UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

State • 2022

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1. **The equation for a thin lens is $\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}$. Find the value of d_i if $f=18$ cm and $d_o=12$ cm.**

   (A) –36 cm (B) –30 cm (C) –24 cm (D) –18 cm (E) –12 cm

2. **Six times the complement of $\angle A$ is 4 greater than twice the supplement of $\angle A$. $m\angle A =$ _____.**

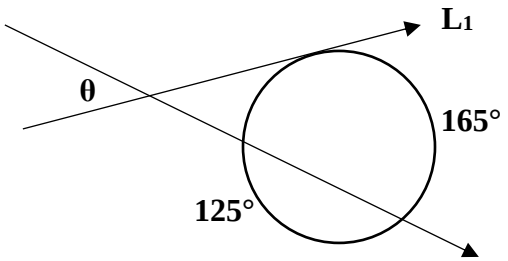
   (A) 40° (B) 42° (C) 44° (D) 46° (E) 48°

3. **Consider three consecutive odd numbers such that five times the third is 21 less than three times the sum of the first two. What is the square of the largest number?**

   (A) 625 (B) 729 (C) 961 (D) 1225 (E) 1521

4. **Find the value of θ. Ray L_1 is tangent to the circle.**

   (A) 46.0°
   (B) 46.5°
   (C) 47.0°
   (D) 47.5°
   (E) 48.0°



Problem 4

5. **Lib is taking a three-day trip to see her parents in Wyoming. On day one, she traveled 480 miles at an average speed of 65 mph. On day 2, she traveled at an average speed of 72 mph. On day 3 she traveled 420 miles at an average speed 68 mph. If she averaged 68.14 mph for the entire trip, how far did she travel on day 2? (nearest mile)**

   (A) 446 mi (B) 449 mi (C) 452 mi (D) 455 mi (E) 458 mi

6. **Twelve men can paint three Nash-Phillips homes in 60 hours. Ten men are sent to paint four Nash-Phillips homes. If these men work for forty-five hours and then half of them quit, how long will it take the remaining men to finish the job? (nearest whole number)**

   (A) 90 hr (B) 93 hr (C) 96 hr (D) 99 hr (E) 102 hr

7. **The time on my clock shows that it is exactly 8:18. Find the obtuse angle formed by the hour hand and the minute hand. (nearest whole number)**

   (A) 137° (B) 139° (C) 141° (D) 143° (E) 145°

8. **Last fall, Professor C attended the Dallas Cowboys game against Denver with his wife and two sons. Parking cost $40. For this game, they got a "Great Deal" on Club Main, row 1 tickets. They only cost $2887 each. Each of them had a Green Chile Kobe Burger, which cost $13.50 each. Each of them got a coke for $6 each and fries for $5 each. If the cost of gas to drive from Salado to Dallas and back was $27.77, what was the total cost of attending the game?**

   (A) $11,705.77 (B) $11,709.77 (C) $11,713.77 (D) $11,718.77 (E) $11,723.77

9. Rocket told his computer science students that their semester grades will be weighted by counting the average of their six test grades as 81% of their numerical grade and their semester exam grade as the other 19%. Joe has the following test scores: 86, 91, 85, 95, 91 and 88. Of the following, which is the lowest score Joe can make on the semester exam in order to make an "A" for the semester if Rocket will give you an "A" if your weighted average is 89.5 or higher?

(A) 89 **(B) 90** **(C) 91** **(D) 92** **(E) 93**

10. At the Tokyo Olympics held last summer, Grant Fisher of the USA placed sixth in the 10,000 meters. The Stanford alum's time of 27:46.39 was only three seconds slower than the time of Ethiopia's Selemon Barega, who won the gold medal. What was Grant's average pace? (minute, nearest second / mile)

(A) (4 min 24 sec) / mi **(B) (4 min 26 sec) / mi** **(C) (4 min 28 sec) / mi**
(D) (4 min 30 sec) / mi **(E) (4 min 32 sec) / mi**

