A+ Math and A+ Number Sense

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Before We Get Started...

- Everyone should remain muted
- Submit questions through chat
- Attendance for CPE credit
A+ Math and A+ Number Sense

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Azle ISD – 1974 to 2017

7th grade math, high school math & Physics teacher

ETS Physics reader – 10 years

Married – 4 children & 3 grandchildren

Co-founder Texas Math and Science Coaches Association (TMSCA)

Coached all 4 UIL HS math & science (STEM) contests + slide rule

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Middle School Mathematics

Students begin taking math in elementary school and continue taking it in high school. Learning to complete math problems quickly is a valuable skill in all facets of life including engineering, accounting, completing a tax return and even grocery shopping. This contest includes problems covering, but not limited to: numeration systems, arithmetic operations involving whole numbers, decimals, rational numbers, exponents, order of operations, probability, statistics, number theory, simple interest, measurements and conversions. Geometry and algebra problems may be included as appropriate for the grade level.

Problem types are not bound by the current Texas essential knowledge and skills (TEKS) elements primarily because UIL
Middle School Mathematics

contest problems tend to extend the knowledge requirements of the general classroom.

The contest, designed for students in grade 6, 7 and 8, consists of 50 multiple-choice problems with a time limit of 40 minutes. Calculators are not allowed to be used by the student during the contest.

Please note that since no calculators are allowed the calculations themselves tend to be ones where a mental math shortcut may be helpful to know to be able to solve the problems in the time allotment for the contest. As such a student that practices and learns the various Number Sense mental math shortcuts will have a decided advantage in solving these
Middle School Mathematics

mathematics contest problems in less time. Also note that the breadth of material covered by this contest makes solving all 50 problems somewhat challenging.
Sample Mathematics Problems

(1) Calculate: \(\frac{1-\frac{1}{2}}{2}\).

A) 1    B) \(\frac{1}{5}\)    C) \(\frac{1}{4}\)    D) \(\frac{1}{3}\)    E) \(\frac{1}{2}\)

\[1 - \frac{1}{2} = \frac{1}{2}\]
\[\frac{1}{2} \div 2 = \frac{1}{4}\]

(C)
Sample Mathematics Problems

(2) Which of the following has more than five sides?
   A) pentagon
   B) square
   C) triangle
   D) hexagon
   E) rhombus

   (D)
There are 6 students who took part in a mathematics contest. They attained scores of 89, 92, 86, 91, 94, and 88 respectively. What is the average score of the six students?

A) 88  B) 89  C) 90  D) 91  E) 92

\[
\frac{89+92+86+91+94+88}{6} = \frac{540}{6} = 90
\]

90 (C)
Sample Mathematics Problems

(4) What is the product of the prime numbers greater than 15 and less than 20?

A) 37    B) 323    C) 17    D) 19    E) None of these

\[ 17 \times 19 = \]

\[ 323 \]  (B)

You should know (memorize) your primes up to 100!
Sample Mathematics Problems

(5) If the price of a barrel of oil drops from $50 to $45, then the percent decrease is:

A) 5%  B) 10%  C) 11 1/9%  D) 1/9 %  E) 0.1%

\[ \% \text{ decrease} = \frac{45 - 50}{50} \times 100\% \]

\[ \% \text{ decrease} = (–10\%) \quad (B) \]
Sample Mathematics Problems

(6) If one acute angle of a right triangle is 38°, then the other acute angle is:

A) 38°  B) 25°  C) 52°  D) 90°  E) 142°

Acute angles of right triangle sum to 90°.

\[ 90° - 38° = 52° \]  (C)
Sample Mathematics Problems

(7) Which of these numbers is an odd number?

A) $1^{2016} + 1$     D) $4^7 + 4$
B) $2017^2 + 2$       E) $11^{15} + 13$
C) $3^6 + 3$

The only way to get an odd answer when adding two numbers is to add an odd plus even number.

Since $7^2 = 49$ (an odd number), $2017^2$ is an odd number. So therefore $2017^2 + 2$ is odd.

(B)
Sample Mathematics Problems

(8) A 70-foot tree has a 35-foot shadow. If the building next to the tree has an 80-foot shadow, how tall is the building?

A) 40 ft.  B) 115 ft.  C) 120 ft.  D) 125 ft.  E) 160 ft.

\[
\frac{70\text{ft}}{35\text{ft}} = \frac{x\text{ft}}{80\text{ft}}
\]

\[x = 160 \text{ (E)}\]
Sample Mathematics Problems

(9) What is the value of $x$ in the figure to the right?

A) $72^\circ$

B) $108^\circ$

C) $138^\circ$

D) $158^\circ$

E) $162^\circ$

Given:

- $60^\circ + \alpha = 90^\circ \rightarrow \alpha = 30^\circ$
- $180^\circ - 48^\circ = \theta \rightarrow \theta = 132^\circ$
- $180^\circ - (\alpha + \theta) = \Omega \rightarrow \Omega = 18^\circ$
- $180^\circ - 18^\circ = x \rightarrow x = 162^\circ$

Since the sum of interior angles for a quadrilateral equals $360^\circ$:

$60^\circ + 90^\circ + 48^\circ + x^\circ = 360^\circ$

$\Rightarrow x = 162^\circ$

(E)
Sample Mathematics Problems

(10) The area of each circle shown to the right is $9\pi$. What is the perimeter of the rectangle?

