# Intermediate Session: Calculator Applications Contest

State of the Contest

Significant Digits Workshop

# 2020 Contest Status

# The 2020 Contest

- Tests A and B are written for UIL invitational meet use.
- Tests C, D, E and G are written for TMSCA use.
- Tests F (district), H (region), and I (state)
  are written for official UIL contest use.

•All tests have been written and answer keys tabulated.

# The 2020 Contest

- All tests have at least one integer, $ and SD problem.
- Last page crunchers are at the bottom of the page.
- There is no clearing of calculators.
- *There are no percent difference problems.*

*** http://www.uiltexas.org/academics/stem/calculator-applications**

# Summary of Percent Problems

| | Numerator | Basis | +/- |
|---|---|---|---|
| Change | New-Old | Old | + or - |
| Error | Approximate-Exact | Exact | + or - |
| Increase | Larger-Smaller | Smaller | + |
| Decrease | Larger-Smaller | Larger | + |

$$\% = 100\left[\frac{Numerator}{|Basis|}\right]$$

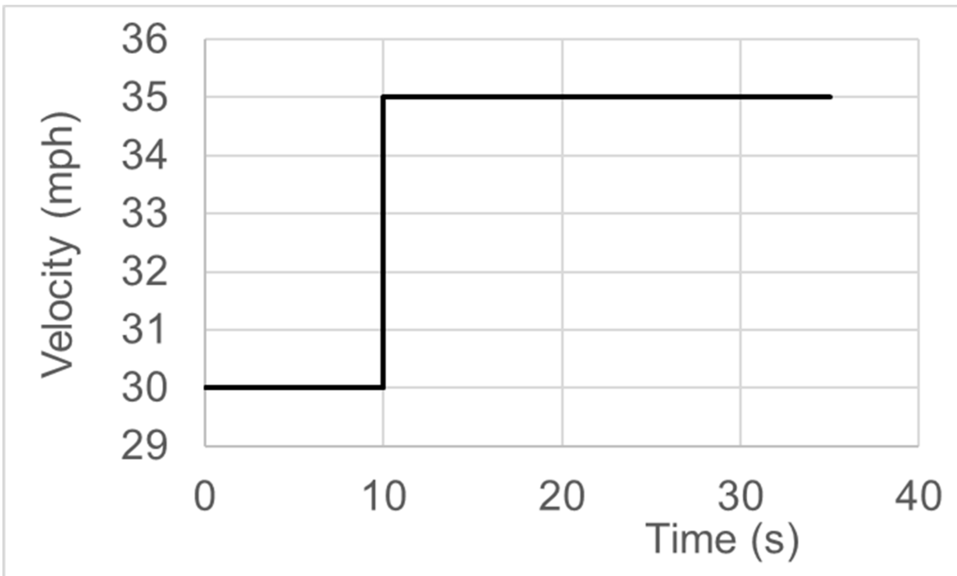
# Acceleration Stated Problems

All kinds of objects in the real world move: footballs, gears, bikes, people, satellites, bullets, planets, cars… If an object changes its speed (a car slows down as it enters a school zone) or direction (a ship changes heading from true north by $10^{\circ}$), we say that the object either accelerates (speeds up) or decelerates (slows down).

