UNIVERSITY INTERSCHOLASTIC LEAGUE

Mathematics

Invitational A • 2022

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1. **In a recent game, Kevin made 4 out of 11 three-point shots, 6 out of 13 two-point shots, and 7 out of 8 free throws (one point each). How many points did he score?**

   (A) 29 (B) 30 (C) 31 (D) 32 (E) 33

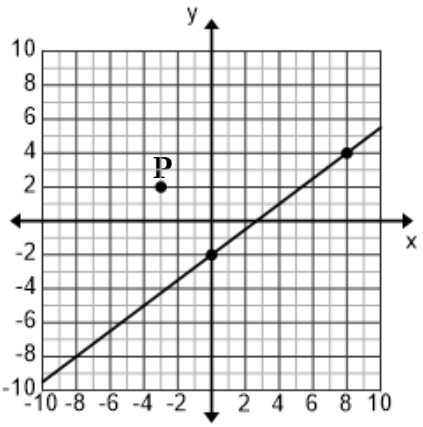
2. **Tom selected his favorite number and doubled it, then he added 12, next he divided by 4, then he multiplied by 3, and finally he subtracted 11. If the result was 22, what is Tom's favorite number?**

   (A) 16 (B) 20 (C) 24 (D) 28 (E) 32

3. **Karen's Kennel will keep your dog when you are out of town. They charge a basic fee of $9.95 per day, plus $1.95 per meal for premium dog food, and $29.95 per hour to exercise your dog. Find the cost to leave a dog with Karen for two weeks if you want your dog to receive three meals each day and you want your dog to be exercised 45 minutes each day.**

   (A) $534.57 (B) $535.68 (C) $536.79 (D) $537.90 (E) $539.01

4. **Consider a line, $y = f(x)$, that contains the point $P(-3, 2)$ and is also perpendicular to the line shown. Which point lies on the line $y = f(x)$?**



   (A) $(-4, 3)$
   (B) $(-2, 1)$
   (C) $(2, -5)$
   (D) $(4, -7)$
   (E) $(6, -10)$

5. **Joe paid $5,000 down on a new RAV 4, which had a total cost of $25,700. The dealership is offering to finance the remainder of the cost, interest free, over 48 months. What will be the amount of each monthly payment?**

   (A) $431.25 (B) $433.50 (C) $435.75 (D) $438.00 (E) $440.25

6. **Lisa accidentally spilled a quart of water into her swimming pool. The pool is rectangular in shape and measures 12 ft by 20 ft. By how much did the water level in the pool increase? (nearest hundred thousandth)**

   (A) 0.00167 in (B) 0.00334 in (C) 0.00668 in (D) 0.0134 in (E) 0.0267 in

7. **During the 1970 season, Gary completed 62.5% of his passes. During the first quarter of a game against Monterey, he attempted 4 passes. Find the probability that he completed 3 of these passes. (nearest tenth of a percent)**

   (A) 32.2% (B) 34.4% (C) 36.6% (D) 38.8% (E) 41.0%

8. **If $h(x) = -x^2 + bx + c$, and has zeros at $x = 3$ and $x = 7$, then the maximum value of $h(x)$ is _____.**

   (A) 3 (B) 3.25 (C) 3.5 (D) 3.75 (E) 4

9. The graph of the function $f(x)=\frac{6x+4}{2x+5}$ has asymptotes $x=d$ and $y=b$ and a zero at $x=c$. $d+b+c=$ _____.

(A) $-\frac{1}{2}$ **(B)** $-\frac{1}{3}$ **(C)** $-\frac{1}{6}$ **(D)** $\frac{1}{6}$ **(E)** $\frac{1}{3}$

10. Mary has ten pictures of her grandma. She plans to choose six of these, frame them, and hang them on her wall. If she hangs these six in one row, how many different arrangements are possible?

