1. **Evaluate: $3! + 4^2 \div 5 - (1 \times 6^0 + 7)$**

   (A) -3.6 (B) -0.4 (C) 1.2 (D) 5.8 (E) 10.4

2. **Ma Bell's telephone company charges a \$15.50 per month base fee plus 1.5¢ per minute for local calls and 25¢ per minute for long distance calls. What would the bill be if 250 minutes in local calls and 50 minutes in long distance calls were made during the month?**

   (A) \$17.80 (B) \$31.75 (C) \$54.25 (D) \$65.50 (E) \$78.75

3. **Let U (universal set) = {p, r, o, b, l, e, m, a, t, i, c}, R = {r, a, t, i, o}, and T = {t, a, b, l, e}. Let $S = T^C \cup R$. Set S contains how many distinct elements?**

   (A) 2 (B) 3 (C) 7 (D) 8 (E) 10

4. **Five-eighths is the same part of two-thirds as three-tenths is of _____.**

   (A) $\frac{9}{32}$ (B) $\frac{15}{16}$ (C) $\frac{18}{25}$ (D) $\frac{12}{19}$ (E) $\frac{8}{25}$

5. **Which of the following linear equations is perpendicular to the line through the points $(-3, 2)$ and $(5, -1)$?**

   (A) $8x + 3y = 10$ (B) $x - 2y = -6$ (C) $8x - 3y = 10$ (D) $3x - 8y = -22$ (E) $x + 2y = 6$

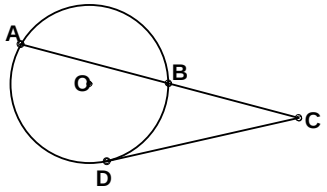
6. **Les Cash, Lotta Dough, and Noah Scents spent \$112.00 at the mall. Les spent half of the amount Noah spent and Lotta spent \$10.00 more than Les spent. How much did Noah spend?**

   (A) \$51.00 (B) \$50.00 (C) \$41.00 (D) \$35.50 (E) \$25.50

7. **If $3x^2 + 4x - 7 = (3x - a)(x - b)$, where a, b are integers, then $a + b = ?$.**

   (A) -8 (B) -6 (C) 6 (D) 7 (E) 8

8. **Given the circle with center O shown. Find AB if BC = 5 cm and CD = 6.5 cm.**



   (A) 12.25 cm (B) 11.5 cm (C) 8.45 cm (D) 3.45 cm (E) 3.05 cm

9. **If $\angle ABC$ and $\angle CBD$ are complementary and $\angle ABC$ and $\angle CBE$ are supplementary, find $m\angle CBE$ if $m\angle CBD = 42°$.**

   (A) 132° (B) 138° (C) 140° (D) 142° (E) 148°

10. Phil It-Upp has a gasoline tank in the shape of a right cylinder. The diameter of the tank is 4 feet. The capacity of the tank is 100 gallons, but it is only three-fourths full. How deep is the gasoline in the tank? (nearest 1/4 inch)

(A) 8.25 in (B) 8.75 in (C) 9 in (D) 9.25 in (E) 9.5 in

11. Find the 9^{th} term of the given the arithmetic sequence: { — 5, — 1.5, 2, a, b, c, ...}.

(A) 23 (B) 24.5 (C) 26.5 (D) 28 (E) 31.5

12. Using the following pattern of numbers, determine the median term of row 14.

1	(row 0)
1 1	(row 1)
1 2 1	(row 2)
1 3 3 1	(row 3)
1 4 5 4 1	(row 4)
1 5 7 7 5 1	(row 5)
1 6 9 10 9 6 1	(row 6)
...	(row ...)

(A) 52 (B) 50 (C) 47 (D) 46 (E) 47

13. If $\frac{2x-3}{4x-1} + \frac{Ax-B}{3x-2} = \frac{2x(13x+3)}{12x^2-11x+2}$, where A and B are constants, then A + B = ?

(A) 11 (B) 5 (C) 1 (D) — 1 (E) — 6

14. Kandy Korn bought 3 lbs of chocolates and 4 lbs of nuts for $24.00 at the *Saul T. Suite* store. Ima Nutt bought 1 lb of chocolates and 6 lbs of nuts for $15.00 at the same store. How much would it cost to buy 2 lbs of chocolates and 5 lbs of nuts at the *Saul T. Suite* store?

(A) $37.50 (B) $15.00 (C) $33.00 (D) $39.00 (E) $19.50

15. Let f(x) = 1 — 2sin(3π(x + 4). Find the sum of the vertical displacement, the amplitude, the period and the frequency.