# Acceleration

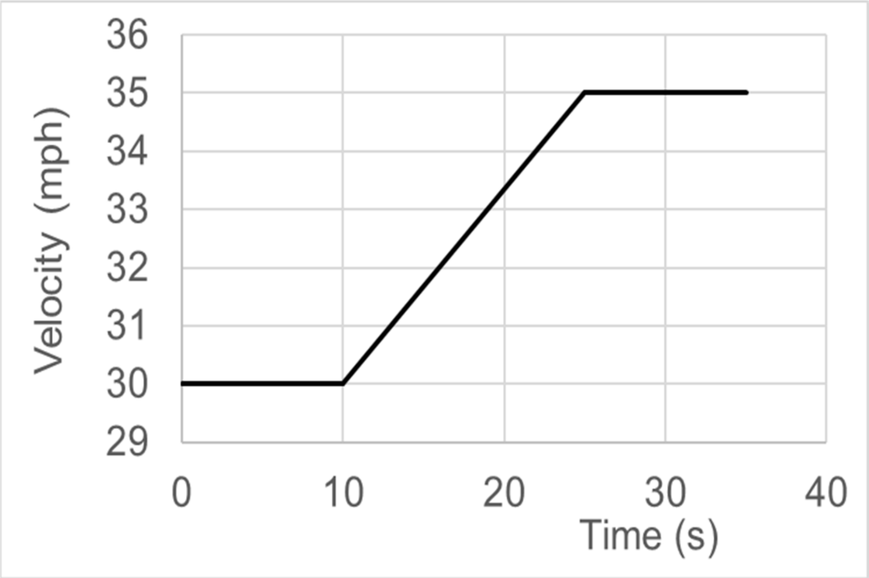
We could imagine that an object goes immediately from a starting velocity $v_o$ to some final velocity $v_f$ at some time $\tau$. Mathematically, this would look like this:

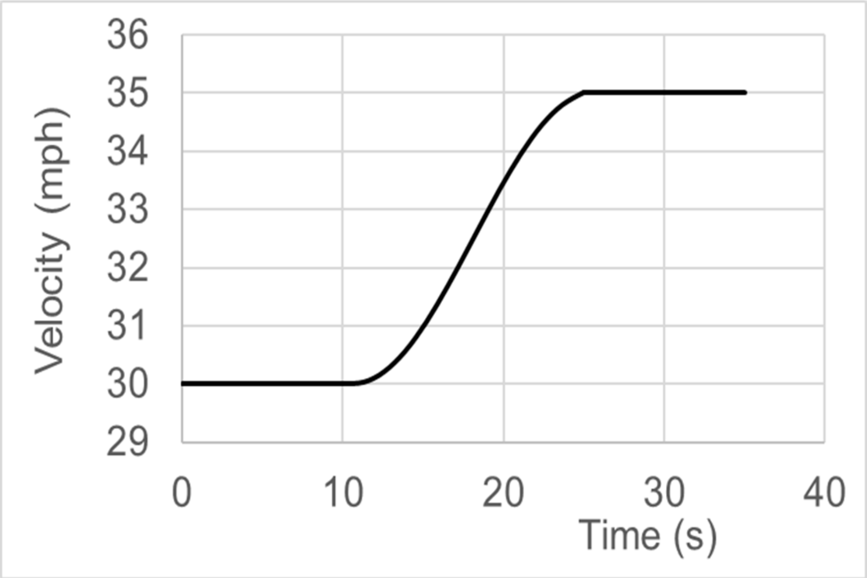
$$v = \begin{cases} v_o \; for \; t < \tau \\ v_f \; for \; t \geq \tau \end{cases}$$



# Acceleration

Such instantaneous velocity changes never happen in the real world.  The velocity slowly changes over a time interval.  Here are any number of ways the change could occur.  Two common models are linear acceleration and sigmoidal acceleration:



# Acceleration

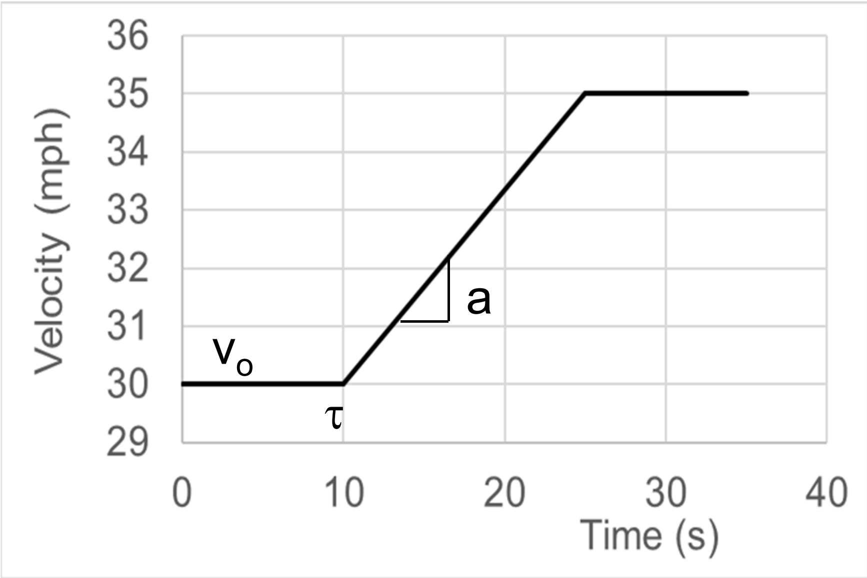
Most acceleration on the UIL Calculator Applications Contest is linear. We can write an equation from the plot for the velocity of the general form:

