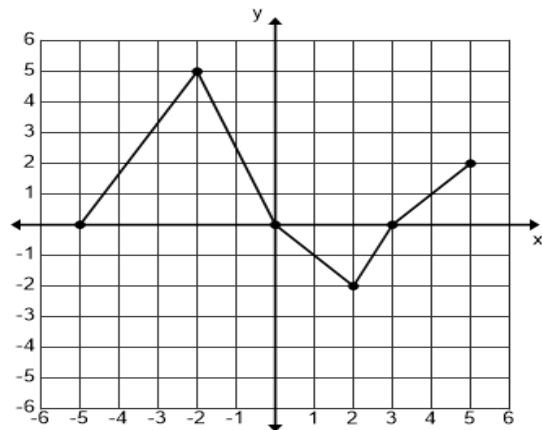


Problems 43, 44

45-46. Consider the graph of  $y = f(x)$ .

45. The function  $f(x)$  is continuous for  $-5 \leq x \leq 5$ . The graph of  $y = f(x)$  consists of five line segments as shown. The average value of  $f(x)$  on the interval  $-5 \leq x \leq 5$  is \_\_\_\_\_.

- (A)  $\frac{11}{10}$             (B)  $\frac{23}{20}$             (C)  $\frac{6}{5}$   
(D)  $\frac{5}{4}$             (E)  $\frac{13}{10}$



Problems 45, 46

46. Find the value of  $\int_{-5}^5 f'(x)dx$ . (nearest tenth)

- (A) 1.7            (B) 2.0            (C) 2.3            (D) 2.6            (E) 2.9

47. Let  $F(x)$  be an antiderivative of  $\frac{x^2 + 2\cos(x)}{x^3 - 7}$ . If  $F(4) = 3$ , then  $F(12) =$  \_\_\_\_\_.  
(nearest hundredth)

- (A) 3.92            (B) 4.03            (C) 4.14            (D) 4.25            (E) 4.36

48. Given:  $\int_2^{-6} f(x)dx = -7$  and  $\int_2^8 f(x)dx = 9$ .  $\int_{-6}^8 f(x)dx = \underline{\hspace{2cm}}$ .

- (A) -2                      (B) 2                      (C) 10                      (D) 16                      (E) 22

49. Given:  $3x^2 - 2xy + y^2 = 6$ . At the point  $P(1, b)$ ,  $b < 0$ ,  $\frac{dy}{dx} = \underline{\hspace{2cm}}$ . (nearest tenth)

- (A) 1.6                      (B) 1.8                      (C) 2.0                      (D) 2.2                      (E) 2.4

50-52. The position of a particle is given by the parametric equations  $x(t) = e^{-3t}$  and  $y(t) = \ln(2t^2 + 5)$  for  $0 \leq t \leq 10$ .

50. Find the velocity vector when  $t = 8$ . (nearest hundredth)

- (A)  $\langle 3.19, 0.36 \rangle$     (B)  $\langle 3.22, 0.33 \rangle$     (C)  $\langle 3.25, 0.30 \rangle$     (D)  $\langle 3.28, 0.27 \rangle$     (E)  $\langle 3.31, 0.24 \rangle$

51. Find the speed of the particle at  $t = 8$ . (nearest tenth)

- (A) 2.9                      (B) 3.1                      (C) 3.3                      (D) 3.5                      (E) 3.7

52. Find the total distance traveled by the particle from  $t = 2$  to  $t = 4$ . (nearest hundredth)

- (A) 1.51                      (B) 1.62                      (C) 1.73                      (D) 1.84                      (E) 1.95

Test #	1	2	3	4	5	6	7	8	9
Score	274	310	328	337	355	319	337	391	346

Dhylan took nine old number sense tests this week to prepare for Saturday's UIL meet. His scores are listed in the table above. Use the table above for problems 53 and 54.