11. Consider a circle with center O. Points A and B lie on the circle such that the length of chord $\overline{AB}$ is 48 and the length of segment $\overline{OB}$ is 26. Find the length of minor arc AB. (nearest tenth)

(A) 60.0 **(B) 60.3** **(C) 60.6** **(D) 60.9** **(E) 61.2**

12. A total of 60 people were on the bus transporting The Brock Eagles to their game with Lorena. The average age of the 8 cheerleaders was exactly 17 years, the average age of the 12 coaches was exactly 38 years and the overall average age of the 60 people on the bus was exactly 21 years. What was the average age of the 40 football players? (nearest tenth)

(A) 16.5 yr **(B) 16.7 yr** **(C) 16.9 yr** **(D) 17.1 yr** **(E) 17.3 yr**

13. At the Hunter's Swap Meet, 7 quail were worth 2 ducks and 1 duck was worth 5 doves. How many quail would you get for 30 doves?

(A) 21 **(B) 24** **(C) 28** **(D) 32** **(E) 35**

14. Consider a triangle with side lengths 16, 30, 34. What is the length of the shortest altitude of the triangle? (nearest tenth)

(A) 13.3 **(B) 13.5** **(C) 13.7** **(D) 13.9** **(E) 14.1**

15. Consider an equilateral triangle inscribed in a circle. If the perimeter of the triangle is equal to the area of the circle, what is the area of the circle minus the area of the triangle? (nearest tenth)

(A) 5.0 **(B) 5.2** **(C) 5.4** **(D) 5.6** **(E) 5.8**

16. A card was drawn from a full deck of 52 cards. What are the odds that the card was a queen or a heart or the queen of hearts?

(A) $\frac{4}{9}$ **(B) $\frac{9}{20}$** **(C) $\frac{23}{50}$** **(D) $\frac{17}{35}$** **(E) $\frac{1}{2}$**

17. Coach Griffin has 18 players on his baseball team. How many different nine-person batting orders are possible?

(A) 48,620 **(B) 128,720** **(C) 1,682, 400** **(D) 886,723,500** **(E) 17,643,225,600**

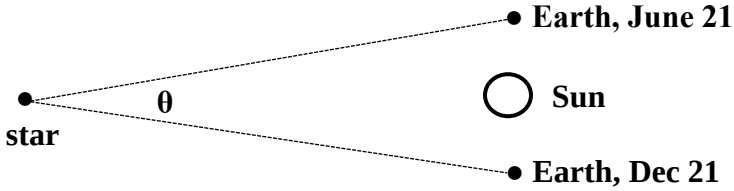
18. Consider a regular hexagon with an inscribed circle and a circumscribed circle. If the area of the circumscribed circle is 452 cm^2, what is the area of the inscribed circle? (nearest square centimeter)

(A) 327 cm^2 **(B) 331 cm^2** **(C) 335 cm^2** **(D) 339 cm^2** **(E) 343 cm^2**

19. There is a row of 5 empty chairs in a room. Three boys and two girls enter the room and are told to sit in the chairs. How many different seating arrangemnets are possible if the girls are not allowed to sit next to each other?

(A) 60 **(B) 72** **(C) 84** **(D) 96** **(E) 120**

20. The distance from Earth's center to the Sun's center is 1.496×10^8 km. If $\theta = 7.5\times10^{-6}$ rad, the distance from the Sun to the star is _____ km. (two significant digits)



(A) 3.1×10^{13} **(B) 3.4×10^{13}** **(C) 3.7×10^{13}** **(D) 4.0×10^{13}** **(E) 4.3×10^{13}**

21. If $f(x) = \frac{1}{32}x - 5$ and $h(x) = x^5$, then $\left(f^{-1} \circ h^{-1}\right)(243) =$ _____.

(A) 96 **(B) 176** **(C) 256** **(D) 336** **(E) 416**

22. If 0.5 is a zero of $6x^3 - 9x^2 + kx + 6 = 0$, then k = _____.

(A) −11 **(B) −9** **(C) −7** **(D) −5** **(E) −3**

23. Becci placed $123,456.78 into an account that earns 6.45% annual interest compounded monthly. How long will it take for the balance in her account to triple? (nearest whole number)