$$v = v_o + a(t - \tau)$$

where a is the “acceleration”. If the object slows down, the acceleration (a) is negative.

For the graph, $v_o$ = 30 mph, $\tau$ = 10 s and a = (5 mph)/(15 s) = 1/3 mph/s:

$$v = 30\ mph + \frac{(t-10\ s)\ mph}{3\ s}$$



# Acceleration

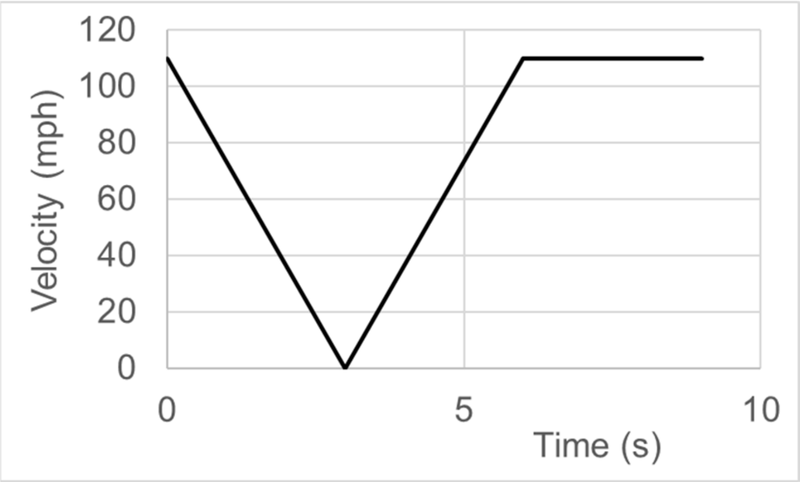
A degenerate form of the acceleration equation is when an object accelerates from rest (i.e., $v_o = 0$) and when the acceleration starts at t = 0 ($\tau = 0$):

$$v = v_o + a(t - \tau) = at$$

where a is the “acceleration”. If the object slows down, the acceleration is negative.

# Acceleration

16G-67. Every Formula 1 racing car can decelerate from 110 mph to zero and then accelerate back to 110 mph, all in less than 6 s. Assuming deceleration and acceleration are equal, what minimum, positive acceleration does this represent? (ft/s$^2$)