53. Find the sum of the mean, median and mode of the scores.

- (A) 1005                      (B) 1006                      (C) 1007                      (D) 1008                      (E) 109

54. How many of the scores are classified as outliers?

- (A) 0                      (B) 1                      (C) 2                      (D) 3                      (E) 4

55. The Dublin Bottling Company produces bottles of Crème Soda and stores them in a huge warehouse until they are shipped. The mean amount of soda in each bottle is 2.00 liters with a standard deviation of 0.02 liters. The Brownwood Safeway placed a large order and when the shipment arrived, the store manager randomly selected 12 bottles and measured the amount of soda in each bottle. If three or more bottles have less than 1.96 liters, the shipment will be rejected. Find the probability that this order was rejected. Assume the amounts of soda in the bottles are independent of each other. (nearest ten-thousandth)

- (A) 0.0022                      (B) 0.0033                      (C) 0.0044                      (D) 0.0055                      (E) 0.0066



Miles per Week	44	50	54	59	66	69
5-K time (minutes)	18.2	17.5	17.3	16.9	16.2	15.9

Aryan keeps a record of his weekly mileage and his 5-K times at cross country meets. Use the table above for problems 56 and 57.

56. Aryan plotted the data and observed a strong, negative, linear relationship between his weekly mileage and his 5-K times. Statistical software generated a LSRL. Find the value of the residual for the data point (50 miles per week, 17.5 minutes). (nearest thousandth)

- (A)  $-0.127$       (B)  $-0.123$       (C)  $-0.119$       (D)  $-0.115$       (E)  $-0.111$

57. Find the time predicted by the LSRL if Aryan increases his mileage to 80 miles per week. (nearest hundredth of a minute)

- (A) 14.95      (B) 14.98      (C) 15.01      (D) 15.04      (E) 15.07

58. A survey asked a random sample of 500 California high school students to name their favorite college mascot. From the sample, 39% selected Willie the Wave from Pepperdine University. Construct a 90% confidence interval for the proportion of all California high school students who would select Willie the Wave as their favorite college mascot. (nearest ten-thousandth)

- (A)  $\{.3511, .4229\}$    (B)  $\{.3521, .4239\}$    (C)  $\{.3531, .4249\}$    (D)  $\{.3541, .4259\}$    (E)  $\{.3551, .4269\}$

Score	1	2	3	4	5
Probability	.02	.06	.14	.33	.45

A large number of students from the Idaho Falls STEM Academy take the AP Electricity and Magnetism exam each spring. Based on results over the last ten years, the counselor created the table above. Define  $X$  to be the AP score on the AP Electricity and Magnetism exam for a randomly selected student. Use the table above for problems 59 and 60.

59. Compute the mean of the random variable  $X$ . (nearest hundredth)

- (A) 4.07      (B) 4.09      (C) 4.11      (D) 4.13      (E) 4.15

60. Compute the standard deviation of the random variable  $X$ . (nearest thousandth)

- (A) 0.985      (B) 0.989      (C) 0.993      (D) 0.997      (E) 1.001

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**University Interscholastic League  
MATHEMATICS CONTEST  
HS • Invitational A • 2024  
Answer Key**

<b>1. B</b>	<b>21. D</b>	<b>41. C</b>
<b>2. D</b>	<b>22. E</b>	<b>42. B</b>
<b>3. A</b>	<b>23. C</b>	<b>43. A</b>
<b>4. C</b>	<b>24. D</b>	<b>44. E</b>
<b>5. E</b>	<b>25. B</b>	<b>45. B</b>
<b>6. C</b>	<b>26. C</b>	<b>46. B</b>
<b>7. A</b>	<b>27. D</b>	<b>47. C</b>
<b>8. E</b>	<b>28. C</b>	<b>48. D</b>
<b>9. E</b>	<b>29. C</b>	<b>49. C</b>
<b>10. C</b>	<b>30. B</b>	<b>50. E</b>
<b>11. B</b>	<b>31. C</b>	<b>51. C</b>
<b>12. B</b>	<b>32. C</b>	<b>52. D</b>
<b>13. A</b>	<b>33. C</b>	<b>53. C</b>
<b>14. A</b>	<b>34. E</b>	<b>54. A</b>
<b>15. C</b>	<b>35. D</b>	<b>55. A</b>
<b>16. D</b>	<b>36. C</b>	<b>56. B</b>
<b>17. E</b>	<b>37. D</b>	<b>57. A</b>
<b>18. A</b>	<b>38. A</b>	<b>58. D</b>
<b>19. D</b>	<b>39. B</b>	<b>59. D</b>
<b>20. C</b>	<b>40. A</b>	<b>60. D</b>