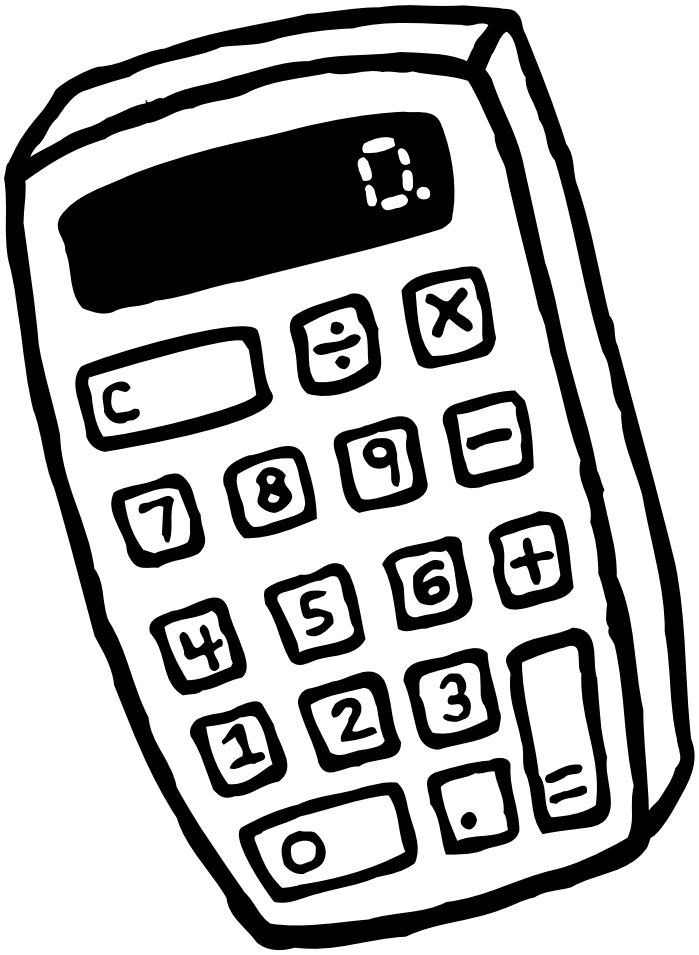
UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational A • 2019



DO NOT TURN THIS PAGE UNTIL

YOU ARE INSTRUCTED TO DO SO!

1. **Evaluate:** $1 + 11^2 \div (2 + 9) + 1 \times 9$

   (A) 79 **(B)** $27\frac{9}{11}$ **(C)** 21 **(D)** $20\frac{2}{11}$ **(E)** 11

2. **Let A = {a, c, u, t, e}, O = {o, b, t, u, s, e}, and R = {r, i, g, h, t}. The number of elements in $(A \cup R) \cap O$ is:**

   (A) 1 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

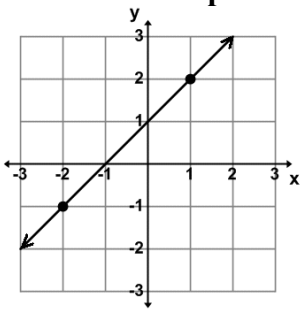
3. **111A09201B ÷ 9 has a remainder of 5. Find the least value of A + B.**

   (A) 8 **(B)** 7 **(C)** 6 **(D)** 5 **(E)** 4

4. **Mae B. Tulong had twelve yards of rope. She cut off a length of rope that was 2 yards 1 foot 8 inches long. Then she divided the remaining length of rope into four equal parts. How long was each of the four equal parts of rope?**

   (A) 1 yd 1' 4" **(B)** 1 yd 2' 8" **(C)** 2 yds 0' 11" **(D)** 2 yds 2' 1" **(E)** 2 yds 1' 1"

5. **Which of the following points lies on a line parallel to the line shown and containing point (0, 3)?**



   (A) (9, 6) **(B)** (7, 11) **(C)** (11, 15) **(D)** (— 7, — 4) **(E)** (— 12, — 12)