$$v = at$$

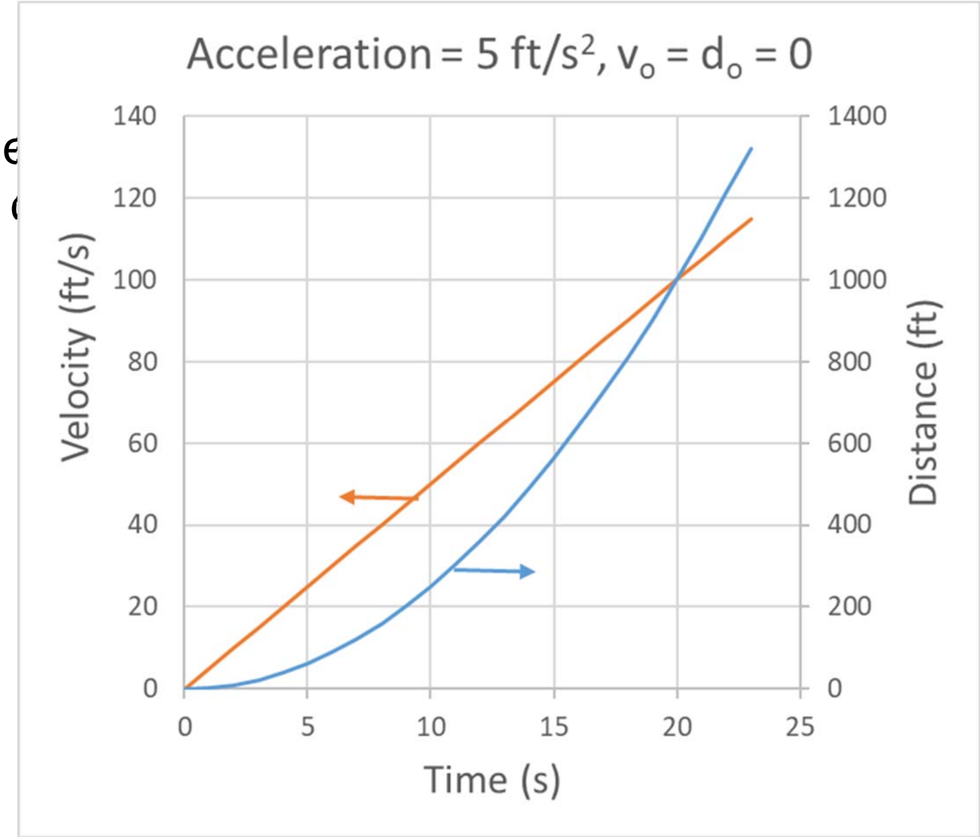
110 mph = a(3 s)

$$a = \frac{110\ mi}{(3\ s)\ hr}\left\{\frac{5280\ ft}{mi}\right\}\left\{\frac{hr}{3600s}\right\}$$

a = 53.8 ft/s$^2$

# Acceleration – Distance Traveled

In calculus terms, velocity v is the
acceleration a is the time rate of c
acceleration starting at t = 0:




$$a = a$$

$$v = \int a\,dt = v_o + \ at$$

$$\mathrm{d} = \int v\,dt = \int (v_o + \ at)dt = \ d_o + \ v_o t + \frac{1}{2}at^2$$

# Acceleration – Distance Traveled

Example: A car accelerates from rest at 25 ft/s$^2$. What is the car's velocity after it has traveled 150 ft? (mph)

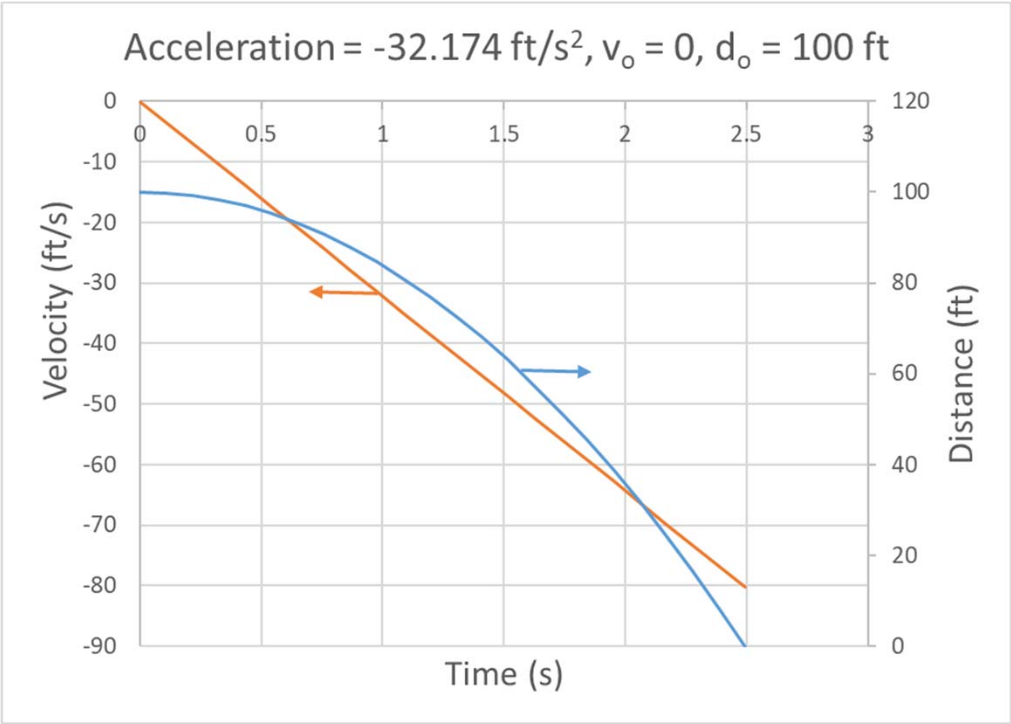
$$\mathrm{d} = d_o + v_o t + \frac{1}{2}at^2 = (0) + (0)t + \frac{1}{2}\left(25\frac{ft}{s^2}\right)t^2 = 12.5\frac{ft}{s^2}(t^2) = 150\ ft$$