(A) 210 (B) 37,958 (C) 75,705 (D) 113,453 (E) 151,200

11. A sheet of graph paper consists of squares, each with an area of 1.00 cm^2. A small bug starts at point $A(5,7)$ and walks to point $B(-6,-1)$. From there the bug walks to point $C(3,-8)$. Find the total distance traveled by the bug. (nearest tenth)

(A) 24.1 cm (B) 24.4 cm (C) 24.7 cm (D) 25.0 cm (E) 25.3 cm

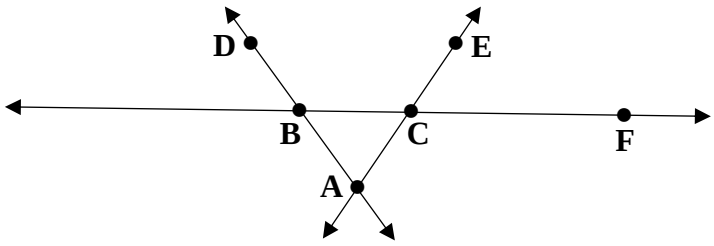
12. Consider a line segment with endpoints $P(-5,7)$ and $Q(4,-9)$. Which of the following points lies on the perpendicular bisector of $\overline{PQ}$?

(A) $\left(-6,-\frac{129}{32}\right)$ **(B)** $\left(-3,-\frac{75}{32}\right)$ **(C)** $\left(3,\frac{31}{32}\right)$ **(D)** $\left(6,\frac{87}{32}\right)$ **(E)** $\left(9,\frac{141}{32}\right)$

13. The circumference of the Earth at sea level along the equator is 4.0074×10^7 m. Cindy is at rest in a city at sea level and located at 40° north latitude. Find her linear speed due to the rotation of the Earth. (nearest whole number)

(A) 335 m/s (B) 340 m/s (C) 345 m/s (D) 350 m/s (E) 355 m/s

14. Find the $m\angle CBD$ if $m\angle ECF=65°$ and $m\angle CAB=59°$.



(A) 120° (B) 122° (C) 124°
(D) 126° (D) 128°

15. Consider ΔDEF with vertices $D(-3,8)$, $E(-1,2)$ and $F(10,6)$. If the coordinates of the centroid of ΔDEF are (a,b), then $a+b=$ _____.

(A) $7\frac{1}{3}$ **(B)** $7\frac{2}{3}$ **(C) 8** **(D)** $8\frac{1}{3}$ **(E)** $8\frac{2}{3}$

16. Consider rhombus ABCD with AB = 16 cm and $m\angle ABC=60°$. If circle O has the same area as rhombus ABCD, find the circumference of circle O. (nearest whole number)

(A) 49 cm (B) 51 cm (C) 53 cm (D) 55 cm (E) 57 cm

17. Point $A(3,-4)$ is rotated 90° counterclockwise about the origin to point B. Point B is reflected across the y-axis to point C. Point C is translated vertically upward 4 units to point $D(a,b)$. $a+b=$ _____.

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

18. Consider ΔSTR with point V on side $\overline{ST}$ and point U on side $\overline{SR}$. Line $\overleftrightarrow{VU}$ is parallel to line $\overleftrightarrow{TR}$. If $SV=6,\ VT=12,$ and $VU=4$, then TR = _____.

(A) 6 **(B)** 8 **(C)** 9 **(D)** 12 **(E)** 15

19. Consider ΔABC with point D on $\overline{AC}$ such that $\overline{BD}\perp\overline{AC}$. If $AB=10,\ BC=17,$ and $AD=8$, then the area of ΔABC = _____. (nearest whole number)

(A) 68 **(B)** 72 **(C)** 76 **(D)** 80 **(E)** 84

20. Points A, B, C and D lie on circle O such that chord $\overline{AC}$ intersects chord $\overline{BD}$ at point E. If $AE=24,\ BE=16,$ and $DE=36$, then CE = _____. (nearest tenth)

(A) 22.8 **(B)** 23.2 **(C)** 23.6 **(D)** 24.0 **(E)** 24.4

21. Find the area of a triangle with vertices $A(2,8)$, $B(-6,-4)$, and $C(6,-2)$.

(A) 56 **(B)** 58 **(C)** 60 **(D)** 62 **(E)** 64

22.

x	-3	-2	-1	1	2	3
f(x)	2	8	6	2	12	38

Use the table to find the value of $f(-4)$.

(A) -20 **(B)** -18 **(C)** -16 **(D)** -14 **(E)** -12

23. The relative intensity of a sound, β in dB, is calculated using the formula $\beta=10\log\left(\frac{I}{10^{-12}}\right)$ where I = the intensity of a sound in W/m^2. If your eardrums will burst when the relative intensity of a sound reaches 160 dB, what is the intensity of such a sound?