6. **Let $4x^2 + 17x - 15 = (ax + b)(cx + d)$. Find $a + b + c + d$.**

   (A) 3 **(B)** 6 **(C)** 7 **(D)** 2 **(E)** 12

7. **Let $(2x - 1)^2(2x + 1) = ax^3 + bx^2 + cx + d$. Find $a + b + c + d$.**

   (A) 1 **(B)** 3 **(C)** 7 **(D)** 11 **(E)** 15

8. **Simplify:** $\left(\frac{x^2 - 3x - 10}{x^2 + 2x - 35}\right) \div \left(\frac{x^2 + 9x + 14}{x^2 + 4x - 21}\right)$

   (A) $x - 3$ **(B)** $\frac{x + 7}{x - 5}$ **(C)** $\frac{x - 3}{x + 7}$ **(D)** $\frac{x^2 + 4x + 4}{x^2 + 4x - 21}$ **(E)** $\frac{x - 7}{x + 3}$

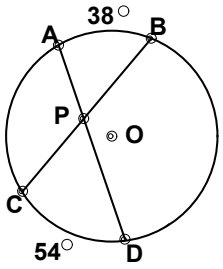
9. **Leo Oiler drew a polyhedron with 7 faces and 11 edges. How many vertices does it have?**

   (A) 2 **(B)** 6 **(C)** 13 **(D)** 16 **(E)** 20

10. Two lines are _____?_____ if and only if the product of their slopes is — 1.

(A) parallel (B) skew (C) collinear (D) perpendicular (E) intersecting

11. Find m∠APB. (drawing is not to scale)



(A) 76° (B) 46° (C) 44° (D) 16° (E) 80°

12. A right cylinder can of *PaPi* Spinach has a diameter length of 4" and a height of 5". What is the total surface area of the spinach can? (nearest tenth)

(A) 88.0 in^2 (B) 56.5 in^2 (C) 75.4 in^2 (D) 44.0 in^2 (E) 62.8 in^2

13. If $\frac{3x+2}{x-1} - \frac{x-3}{2x+1} = \frac{ax^2+bx+c}{dx^2+ex+f}$, then $a+b+c+d+e+f$ equals:

(A) 21 (B) 19 (C) 18 (D) 16 (E) 15

14. If $a_1 = 1$, $a_2 = 3$, $a_3 = -5$ and $a_n = a_{n-1} + a_{n-3} - a_{n-2}$, where $n \geq 4$, then a_6 equals:

(A) 11 (B) 9 (C) 3 (D) — 1 (E) — 7

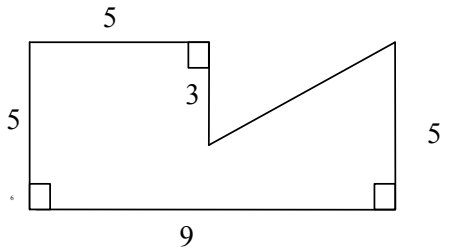
15. Let $x - 3y = 5$ and $2y + z = 3$ and $2 - z = x$. Find $x + yz$.

(A) — 103 (B) — 63 (C) — 8 (D) 13 (E) 77

16. Which of the following mathematicians is noted for his work with sets, probability, and logic?

(A) John Venn (B) Leonard Euler (C) Euclid (D) Alan Turing (E) John Napier

17. Find the perimeter this hexagon? All lengths are in cm.



(A) 27 cm (B) 31 cm (C) 32 cm (D) 36 cm (E) not enough data

18. Thirty seniors took the state math test last year. Twenty-two of them were boys and eight were girls. All of them had an equal chance to win one of the top three medals. What was the probability that two girls and one boy won one of the top three medals? (nearest whole percent)

(A) 1% **(B) 2%** **(C) 15%** **(D) 30%** **(E) 46%**

19. Given: $f(x) = 3 - 2\sin(x + 4)$, where the domain is $\{x \mid x \in \text{Reals}\}$ and the range is $\{f(x) \mid a \leq f(x) \leq b \text{ and } y \in \text{Reals}\}$. Which of the following is not in the range?

(A) 1.5 **(B) 3.124** **(C) 2.04** **(D) 5.333...** **(E) 4.75**

20. The expression $(1 - \cos\theta)(1 + \cos\theta)(1 + \cot^2\theta)$ is equivalent to:

(A) $\csc^2\theta$ **(B) $\frac{1}{\sec\theta}$** **(C) 1** **(D) $\frac{1}{\csc\theta}$** **(E) $\sin^2\theta + 1$**

21. $e^{3i} = \cos(3) + i\sin(3)$ is an example of _____ formula.