(A) 14 yr **(B) 15 yr** **(C) 16 yr** **(D) 17 yr** **(E) 18 yr**

24. Find the period of the function $f(x) = -3 + 2\sec\left(3x - \frac{\pi}{4}\right)$.

(A) $\frac{2\pi}{3}$ **(B) π** **(C) $\frac{4\pi}{3}$** **(D) $\frac{5\pi}{3}$** **(E) 2π**

25. Given: $\cos\theta = \frac{12}{13}$, $\sin\alpha = -\frac{15}{17}$. If both θ and α are in quadrant IV, then $\sec(\theta - \alpha) =$ _____. (nearest hundredth)

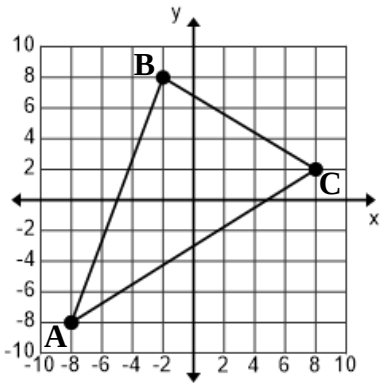
(A) 1.17 **(B) 1.20** **(C) 1.23** **(D) 1.26** **(E) 1.29**

26. $m\angle BAC =$ _____. **(nearest tenth)**

(A) 37.1° **(B) 37.4°** **(C) 37.7°** **(D) 38.0°** **(E) 38.3°**

27. Point D (not shown) lies on $\overline{AC}$ and $\overline{BD} \perp \overline{AC}$. Find AD. (nearest tenth)

(A) 12.7 **(B) 13.0** **(C) 13.3** **(D) 13.6** **(E) 13.9**



Problems 26, 27

28. A boat is traveling due south, parallel to the shoreline. It is traveling at a constant speed of 15 mph. At t = 0, the bearing from the boat to a tall motel at the shoreline was W 72° S. At t = 20 minutes, the bearing was W 36 ° S. Find the shortest distance from the shoreline to the boat. (nearest tenth)

(A) 1.3 mi **(B) 1.5 mi** **(C) 1.7 mi** **(D) 1.9 mi** **(E) 2.1 mi**

29. Which of the following are the three cube roots of $1+i$ in polar form ?

I. $\sqrt[6]{2}\text{cis}(15°)$ **II.** $\sqrt[6]{2}\text{cis}(45°)$ **III.** $\sqrt[6]{2}\text{cis}(135°)$ **IV.** $\sqrt[6]{2}\text{cis}(225°)$ **V.** $\sqrt[6]{2}\text{cis}(255°)$

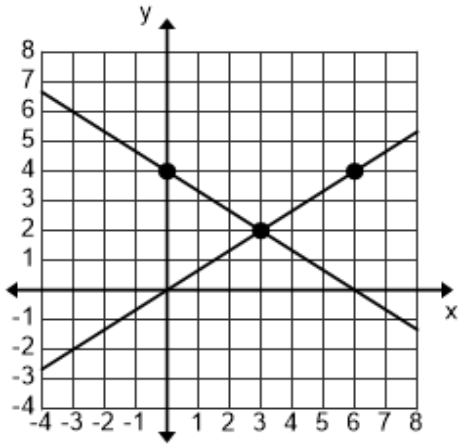
(A) I, II, III **(B) II, III, IV** **(C) I, III, V** **(D) II, III, IV** **(E) III, IV, V**

30. Find the acute angle formed by the two intersecting lines shown on the right. (nearest tenth)

(A) 67.0° **(B) 67.2°** **(C) 67.4°** **(D) 67.6°** **(E) 67.8°**

31. The asymptotes of a hyperbola are graphed on the right. If the equation of the hyperbola centered at (h, k) is $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ and one of the foci is the point $(h, k+c)$. $a+b+c =$ _____. (nearest tenth)