(A) 0.0001 W/m^2 **(B)** 0.01 W/m^2 **(C)** 1 W/m^2 **(D)** 100 W/m^2 **(E)** 10,000 W/m^2

24. If $f(x)=2x^3+6x^2+kx-10$ and $f(-3)=-22$, then $f(1)=$ _____.

(A) 2 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 10

25. If $s(x)$ is the slant asymptote of the function $f(x)=\frac{x^2+x+6}{x-1}$, then $s(6)=$ _____.

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

26. Consider the orbit of Mars around the Sun. The distance from Mars to the Sun at perihelion is 206.617×10^6 km and the distance at aphelion is 249.229×10^6 km. Find the eccentricity of the orbit of Mars. (nearest hundred-thousandth)

(A) 0.0935 (B) 0.1843 (C) 0.2751 (D) 0.3659 (E) 0.4567

27. A superball is dropped from a height of 12 feet. Each time it hits the floor, it bounces $\frac{11}{12}$ of its previous height. Find the total distance traveled by the ball.

(A) 242 ft (B) 254 ft (C) 264 ft (D) 276 ft (E) 288 ft

28. Simplify the expression. $\frac{1+\csc(x)}{\cos(x)+\cot(x)}$

(A) sec(x) (B) csc(x) (C) sin(x) (D) tan(x) (E) $1-\csc(x)$

29. Consider the formula $y = y_0 + v_0 t + \frac{1}{2}at^2$ for the height of a projectile at time t, t in seconds. If $a = -32.174$ ft/s^2 and the projectile was launched straight up from the ground at 200 ft/s, how high will it go? (nearest whole number)

(A) 618 ft (B) 622 ft (C) 626 ft (D) 630 ft (E) 634 ft

30. Cobalt 57 has a half life of 271.8 days. If Karen has 12 g today, how many days from now will it be until she has only 2 g? (nearest tenth)

(A) 702.6 (B) 704.7 (C) 706.8 (D) 708.9 (E) 711.0

31. An airplane is flying horizontally and it is at an altitude of 5.46 miles when it passes over an observer on the ground. At the instant that the angle of elevation from the observer to the plane is $33° 33' 33''$, what is the distance from the observer to the plane? (nearest foot)

(A) 52,144 ft (B) 52,151 ft (C) 52,158 ft (D) 52,165 ft (E) 52,172 ft

32. Which of the following is one of the four fourth roots of the complex number $81i$? (answers are in standard form and have been rounded to the nearest hundredth)

(A) $-2.61 - 0.99i$ (B) $-2.65 - 1.03i$ (C) $-2.69 - 1.07i$ (D) $-2.73 - 1.11i$ (E) $-2.77 - 1.15i$

33. At the NCTC mathematics contest, the top 10 students received cash prizes. First place received $350, second place received $315, third place received $280, fourth placed received $245, and so on. What was the total amount of prize money awarded?

(A) $1910 (B) $1915 (C) $1920 (D) $1925 (E) $1930

34. If the focus of the parabola $y^2 + 6y + 4x + 21 = 0$ is (a, b), then $a + b =$ _____.

(A) -7 **(B)** -6 **(C)** -5 **(D)** -4 **(E)** -3

35. Find the rectangular coordinates of the point given in polar coordinates. $\left(6, \frac{13\pi}{9}\right)$

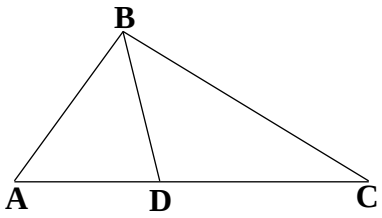
(all answers are rounded to the nearest hundredth)

(A) $(-0.96, -5.83)$ **(B)** $(-1.00, -5.87)$ **(C)** $(-1.04, -5.91)$ **(D)** $(-1.08, -5.95)$ **(E)** $(-1.12, -5.99)$

36. Find the distance from the point $(1, 2, 3)$ to the plane $x + y - 2z = 6$. (nearest hundredth)

(A) 3.45 **(B)** 3.67 **(C)** 3.89 **(D)** 4.11 **(E)** 4.33

37. $m\angle ABD = m\angle CBD,\ AB = 10,\ BC = 16,\ DC = 10$
Find AD.