(A) $5\frac{1}{6}$ (B) $1\frac{2}{3}$ (C) 6 (D) 10 (E) $1\frac{1}{6}$

16. Determine the range of(x) = 1 — 2cos[3π(x + 4)].

(A) [— 2, 2] (B) [0, 4] (C) [— 1 , 2] (D) [1, 4] (E) [— 1, 3]

17. Find a + b + c + d given the Fibonacci characteristic sequence: 6, a, b, 28, c, 73, d, ...

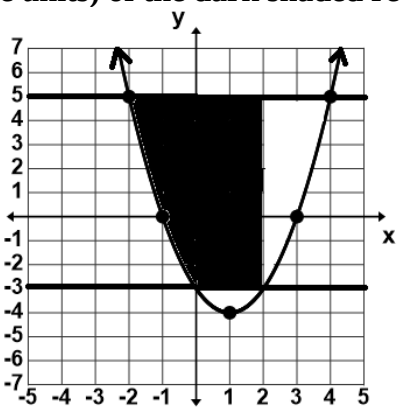
(A) 107 (B) 146 (C) 191 (D) 225 (E) 298

18. Omitted Problem.

19. Rose Thorn's assignment for her horticulture class was to plant birches and roses. While each girl planted 3 roses, each boy planted 1 birch. At the end of the day, 24 plants had been planted. How many girls were in the class if the number of boys in the class was the same as the number of girls?

(A) 4 **(B) 6** **(C) 8** **(D) 12** **(E) 18**

20. The area (in square units) of the dark shaded region shown is:



(A) $37\frac{1}{3}$ **(B)** $34\frac{2}{3}$ **(C) 28** **(D)** $26\frac{2}{3}$ **(E)** $25\frac{1}{3}$

21. A function, $g(x) = x^2 + bx + c$, exists such that $g(-1) = 1$ and $g(2) + g(-3) = -1$. Find $g(4)$.

(A) — 14 **(B) 24** **(C) — 54** **(D) 12** **(E) — 28**

22. The series 6, 4, 3, ... is a harmonic series. Find the sum of the 4^{th} and 6^{th} terms of the series.

(A) $2\frac{1}{4}$ **(B)** $3\frac{5}{7}$ **(C)** $4\frac{2}{5}$ **(D)** $4\frac{4}{35}$ **(E)** $6\frac{4}{35}$

23. Which of the following mathematicians is associated with the "Stepped Reckoner", the first hand cranked calculator that could perform all four arithmetic operations?

(A) Erastosthenes (B) Gottfried Leibniz (C) Diophantus (D) Benoit Mandelbrot (E) Theano

24. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. How many digits are in f_{31}?

(A) 3 **(B) 4** **(C) 5** **(D) 6** **(E) 7**

25. Reed Moore's book store is packaging five books to a bundle. He has history books, sports books, religious books, DIY books, children's books, and comic books. How many different bundles of books can Reed package?

(A) 126 **(B) 720** **(C) 252** **(D) 360** **(E) 462**

26. Find k if LCM(24, k) = 192 and GCF(24, k) = 8.

(A) 16 **(B) 32** **(C) 48** **(D) 64** **(E) 96**

27. Find the number of positive integral divisors of 2,025.

(A) 15 (B) 13 (C) 11 (D) 10 (E) 8

28. Determine the value of k so that $6x^2 - 4x + k = 0$ has one real root.

(A) 2 (B) $1\frac{1}{2}$ (C) 1 (D) $\frac{2}{3}$ (E) $\frac{1}{4}$

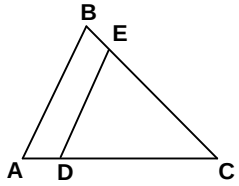
29. If $2x + 3y = 4$, $4x - y = 6$ and $x + ky = -1$, then k equals:

(A) -9 (B) -5 (C) $1\frac{6}{7}$ (D) $1\frac{2}{7}$ (E) 10

30. Rusty Yought sailed his boat from his dock 20 km in the direction N30°W. Then he changed course and sailed 14 km in the direction N45°E. How far was Rusty from his dock after sailing the 34 km? (nearest km)

(A) 17 km (B) 21 km (C) 27 km (D) 36 km (E) 42 km

31. $\triangle ABC$ and $\triangle CDE$ exist such that AB $\parallel$ DE, AB = 6", BC = 4" and BE = 1". Find DE.



(A) 1.5" (B) 4" (C) 4.5" (D) 5" (E) not enough information given

32. Point P (3, $-$ 2) is the midpoint of the line segment with endpoints Q ($-$ 1, y) and R (x, $-$ 11). Find $x + y$.

(A) 0.5 (B) 6.5 (C) 14 (D) 16 (E) 21

33. Which of the following points of concurrency does not lie on the line of Euler in a scalene triangle?

(A) incenter (B) centroid (C) orthocenter (D) circumcenter (E) all of them

34. Find the range of the function $y = |5 - 3x| + 2$ given that the domain is restricted to $\{x \in \text{Reals} \mid -2 \leq x \leq 5\}$.