$$t = \sqrt{\frac{150\ ft}{12.5\frac{ft}{s^2}}} = 3.46\ s$$

$$v = v_o + at = (0) + \left(25\frac{ft}{s^2}\right)(3.46\ s) = 86.6\ \frac{ft}{s}\left\{\frac{mi}{5280\ ft}\right\}\left\{\frac{3600\ s}{hr}\right\} = 59.0\ mph$$

# Acceleration - Gravity

On earth, gravity causes objects to accelerate to the ground. The measured acceleration is given the variable g which is by contest rules defined to be -32.174 ft/s$^2$. The negative sign is associated with the convention that we typically measure distance from the ground up as positive.



# Acceleration - Gravity

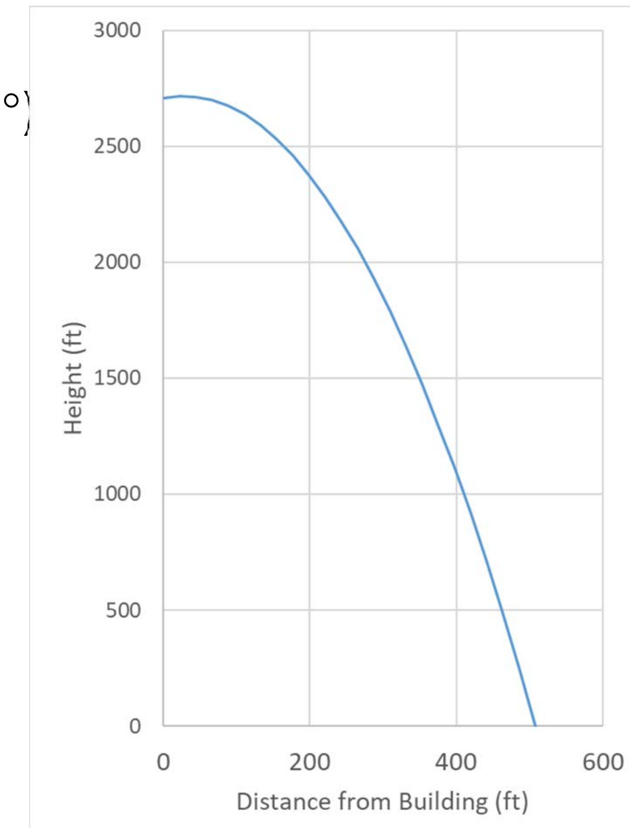
18E-63. Larry tossed a ball off the Burj Khalifa Building in Dubai, the tallest building in the world. The ball had a release angle of +35° relative to the horizontal and a release velocity of 45 ft/s. It hit the ground in 13.8 s. How tall is the building? (ft)

The vertical release velocity is $v_o$ = (45 ft/s) sin(35°)
building height, is $d_o$. So,