(A) 6
(B) 6.25
(C) 6.5
(D) 6.75
(E) 7

38. Find the number that is $\frac{3}{8}$ of the way from $-6\frac{4}{7}$ to $12\frac{2}{7}$.

(A) $\frac{1}{2}$ **(B)** $\frac{4}{7}$ **(C)** $\frac{5}{7}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{6}{7}$

39. Evaluate: $\lim_{x \to \left(\frac{\pi}{2}\right)^+} (\tan x)$

(A) -1 **(B)** 1 **(C)** $-\infty$ **(D)** $+\infty$ **(E)** 0

40. Consider the function $f(x) = \frac{12x}{x^2 + 4}$. Find the x-intercept of the line tangent to $f(x)$ when x = 1.

(A) -1 **(B)** $-\frac{3}{4}$ **(C)** $-\frac{2}{3}$ **(D)** $-\frac{1}{2}$ **(E)** $-\frac{1}{3}$

41. Consider the function $f(x) = -x^4 + 2x^2$. Which of the values in the interval $(-3, 2)$ satisfy the mean value theorem for $f(x)$ on $[-3, 2]$? (nearest thousandth)

(A) -1.979 **(B)** -1.865 **(C)** -1.751 **(D)** -1.637 **(E)** -1.523

42. Given: $\int_3^8 f(x)dx = 12$ and $\int_3^8 g(x)dx = -4$. Evaluate $\int_3^8 [3f(x) - 2g(x)]dx$.

(A) 16 (B) 23 (C) 30 (D) 37 (E) 44

43. Consider the graph of $f(x) = e^{2x}\ln(x)$. Find the y-intercept of the line tangent to $f(x)$ at the point (a, b) if b = 2. (nearest tenth)

(A) −14.7 (B) −14.4 (C) −14.1 (D) −13.8 (E) −13.5

44. The rate of change of the number of bunnies $B(t)$ in a population of bunnies is directly proportion to $1000 - B(t)$, where t is the time in years. At t = 0, the population is 100. When t = 3, the population has increased to 300. Find the population when t = 4.

(A) 356 (B) 367 (C) 378 (D) 389 (E) 400

45. Find the volume of the solid generated when the region bounded by the graphs of $y = \sqrt{3x}$ and $y = .6x^2$ is revolved around the line $x = -3$. (nearest whole number)

(A) 29 (B) 32 (C) 35 (D) 38 (E) 41

46. Find the third degree Maclaurin polynomial for $f(x) = \sin(x)$. Then calculate the magnitude of the exact error when evaluating $f(0.8)$ using this polynomial. (nearest hundred-thousandth)

(A) 0.00203 (B) 0.00236 (C) 0.00269 (D) 0.00302 (E) 0.00335

47. When analyzing home prices in Denton, these are considered resistant to extreme values.
I. mean II. median III. standard deviation IV. IQR

(A) I, II (B) I, III (C) III, IV (D) II, IV (E) I, II, III, IV

48. A large sample of students at Newberg High School took an IQ test and it was first reported that the mean score was 100 and the standard deviation was 15. However, an error was found in the scoring and the corrected IQ score is found by taking each individual score and adding 2 and then multiplying by 1.12. Find the mean and standard deviation of the corrected IQ scores.

(A) 102, 17 (B) 114.24, 15 (C) 114.24, 16.8 (D) 114.24, 17 (E) 114.24, 19.04

Cookies	0	2	6	12	20	30
Score	284	276	262	232	208	164

49. The number sense team at Sabine High School performed an experiment. Each person ate a specified number of chocolate chip cookies and then they took a practice test. When examining the data, it appeared that the test scores decreased linearly. The table above shows the results of one of the students. Use the data in the table to create an appropriate model and predict this student's score if she eats 27 cookies.

(A) 173 (B) 175 (C) 177 (D) 179 (E) 181

50. The basic principle/principles of designing an experiment include which of the following?
I. Control II. Random Assignment III. Replication

(A) I only (B) II only (C) I, II only (D) II, III only (E) I, II, III

Event	Probability
Runs over 40 miles per week	0.36
Runs over 40 miles per week and gets injured	0.12
Does not run over 40 miles per week and does not get injured	0.57

51. An adult male is randomly selected from a large group of runners. Use the information above to find the probability that the individual selected gets injured given that he runs over 40 miles per week.