(A) Aryabhata's **(B) Bayes'** **(C) Cantor's** **(D) Diophantus'** **(E) Euler's**

22. If $f(x) = x^2 - 3x + 2$ and $g(x) = 2x^2 - x + 3$, then $g(f(4)) = ?$

(A) 69 **(B) 182** **(C) 4** **(D) 13** **(E) 870**

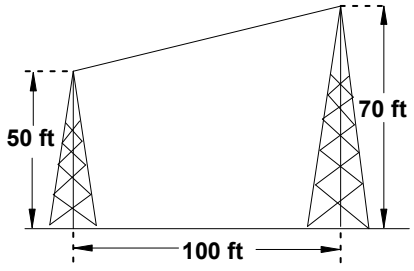
23. $(8x^3 - 4x^2 - 2x + 1) \div (2x + 1)$ has a remainder of _____.

(A) -1 **(B) -0.5** **(C) 0** **(D) 0.5** **(E) 1**

24. Find the absolute value difference between coefficients of the x^2y^3 term and the x^3y^2 term in the expansion of $(3x + 2y)^5$.

(A) 36 **(B) 144** **(C) 330** **(D) 360** **(E) 720**

25. A cable is connected from the shorter tower to the taller tower. What is the minimum length of the cable? (nearest inch)



(A) 102'3" **(B) 102'0"** **(C) 101'8"** **(D) 101'6"** **(E) 101'0"**

26. Find the area bounded by $f(x) = x^3$, $f(y) = -2$, and $f(y) = 1$. (square units)

(A) -3.75 **(B) -3** **(C) -1** **(D) 4** **(E) 4.25**

27. What is the slope of the secant line to the graph of $f(x) = 2x^2 + 3x - 4$ passing through the points (1, m) and (— 3, n)?

(A) 1.5 (B) 1 (C) — 1 (D) — 1.5 (E) — 2

28. Find the 20^{th} term given the sequence: 3, 8, 15, 24, 35, 48, ...

(A) 483 (B) 420 (C) 380 (D) 399 (E) 440

29. A box of golf balls contains 6 white ones, 4 pink ones, and 2 blue ones. Three balls are randomly drawn from the box, without replacement. What are the odds that they are all the same color?

(A) 12% (B) 60% (C) 10% (D) 6% (E) 55%

30. The Shawk Electric Company charges a monthly base fee of $10.50 and a usage fee of 8¢ per kilowatt hour used. The company offers a $25.00 credit if the kilowatt usage is over 1200 kWh. How much would the bill be before taxes if the monthly usage was 1450 kWh.

(A) $197.50 (B) $124.50 (C) $121.50 (D) $101.50 (E) $81.50

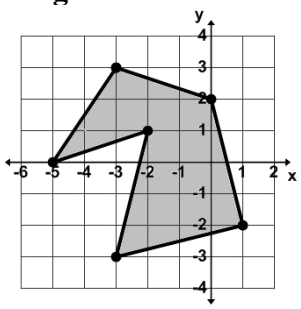
31. Two billion three hundred four million five thousand sixty-seven is added to twenty-three million four hundred fifty-two thousand six hundred seven. Which of the following digits appears the most in the sum?

(A) 2 (B) 4 (C) 5 (D) 6 (E) 7

32. Soh Yung is 3 times as old as her sister Tu Yung. In 4 years Soh will only be twice as old as Tu. What will the sum of their ages be in 10 years?

(A) 14 (B) 16 (C) 22 (D) 26 (E) 36

33. Find the area of the shaded figure.



(A) 11.5 units^2 (B) 14 units^2 (C) 17.5 units^2 (D) 18 units^2 (E) 19 units^2

34. The length of the base of △PQR is 40 cm. and the height is 60 cm. △ICU is formed by cutting off 25% of the base of △PQR and adding 20% of the height of △PQR. The area of △ICU is what percent of △PQR?