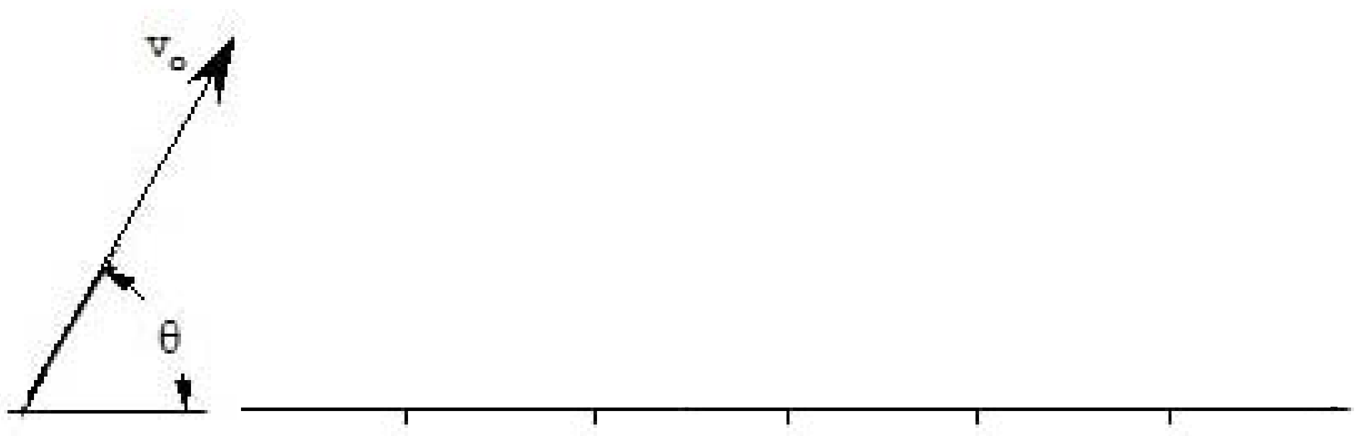
$$d = 0 = d_0 + v_0 t + \frac{1}{2} g t^2$$

$$0 = d_0 + \left(25.8 \frac{ft}{s}\right)(13.8\ s) + \frac{1}{2}(-32.174 \frac{ft}{s^2})(13.8\ s)^2$$