(A) 8.0 **(B) 8.2** **(C) 8.4** **(D) 8.6** **(E) 8.8**



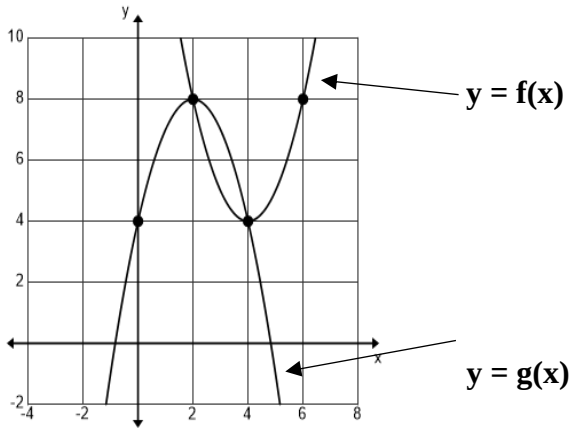
Problems 30, 31

32. The equation of the directrix of the graph of $y = f(x)$ is $y =$ _____.

(A) $\frac{63}{16}$ **(B)** $\frac{31}{8}$ **(C)** $\frac{15}{4}$ **(D)** $\frac{7}{2}$ **(E) 3**

33. The distance from the focus of the graph of y = f(x) to the focus of the graph of y = g(x) is _____. (nearest tenth)

(A) 3.6 **(B) 3.8** **(C) 4.0** **(D) 4.2** **(E) 4.4**



Problems 32, 33

34. Convert the rectangular equation $xy = 8$ to a polar equation.

(A) $r^2 = 8\sec(\theta)$ **(B)** $r^2 = 16\csc(2\theta)$ **(C)** $r^2 = 8\csc(\theta)$ **(D)** $r^2 = 16\sec(2\theta)$ **(E)** $r^2 = 8\cot(\theta)$

35. Just as an airplane leaves the runway, the plane's angle of ascent is 16° and its speed is 200 mph. If it continues on this path at this speed, how long will it take the plane to reach an altitude of 8,000 ft? (nearest second)

(A) 87 s **(B) 90 s** **(C) 93 s** **(D) 96 s** **(E) 99 s**

36. The partial fraction decomposition of $\dfrac{6x^2 + 5x + 12}{x^3 + 3x}$ **is** $\dfrac{A}{x} + \dfrac{Bx + C}{x^2 + 3}$**.** $A + B + C =$ _____.

(A) 11 **(B) 12** **(C) 13** **(D) 14** **(E) 15**

37. Consider the graph produced by the parametric equations $x = 4\sec\theta$ **and** $y = 2\tan\theta$**. Which of the following points is a focus of the graph?**

(A) $\left(-2\sqrt{6}, 0\right)$ **(B)** $\left(-2\sqrt{5}, 0\right)$ **(C)** $\left(-2\sqrt{3}, 0\right)$ **(D)** $\left(-2\sqrt{2}, 0\right)$ **(E)** $(-2, 0)$

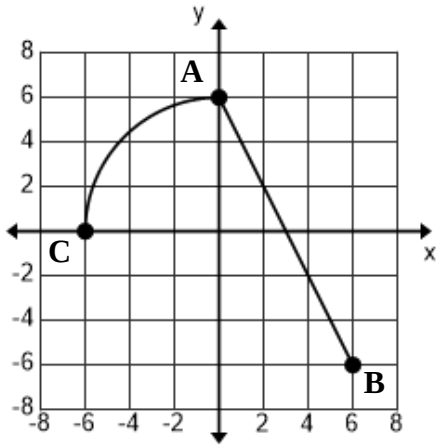
38. $\sqrt{5} + \sqrt{10} + \sqrt{15} + \sqrt{20} + \sqrt{25} + \ldots + \sqrt{220} + \sqrt{225} =$ _____. **(nearest whole number)**