(A) {all Reals} (B) $\{y \mid y \in \{\text{Reals}\}, 2.5 \leq y \leq 12\}$ (C) $\{y \mid y \in \{\text{Reals}\}, -2 \leq y \leq 2\}$
(D) $\{y : y > 2\}$ (E) $\{y \mid y \in \{\text{Reals}\}, 2 \leq y \leq 13\}$

35. Let $f(x) = 3x - 2$, $g(x) = 5x - 1$, $h(x) = 4x$, and $g(f(x)) - f(h(x)) = -27$. Find x.

(A) -12 (B) -9 (C) -6 (D) 3 (E) 6

36. Leo Pisa pulled the ace of hearts, ace of diamonds, two of spades, three of clubs, and the five of hearts from a standard deck of cards. He placed them face down and mixed them up. He randomly turns three cards over face up. What is the probability that the sum of the pip values of the three cards face up is a Fibonacci number? (nearest percent)

(A) 29% **(B)** $33\frac{1}{3}\%$ **(C) 57%** **(D)** $66\frac{2}{3}\%$ **(E) 70%**

37. A right triangle, $\triangle ABC$, exists such that $m\angle ACB = 90°$, M is the midpoint of $\overline{AB}$, BC = 5", and MC = 7.5". Find $m\angle BMC$. (nearest degree)

(A) 23° **(B) 29°** **(C) 39°** **(D) 43°** **(E) 45°**

38. Find the eccentricity of the ellipse, $\frac{(x-1)^2}{9} + \frac{y^2}{5} = 100$**. (nearest hundredth)**

(A) $\frac{1}{15}$ **(B)** $\frac{41}{50}$ **(C)** $\frac{5}{9}$ **(D)** $\frac{2}{3}$ **(E) 1**

39. Let $f(x) = 6x^5 + 33x^4 - 30x^3 + 100$**. Find the sum of the x-values of the critical points of f(x).**

(A) −5.6 **(B) −4.4** **(C) 0** **(D) 0.6** **(E) 2.2**

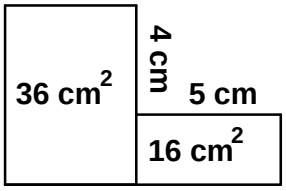
40. The odds of a Big 12 basketball team beating a SEC basketball team is $\frac{3}{7}$**. How many games can they expect to lose if the Big 12 teams play 14 games against the SEC?**

(A) 10 **(B) 9** **(C) 8** **(D) 6** **(E) 4**

41. $\int \left(\frac{1-x^2\sin(x)}{x^2}\right) dx =$ _____ $+ C$**, where C is some arbitrary constant and** $x > 0$**.**

(A) $\frac{2-\cos(x)}{x}$ **(B)** $\ln(x) + \cos(x)$ **(C)** $\cos(x) - \ln(x)$ **(D)** $\frac{-1+x\cos(x)}{x}$ **(E)** $\frac{-\ln(x)-\cos(x)}{x}$

42. Given the areas of the two rectangles shown find the perimeter of the hexagon. (The drawing is not drawn to scale.)



(A) 36 cm **(B) 34.4 cm** **(C) 32.5 cm** **(D) 31.2** **(E) not enough data**

43. Which of the following is/are true about the natural number 3?
1. prime 2. Germain prime 3. Mersenne prime 4. one of the primes of a pair of twin primes

(A) 1 & 3 but not 2 & 4 **(B) 1, 2, & 3 but not 4** **(C) 1, 3, & 4 but not 2**
(D) 1 only **(E) 1, 2, 3, & 4**

44. $2154_7 \div 6_7 \times 5_7 =$ _____ $_7$

(A) 2125_7 **(B)** 2233_7 **(C)** 2323_7 **(D)** 1611_7 **(E)** 1215_7

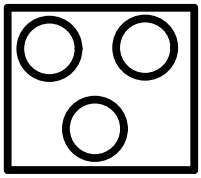
45. If $a_1 = -2$ $a_2 = -1$, $a_3 = 0$, and $a_n = (a_{n-2})(a_{n-3}) + (a_{n-1})$, where $n > 3$ then $a_7 = ?$

(A) -2 **(B)** 0 **(C)** 2 **(D)** 6 **(E)** 8

46. Let vector $u = (-1, 2)$ and vector $v = (3, -5)$. Find the measure of the larger angle with initial side u and terminal side v. (nearest minute)

(A) 184°39' **(B)** 94° 23' **(C)** 85° 36' **(D)** 175° 36' **(E)** 184° 24'

47. Willie Maykit tosses a bean bag at the target board. The board is a square with sides 3 feet long with 12 inch diameter circles cut out of it as shown. What is the probability he tosses the bag into one of the holes? (nearest whole percent)