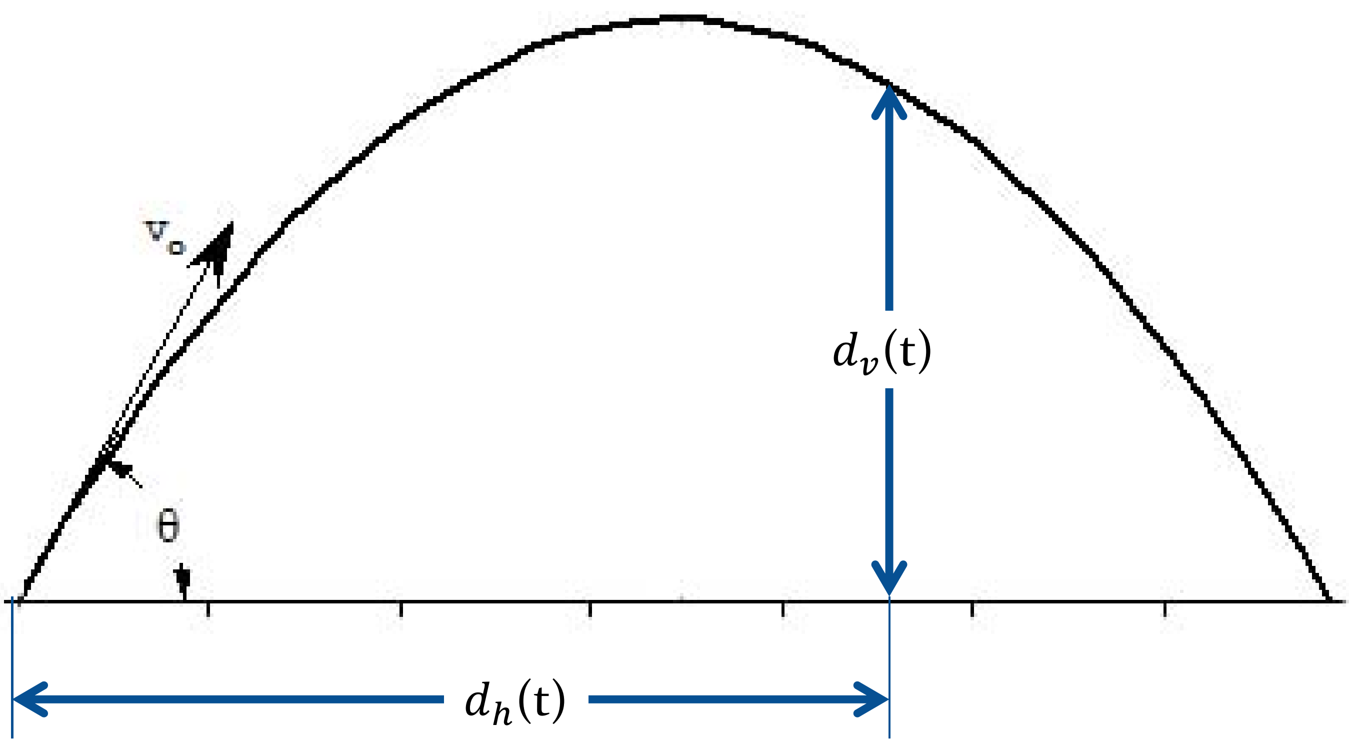
$$d_0 = 2710\ ft$$



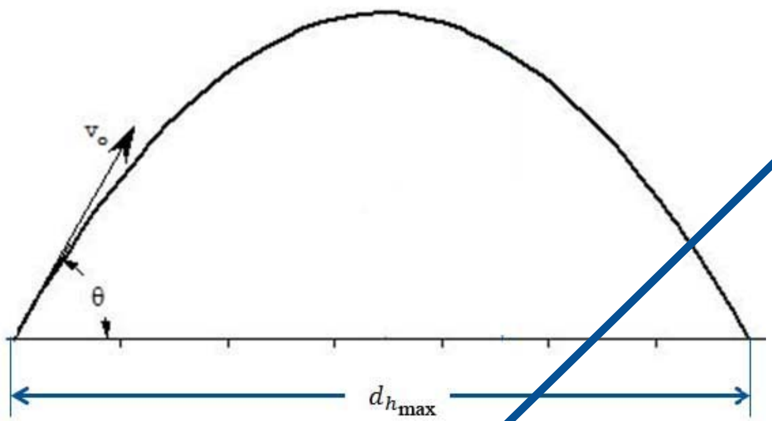
# Trajectory



# Trajectory



# Trajectory




$$g = -32.174\ \frac{ft}{s^2}$$

$$d_h = vt = (v_o\cos(\theta))t$$

$$d_v = d_{ov} + [v_{vo}]t + \frac{1}{2}gt^2$$

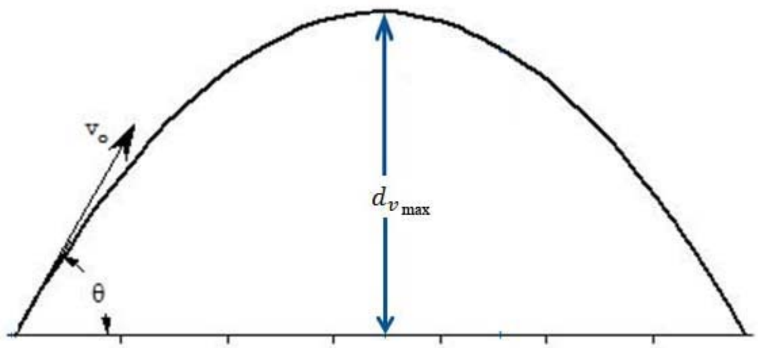
$$d_v = d_{ov} + [v_o\sin(\theta)]t + \frac{1}{2}gt^2$$

If $d_v = d_{ov}$, $[v_o\sin(\theta)]t = -\frac{1}{2}gt^2$ and $t = \frac{-2v_o\sin(\theta)}{g}$