(A) 55% (B) $69\frac{4}{9}\%$ (C) 80% (D) $83\frac{1}{3}\%$ (E) 90%

35. ***PurtyDurty*** **detergent contains 80% soap and 20% bleach.** ***WishyWashy*** **detergent contains 55% soap and 45% bleach. If** ***PurtyDurty*** **is mixed with** ***WishyWashy*****, what percent of the mixture should be** ***PurtyDurty*** **if the final mixture is 35% bleach?**

(A) 60% **(B) 40%** **(C) 35%** **(D) 32%** **(E) 10%**

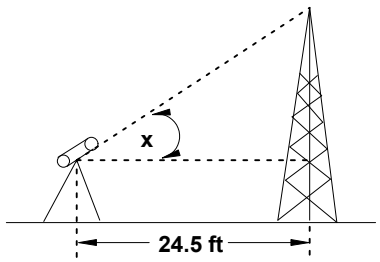
36. A circle with its center at the origin or the Cartesian x-y coordinate system has a radius of 3 units. If you start at (— 3, 0) and travel on the circle $\frac{8\pi}{3}$ radians in a clockwise direction, where on the x-y coordinate plane will you stop at?

(A) Quadrant III **(B) Quadrant IV** **(C) Quadrant I** **(D) Quadrant II** **(E) y-axis**

37. Rusty Pipes has a leaky pipe dripping water onto the floor forming a circular pool. The radius of the pool increases at a rate of 4 cm/min. How fast is the area of the pool increasing when the radius is 5 cm? (nearest cm^2/min)

(A) 126 cm^2/min **(B) 20 cm^2/min** **(C) 314 cm^2/min** **(D) 157 cm^2/min** **(E) 63 cm^2/min**

38. Sir Vayor used his theodolite to measure the height of the tower to be 17 ft 4" tall. His theodolite was 5 ft from the level ground. What angle x did he use to compute the height? (nearest second)



(A) 42° 21' 04" **(B) 26° 43' 15"** **(C) 35° 16' 44"** **(D) 41° 38' 01"** **(E) 24° 30' 00"**

39. Betty Chuzrite selects one letter from each of the sets {a, c, u, t, e} and {o, t, u, s, e}. What is the probability she selects one vowel? (nearest whole percent)

(A) 92% **(B) 84%** **(C) 67%** **(D) 60%** **(E) 48%**

40. Betty Chuzrite selects one letter from each of the sets {a, c, u, t, e} and {o, t, u, s, e}. What is the probability she selects at least one vowel? (nearest whole percent)

(A) 92% **(B) 84%** **(C) 67%** **(D) 60%** **(E) 48%**

41. Betty Chuzrite selects one letter from each of the sets {a, c, u, t, e} and {o, t, u, s, e}. What is the probability she selects two vowels given that a vowel is chosen from the first set? (nearest whole percent)

(A) 92% **(B) 84%** **(C) 67%** **(D) 60%** **(E) 48%**

42. $236_7 + 453_7 - 165_7 =$ _____ $_7$.

(A) 263 **(B) 512** **(C) 524** **(D) 412** **(E) 563**

43. Let P and Q be the roots of $4x^2 + 17x = 15$. Find $(P + Q)(PQ)$.

(A) -63.75 **(B)** 8 **(C)** -81.25 **(D)** 0.5 **(E)** 15.9375

44. Let $\begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix} \times \begin{bmatrix} 2 & 1 \\ -3 & -4 \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$. Find $a + b + c + d$.

(A) -19 **(B)** -7 **(C)** -4 **(D)** 5 **(E)** 7

45. Let $f(x) = ax^2 + bx + 5$ where a and b are integers. If $f(1) = 2$ and $f(2) = 3$, then $f(3) = ?$

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

46. Find $a + b + c + d$ given the Fibonacci characteristic sequence: a, -3, b, -1, c, 0, d, 1, ...

(A) 9 **(B)** 8 **(C)** 7 **(D)** 6 **(E)** 4

47. Evaluate: $\prod_{n=2}^{6} (-1)^{n-2} - n$

(A) 0 **(B)** -5 **(C)** -6 **(D)** -19 **(E)** -360

48. Let $f''(x) = 12x - 6$, $f'(0) = 4$, and $f(0) = -5$. Find $f(1)$.

(A) -2 **(B)** -1 **(C)** 3 **(D)** 4 **(E)** 6

49. How many distinct 4-letter code words can be made from the letters in the word ALGEBRA?

(A) 500 **(B)** 480 **(C)** 420 **(D)** 360 **(E)** 70

50. Find the sum of the first three Mersenne Primes such that the sum is a prime but is not a Mersenne Prime.

(A) 29 **(B)** 31 **(C)** 37 **(D)** 41 **(E)** 43

51. 8,051 is the product of the two prime factors. The sum of these two prime factors is?