(A) 453 **(B) 455** **(C) 457** **(D) 459** **(E) 461**

39. The circle $(x+6)^2 + (y+3)^2 = 5$ **is tangent to the circle** $x^2 + y^2 = 20$**. The common internal tangent is a line with x-intercept** $(a, 0)$ **and y-intercept** $(0, b)$**.** $a + b =$ _____**.** **(nearest tenth)**

(A) –17.0 **(B) –16.5** **(C) –16.0** **(D) –15.5** **(E) –15.0**

For problems 40 and 41, consider the graph of the piecewise function, $y = f(x)$**, which is shown on the right. The domain of** $f(x)$ **is** $-6 \le x \le 6$**.**



40. The y-intercept of the perpendicular bisector of line segment $\overline{AB}$ **is the point** $(0, b)$**.** $b =$ _____.

(A) $-\dfrac{5}{4}$ **(B)** $-\dfrac{4}{3}$ **(C)** $-\dfrac{3}{2}$ **(D)** $-\dfrac{5}{3}$ **(E)** $-\dfrac{7}{4}$

41. $\int_{-6}^{6} f(x)dx =$ _____. **(nearest tenth)**

(A) 28.3 **(B) 29.4** **(C) 30.6** **(D) 31.7** **(E) 32.8**

42. Given: $\int_{-6}^{6} f(x)dx = 24$ **and** $\int_{6}^{10} f(x)dx = 8$. **If** $f(x)$ **is an even function, then** $\int_{0}^{10} f(x)dx =$ _____.

(A) 8 **(B) 12** **(C) 16** **(D) 20** **(E) 24**

43. If $\int_{1}^{x} f(t)dt = \frac{33x}{\sqrt{9x^2 - 12}} + 9$, **then** $\int_{1}^{\infty} f(t)dt =$ _____.

(A) 9 **(B) 12** **(C) 20** **(D) 24** **(E) 42**

44. The area in the first quadrant bounded by the curve $y = 2\cos\left(\frac{x}{4}\right)$, **the line** $x = \frac{3\pi}{2}$, **x-axis, and the y-axis is divided into two equal parts by the line** $x = k$. **Find k. (nearest hundredth)**

(A) 1.92 **(B) 1.94** **(C) 1.96** **(D) 1.98** **(E) 2.00**

45. Find the area of the inner loop of the graph of the limacon $r = 1 + 2\sin\theta$. **(nearest hundredth)**

(A) 0.54 **(B) 0.57** **(C) 0.60** **(D) 0.63** **(E) 0.66**

46. The position of a particle moving in the x-y plane is given by the parametric functions $x(t)$ **and** $y(t)$, **where** $0 \le t \le 2\pi$. **If** $x'(t) = t\cos t$ **and** $y'(t) = 3e^{-5t} + 6$, **what is the slope of the line tangent to the graph of the particle's path when** $t = 3$? **(nearest hundredth)**

(A) –3.68 **(B) –2.02** **(C) 0.87** **(D) 2.33** **(E) 3.72**

47. The function f is defined by $f(x) = 6x + e^{2x}$. **If** $g(x) = f^{-1}(x)$ **for all x, what is the value of** $g'(1)$? **(nearest thousandth)**

(A) 0.125 **(B) 0.250** **(C) 0.500** **(D) 0.750** **(E) 1.000**

Use this information for problems 48 and 49.
The position of a particle traveling along the x-axis is given by $x(t) = \sin(t)$, **for** $0 \le t \le 2\pi$.

48. Find the position of the particle, along the x-axis, when the velocity of the particle is $\frac{\sqrt{2}}{2}$ **to the left, and the acceleration of the particle is** $\frac{\sqrt{2}}{2}$ **to the right.**

(A) –1 **(B)** $-\frac{\sqrt{2}}{2}$ **(C) 0** **(D)** $\frac{\sqrt{2}}{2}$ **(E) 1**

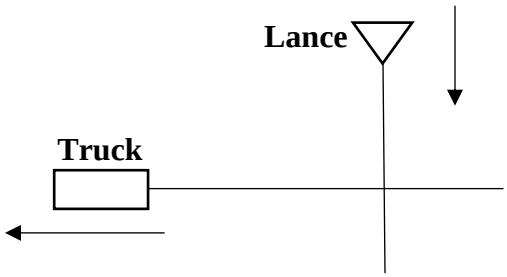
49. Find the total distance traveled by the particle from t = 0 to t = 2π.