So $d_h = d_{hmax} = [v_o\cos(\theta)]\left[\frac{-2v_o\sin(\theta)}{g}\right]$

$$\boxed{d_{hmax} = \frac{-v_o^2[2\sin(\theta)\cos(\theta)]}{g} = \frac{-v_o^2\sin(2\theta)}{g}}$$

# Trajectory




To get $d_{vmax}$ , note that $d_v$ = $d_{vmax}$ when the time is half the maximum value:

$$t = \frac{-v_o \sin(\theta)}{g}$$

Substituting into $d_v = d_{ov} + [v_o \sin(\theta)]t + \frac{1}{2}gt^2$

$$d_{vmax} = d_{ov} + [v_o \sin(\theta)]\left[\frac{-v_o \sin(\theta)}{g}\right] + \frac{1}{2}g\left[\frac{-v_o \sin(\theta)}{g}\right]^2$$

$$\boxed{d_{vmax} = d_{ov} - \left[\frac{v_o^2 sin^2(\theta)}{2g}\right]}$$

# Trajectory 1

Kate tosses a ball off a building with an initial vertical velocity of 25 ft/s. If it hits the ground in 3.4 s, how tall is the building? (ft)

$$d_v = d_{ov} + [v_o \sin(\theta)]t + \frac{1}{2} g t^2$$

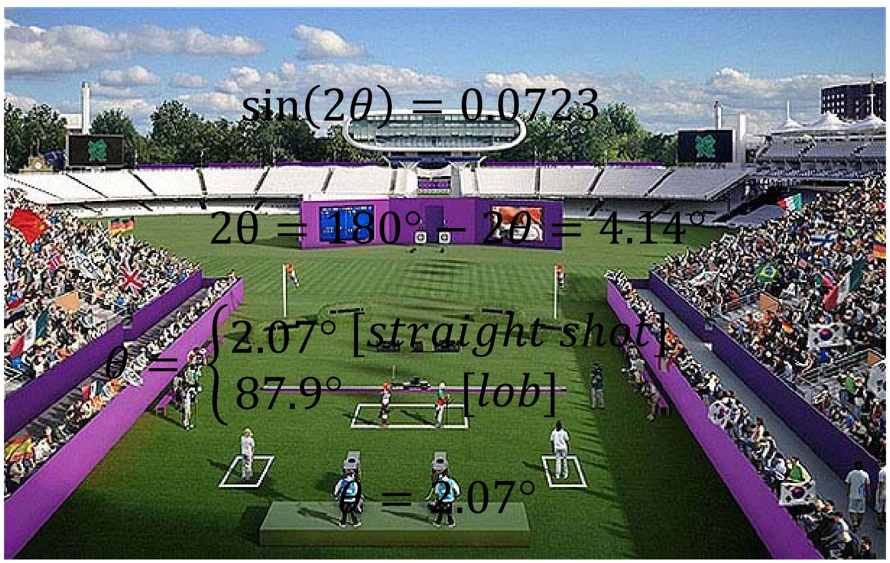
$$0 = d_{ov} + \left[25 \frac{ft}{s}\right](3.4s) + \frac{1}{2}(-32.174 \frac{ft}{s^2})(3.4s)^2$$

$$0 = d_{ov} - 101\, ft$$

$$d_{ov} = 101\, ft$$

# Trajectory 2

12F-37. In Olympic archery, the archer shoots an arrow with an average speed of 320 ft/s at a target 70 meters away.  Assuming the arrow is released at an elevation equivalent to the target's bulls eye, what should the archer's release angle be (positive, less than 45° with 0° parallel to the ground)? (deg)




$$d_{hmax} = \frac{-v_o^2 \sin(2\theta)}{g}$$

$$70\ m \left[\frac{100\ cm}{1\ m}\right]\left[\frac{in}{2.54\ cm}\right]\left[\frac{ft}{12\ in}\right] = 230\ \text{ft}$$

$$230\ ft = \frac{-(320\ ft/s)^2 \sin(2\theta)}{-32.174 ft/s^2}$$

# The End