(A) 86 **(B)** 131 **(C)** 164 **(D)** 177 **(E)** 180

52. Eratosthenes sifted out all of the prime numbers less than 100. How many of these primes contained the digit 9?

(A) 6 **(B)** 9 **(C)** 5 **(D)** 8 **(E)** 7

53. Which of the following is/are not function(s)?

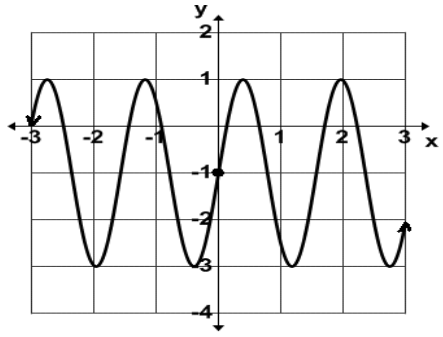
I. {(2,6), (−3,6), (4,9), (2,10)} II. {(1,3), (2,3), (3,3), (4,3)} III. {(−2,2), (−1,1), (0,0), (1,1)}

(A) I only **(B)** II only **(C)** I & II **(D)** II & III **(E)** all of them

54. Which of the following points does not lie on the line containing the point (— 2, — 3) and having a slope of —1.5?

(A) (— 5, 1.5) **(B) (— 8, 6)** **(C) (7 , — 15.5)** **(D) (4, — 12)** **(E) (9, — 19.5)**

55. The graph of f(x) shown below has a frequency of 0.6366197... . Find f(5.7). (nearest tenth)



(A) — 2.1 **(B) — 2.4** **(C) — 2.5** **(D) — 2.6** **(E) — 2.8**

56. Let S = {7, 11, 18). The arithmetic mean of S is A. The geometric mean of S is G. And the harmonic mean of S is H. Which of the following is the correct order of the means A, G, and H from least to greatest?

(A) H, A, G **(B) A, G, H** **(C) G, A, H** **(D) H, G, A** **(E) A, H, G**

57. Les Avridge had quiz grades of 75, 83, 66, 90, 83, 50, 65, and 83. The average of the arithmetic mean, median, mode, and range of his quiz grades is? (nearest whole number)

(A) 62 **(B) 69** **(C) 71** **(D) 72** **(E) 80**

58. Let function *f* be defined as f(x) = 2x — 6 for all real numbers.
Let function *g* be defined as follows for all integers such that $-3 \leq x \leq 3$:

x	— 3	— 2	— 1	0	1	2	3
g(x)	12	15	16	15	12	7	0

Which of the following is true about both functions *f* and *g*?

(A) They reach their maximum value at the same domain value
(B) They have the same y-intercept **(C) They are both odd functions**
(D) They share an x-intercept **(E) none of these are true**

59. Let f(x) = $|x - 5|$. How many of the following statements are always true?

a. $\lim_{x \to 5^+} f(x)$ exists b. $\lim_{x \to 5^-} f(x)$ exists c. f(x) is continuous d. f(x) is differentiable

(A) 0 **(B) 1** **(C) 2** **(D) 3** **(E) 4**

60. Expand $10^B \div (10^{(2B)} - 10^B - 1)$ for B = 2. What is the 20^{th} digit after the decimal place?

(A) 4 **(B) 5** **(C) 7** **(D) 8** **(E) 9**

DO NOT DISTRIBUTE TO STUDENTS BEFORE OR DURING THE CONTEST

University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitation A ● 2019
Answer Key

1. C	**21. E**	**41. D**
2. B	**22. A**	**42. C**
3. A	**23. C**	**43. E**
4. E	**24. D**	**44. B**
5. D	**25. B**	**45. D**
6. C	**26. E**	**46. A**
7. B	**27. C**	**47. E**
8. C	**28. E**	**48. A**
9. B	**29. A**	**49. B**
10. D	**30. D**	**50. D**
11. B	**31. E**	**51. E**
12. A	**32. E**	**52. A**
13. E	**33. D**	**53. A**
14. C	**34. E**	**54. C**
15. A	**35. B**	**55. B**
16. A	**36. C**	**56. D**
17. C	**37. A**	**57. B**
18. C	**38. B**	**58. D**
19. D	**39. E**	**59. D**
20. C	**40. B**	**60. B**

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____