(A) $2\sqrt{2}$ **(B)** π **(C) 4** **(D)** $4\sqrt{2}$ **(E)** 2π

50. Consider the volume of a solid formed using cross sections. The base is the region in the fourth quadrant bounded by the graph of the ellipse $25x^2 + 16y^2 = 400$. Each cross-section is perpendicular to the x-axis and is an isosceles right triangle with a leg as the base. Find the volume of the solid. (nearest tenth)

(A) 29.1 **(B) 29.4** **(C) 29.7** **(D) 33.0** **(E) 33.3**

51. Lance is cycling south on FM 2464 at 30 mph at the same time a truck is traveling west on FM 3131. At the moment Lance is 300 feet north of FM 3131, the truck is 400 feet west of FM 2464 and the distance between Lance and the truck is increasing at 26.4 ft/s. What is the speed of the truck? (nearest tenth)



Problem 51

(A) 42.6 mph **(B) 45.0 mph** **(C) 47.4 mph**
(D) 49.8 mph **(E) 52.2 mph**

52. The illumination from a light source varies inversely with the square of the distance from the light source and directly with intensity of the light source. Two light sources are placed 7.00 m apart. The intensity of light source A is 54 cd and the intensity of light source B is 128 cd. Find the distance from light source A to the darkest point along a line connecting the two light sources. (nearest tenth)

(A) 2.4 m **(B) 2.6 m** **(C) 2.8 m** **(D) 3.0 m** **(E) 3.2 m**

53. In a test of $H_0 : p = 0.37$ against $H_a : p \neq 0.37$, a random sample of size 200 yields a test statistic of $z = 2.25$. The P-value of this test is approximately equal to _____. (nearest thousandth)

(A) 0.012 **(B) 0.024** **(C) 0.036** **(D) 0.048** **(E) 0.060**

54. Suppose the mean SAT math score for students in Idaho is 523 with a standard deviation of 117 and the mean ACT math score is 20.4 with a standard deviation of 7.1. Jim scored 690 on the math section of the SAT test. Of the following, which is the lowest score he can make on the math section of the ACT test in order have done at least as well on the ACT test as he did on the SAT test?

(A) 29 **(B) 30** **(C) 31** **(D) 32** **(E) 33**

55. Mr. King retired and opened a candy store in a small town in New Mexico. He kept careful records and discovered that 72% of all customers purchase chocolate truffles, 48% purchase licorice and 36% purchase both. If a customer is randomly selected, what is the probability that she orders either chocolate truffles or licorice, but not both?

(A) 40% **(B) 42%** **(C) 44%** **(D) 46%** **(E) 48%**

56. Dr. Good specializes in research on Australian wombats. This year, as part of her research, she needs to construct a 98% confidence interval for the average weight of adult wombats. Based on previous research, she estimates that the standard deviation is 2.20 kg. Dr. Good plans to select a random sample of wombats and record their weights. Of the following, which is the smallest sample size that will result in a margin of error of 0.125 kg or less for the confidence interval?

(A) 1240 **(B) 1350** **(C) 1460** **(D) 1570** **(E) 1680**

57. Professor Taylor drinks a bottle of V-2 vegetable juice for breakfast every day. V-2 is a blend of carrot juice and beet juice. The amount of carrot juice in each bottle can be modeled using a normal distribution with a mean of 4.0 ounces and a standard deviation of 0.08 ounces. The amount of beet juice can also be modeled using a normal distribution with a mean of 3.8 ounces and a standard deviation of 0.07 ounces. The Walmart Distribution Center in Sabine received a shipment of over 900,000 bottles. If six bottles are randomly selected, what is the probability that at least two of them will have more beet juice than carrot juice? (nearest ten-thousandth)

(A) 0.0088 (B) 0.0106 (C) 0.0124 (D) 0.0142 (E) 0.0160

Use the table below for problems 58 and 59.