(A) 20% **(B)** 25% **(C)** 26% **(D)** 30% **(E)** 33%

48. Find $m + n$ if $\begin{bmatrix} -1 & \frac{2}{3} \\ 1 & -3 \end{bmatrix} \cdot \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} -5 \\ 8 \end{bmatrix}$

(A) -13 **(B)** -4 **(C)** -2 **(D)** 1 **(E)** 2

49. When $f(x) = x^3 + kx^2 + 5x + 1$ is divided by $x - 2$ the remainder is 7. Find the value of k.

(A) 4 **(B)** 3 **(C)** 1 **(D)** -2 **(E)** -3

50. How many five letter distinguishable code words can be created from the letters in the word ARITHMETIC such that the first letter is an I, the third letter is a consonant and the last letter is a T?

(A) 907,200 **(B)** 840 **(C)** 720 **(D)** 420 **(E)** 210

51. Let $\frac{dy}{dx} = 3x^2 - 6x + 2$, and $y = 4$ when $x = 0$. Find y when $x = -1$.

(A) -2 **(B)** 11 **(C)** -6 **(D)** 1 **(E)** 3

52. Let $f_0 = 0$, $f_1 = 1$, $f_2 = 1$, $f_3 = 2$, $f_4 = 3$, ... be the terms of the Fibonacci sequence. Which of the following is a member of this sequence?

(A) 75,025 **(B)** 671,320 **(C)** 78,152 **(D)** 651,116 **(E)** 69,152

53. Let $f(x) = \begin{cases} x^2 + 2x & \text{if } x \geq 1 \\ 1 + 2\cos(x - 1) & \text{if } x < 1 \end{cases}$. Which of the following is/are true?

1. $\lim_{x \to 1^+} f(x)$ exists 2. $\lim_{x \to 1^-} f(x)$ exists 3. $f(x)$ is continuous

(A) none of these (B) 1 & 2 but not 3 (C) 1 only (D) 2 only (E) 1, 2, & 3

54. $\frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \ldots + \frac{1}{45} + \frac{1}{55} + \frac{1}{66} = ?$

(A) $\frac{21}{22}$ (B) $\frac{5}{6}$ (C) $1\frac{1}{6}$ (D) $1\frac{23}{66}$ (E) $2\frac{1}{33}$

55. Let $f''(x) = -6x + 6$, $f'(2) = 0$, and $f(0) = -4$. Find $f(1)$.

(A) 8 (B) 2 (C) 0 (D) -2 (E) -6

56. Simplify: $\frac{(n-1)!}{(n-2)!} \times \frac{1}{(n)!} \div \frac{n}{(n+1)!}$

(A) $n - 1$ (B) $\frac{n-1}{n+2}$ (C) $n^2 - n$ (D) $\frac{(n-1)^2}{n}$ (E) $\frac{n^2-1}{n}$

57. If $\frac{x+12}{x-8} + \frac{x-8}{x+12}$ is written as the mixed number $A\frac{B}{C}$ then $B = ?$

(A) 96 (B) 32 (C) 400 (D) 16 (E) 200

58. Suppose A, B, and C are positive integers such that $\frac{41}{7} = A + \frac{1}{B + \frac{1}{C+1}}$.

The value of $3A + B + 4C$ equals:

(A) 41 (B) 36 (C) 34 (D) 28 (E) 11

59. If 4 Chops equal 3 Chips and 2 Chips equal 5 Chaps, then how many Chaps does it take to make one Chop?

(A) $1\frac{1}{5}$ (B) $1\frac{7}{8}$ (C) $2\frac{1}{8}$ (D) $1\frac{3}{5}$ (E) $2\frac{2}{3}$

60. How many ordered pairs (a, b) exist such that the four-digit number, a31b, is divisible by both 2 and 3?

(A) 18 (B) 15 (C) 12 (D) 9 (E) 6

University Interscholastic League
MATHEMATICS CONTEST
HS ● District ● 2017
Answer Key

1. C	**21. C**	**41. D**
2. B	**22. D**	**42. B**
3. D	**23. B**	**43. E**
4. E	**24. E**	**44. D**
5. C	**25. C**	**45. D**
6. A	**26. D**	**46. E**
7. B	**27. A**	**47. C**
8. D	**28. D**	**48. B**
9. A	**29. A**	**49. E**
10. E	**30. C**	**50. E**
11. A	**31. C**	**51. A**
12. B	**32. C**	**52. A**
13. D	**33. A**	**53. E**
14. E	**34. E**	**54. B**
15. A	**35. C**	**55. D**
16. E	**36. A**	**56. E**
17. C	**37. C**	**57. C**
18. A	**38. D**	**58. B**
19. B	**39. B**	**59. B**
20. E	**40. A**	**60. B**