(A) 0.09 (B) 0.21 (C) $0.\overline{3}$ (D) 0.43 (E) 0.48

52. Each child of a particular pair of parents has probability 0.36 of being gifted in statistics. If the parents have 6 children, find the probability that exactly 4 of their children are gifted in statistics. (nearest thousandth)

(A) 0.103 (B) 0.127 (C) 0.151 (D) 0.175 (E) 0.199

53. If you reject the null hypothesis when the null hypothesis is true, you have committed a Type _____ error.

(A) I (B) II (C) III (D) IV (E) V

54. $CBA2_{15} - ABC8_{15} =$ _____$_{15}$

(A) 1EC9 (B) 1ECD (C) 1ED3 (D) 1EDA (E) 1EE1

55. Find the sum of the sequence. 9, 8, 17, 25, 42, 67, …, 461, 746

(A) 1935 (B) 1945 (C) 1955 (D) 1965 (E) 1975

56. Jerry's Ice Cream Shoppe in Highland Park has chocolate, vanilla, strawberry, butter pecan, peppermint, licorice, and chocolate mint flavors of ice cream. How many different ways can Mr. Speir purchase a six-scoop bowl of ice cream?

(A) 384 (B) 924 (C) 3003 (D) 84,400 (E) 117,649

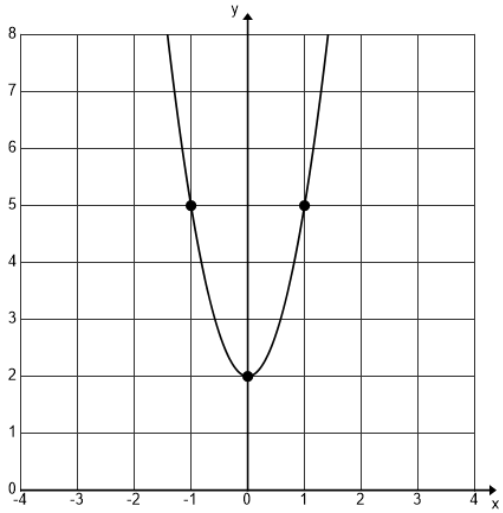
57. According to Wikipedia, this English mathematician "is widely considered to be the father of theoretical computer science and artificial intelligence…. For a time, he led Hut 8, the section that was responsible for German naval cryptanalysis."

(A) Charles Babbage (B) Jon von Neumann (C) Tommy Flowers
(D) Alan Turing (E) Christian Goldbach

58. The number 44 is considered to be which of the following types of numbers?
I. Happy II. Polite III. Odious IV. Extravagant

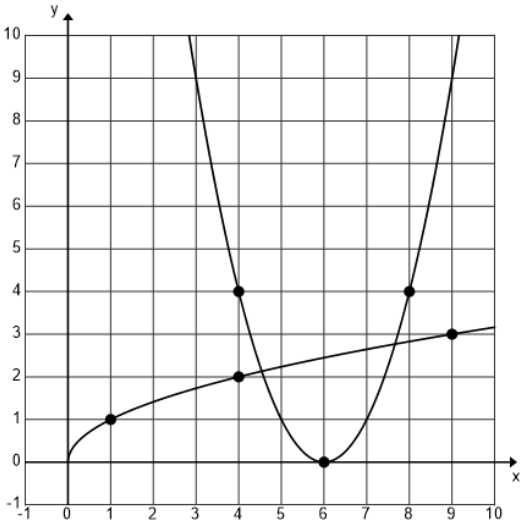
(A) I, II only (B) I, III only (C) I, II, IV only (D) I, III, IV only (E) I, II, III, IV

59. The graph of $g'(x)$ is shown on the right. If $g(-1) = -2$, then $g(2) =$ _____.



(A) 12
(B) 13
(C) 14
(D) 15
(E) 16

60. Find the area bounded by the two curves shown on the right. (nearest tenth)



(A) 4.3
(B) 4.5
(C) 4.7
(D) 4.9
(E) 5.1

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University Interscholastic League
MATHEMATICS CONTEST
HS ● Invitational A ● 2022
Answer Key

1. C	**21. E**	**41. D**
2. A	**22. B**	**42. E**
3. B	**23. E**	**43. D**
4. E	**24. A**	**44. A**
5. A	**25. E**	**45. E**
6. A	**26. A**	**46. C**
7. C	**27. D**	**47. D**
8. E	**28. A**	**48. C**
9. C	**29. B**	**49. C**
10. E	**30. A**	**50. E**
11. D	**31. B**	**51. C**
12. C	**32. E**	**52. A**
13. E	**33. D**	**53. A**
14. C	**34. A**	**54. A**
15. A	**35. C**	**55. B**
16. C	**36. B**	**56. B**
17. C	**37. B**	**57. D**
18. D	**38. A**	**58. E**
19. B	**39. C**	**59. B**
20. D	**40. C**	**60. E**