	Sports Station	Movie Station	Nature Station	Total
Beef	**65**	**55**	**30**	**150**
Turkey	**50**	**60**	**80**	**190**
Veggie	**40**	**65**	**75**	**180**
Total	**155**	**180**	**185**	**520**

58. Market researchers believe that the station on the TVs in a restaurant may affect the choices customers make when they order one of the three plates available at Maria's in Abilene. A study was done in which three TV stations were randomly assigned as treatments and the resulting number of each type of lunch plate ordered was recorded. The table above summarizes the data collected. A chi-square test was used to analyze the results. What was the contribution of the Veggie/Nature cell to the chi-square statistic? (nearest hundredth)

(A) 1.88 (B) 2.00 (C) 2.12 (D) 2.24 (E) 2.36

59. Further analysis produced a chi-square statistic of 28.645416 and a p-value of _____. (3 significant digits)

(A) 0.00000923 (B) 0.0000817 (C) 0.000711 (D) 0.00605 (E) 0.0499

60. Does spending six weeks of the summer attending the Newberry Number Sense camp improve the number sense scores of high school students? To find out, a pre-camp test was administered at the beginning of camp and a post-camp test was administered at the conclusion of camp. The two tests were written by Mr. White and determined to be of equal difficulty by Mr. Barton. The results are shown for the ten students who attended the camp last summer.

Student	1	2	3	4	5	6	7	8	9	10
Pre	122	144	116	148	166	152	202	104	216	188
Post	168	178	124	198	172	156	244	156	262	202

Which of the following procedures is the most appropriate to determine if there is a significant difference in the Pre and Post number sense scores?

(A) Two-proportion z test
(B) Two sample t test for means
(C) Chi-square test for independence
(D) t-test for the slope of a regression line
(E) Matched pairs t test for means

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University Interscholastic League
MATHEMATICS CONTEST
HS ● State ● 2022
Answer Key

1. A	**21. C**	**41. A**
2. C	**22. B**	**42. D**
3. E	**23. D**	**43. C**
4. D	**24. A**	**44. A**
5. B	**25. E**	**45. A**
6. E	**26. B**	**46. B**
7. C	**27. D**	**47. A**
8. C	**28. E**	**48. B**
9. C	**29. C**	**49. C**
10. C	**30. C**	**50. E**
11. E	**31. D**	**51. B**
12. B	**32. C**	**52. D**
13. A	**33. C**	**53. B**
14. E	**34. B**	**54. C**
15. A	**35. E**	**55. E**
16. A	**36. A**	**56. E**
17. E	**37. B**	**57. C**
18. D	**38. C**	**58. A**
19. B	**39. E**	**59. A**
20. D	**40. C**	**60. E**

University Interscholastic League
Mathematics Contest

WRITE ALL ANSWERS WITH
CAPITAL LETTERS

	Score	Initials
Final	_____	_____
2nd	_____	_____
1st	_____	_____

Contestant # _____ **Conference** _____

1. _____	21. _____	41. _____
2. _____	22. _____	42. _____
3. _____	23. _____	43. _____
4. _____	24. _____	44. _____
5. _____	25. _____	45. _____
6. _____	26. _____	46. _____
7. _____	27. _____	47. _____
8. _____	28. _____	48. _____
9. _____	29. _____	49. _____
10. _____	30. _____	50. _____
11. _____	31. _____	51. _____
12. _____	32. _____	52. _____
13. _____	33. _____	53. _____
14. _____	34. _____	54. _____
15. _____	35. _____	55. _____
16. _____	36. _____	56. _____
17. _____	37. _____	57. _____
18. _____	38. _____	58. _____
19. _____	39. _____	59. _____
20. _____	40. _____	60. _____