A) 18  
B) 24  
C) 36  
D) 48  
E) 72

\[ \text{Area}_{\text{CIRCLE}} = \pi (\text{radius})^2 \]

\[ 9\pi = \pi \times (\text{radius})^2 \rightarrow \text{radius} = 3 \]

\[ \text{Diameter} = 2 \times 3 = 6 \]

\[ \text{Length} = 3 \times 6 = 18 \quad \text{& Width} = 6 \]

\[ \text{Perimeter} = 2 \times (18 + 6) = 48 \quad \text{(D)} \]
Number Sense Tips & Problem Solving

Andy Zapata
UIL Elementary & Junior High Number Sense

Individuals are called upon every day to use their ability to make quick mental calculations to make decisions. The development of such abilities should be an integral part of the math curriculum. Concepts covered include, but are not limited to: addition, subtraction, multiplication, division, proportions, and use of mathematical notation.

Students will be given a 10-minute, fill-in-the-blank test which they must complete without doing calculations on paper or on a calculator. Erasures and mark-outs are not permitted.

Every tenth problem is an estimation problem with an integral answer within a 5% range of the correct answer.
In the following slides I will show you the guidelines I use to create the UIL number sense tests for elementary and junior high competitors. I consider elementary students to be in 4th – 6th grades while junior high students are in 7th – 8th grades. These guidelines can also be found at the UIL A+ website. I will also show you some examples of number sense shortcuts. Finally at the end of this presentation I will show you a slide with some example problems for you to solve “MENTALLY” and the answers if time permits.
Elementary Problem Guidelines

Problem 1 – 20
1. Addition, subtraction, multiplication, & division of whole numbers
2. Recognizing place value
3. Rounding off whole numbers
4. Multiplication short-cuts
5. Remainder type problems
6. Even & odd number type problems
7. Expanded notation
8. Sums of whole numbers with regrouping
9. Differences of whole numbers with regrouping
10. Roman numerals/Arabic numbers conversion
Elementary Problem Guidelines

Problems 21 – 40

1. Addition/subtraction of fractions with common denominators
2. Addition, subtraction, multiplication, & division of decimal fractions
3. Comparing decimal fractions
4. Conversion problems (either way): fraction/decimal, percent/fraction, percent/decimal
5. Order of operations
6. More multiplication short-cuts
7. Ratio/proportion
8. Consumer type problems
Elementary Problem Guidelines

Problems 21 – 40 (continued)
9. Problems about prime numbers
10. Greatest common divisor (GCD) & least common multiple (LCM)
11. Conversion problems (either way): length, measurements, time

Problems 41 – 60
1. Addition, subtraction, multiplication & division of fractions and mixed numbers
2. Substitution problems
3. Perimeter/area of: square, rectangle, triangle
4. Radius/diameter of a circle
5. Powers & roots of numbers
Elementary Problem Guidelines

Problems 41 – 60 (continued)
6. Solving simple equations
7. Sequences
8. Sets
9. Word problems
10. Volume of cube/rectangular box
11. Right triangle problems
12. More multiplication short-cuts
13. Base systems conversions
Elementary Problem Guidelines

Problems 61 – 80
1. Addition, subtraction, multiplication & division of integers
2. Inverses
3. Basic geometry facts
4. More area problems
5. Squaring two-digit numbers
6. More multiplication short-cuts
7. More powers and roots of numbers
8. More consumer type problems
9. Inequalities
10. Probability
11. More area problems: parallelogram, rhombus, trapezoid
Problems 61 – 80 (continued)
12. Coordinate geometry - number line
13. More percent type problems

Please note that problem types in one category should not be found in a previous category. However problem types of one category can be found in subsequent categories.

For example one would not expect to see the problem \( \frac{4}{9} + \frac{2}{9} \) in problems 1 – 20. However one can see this problem \( \left( \frac{4}{9} + \frac{2}{9} \right)^2 \) in problems 61 – 80.
Junior Problem Guidelines

Problems 1 – 20
1. Addition, subtraction, multiplication & division of whole numbers, fractions, and decimals
2. Order of operations
3. Use of the distributive property
4. Comparison of fractions & decimals
5. Multiplication short-cuts
6. Squaring numbers
7. Roman numerals/Arabic numbers
8. Mean, median, mode
9. Sums of whole numbers of multiple terms of a finite series
Junior Problem Guidelines

Problems 21 – 40
1. Addition, subtraction, multiplication & division of mixed numbers and integers
2. More multiplication short-cuts
3. Percent problems
4. Conversion problems (either way):
   English/metric, length, area, capacity, time
5. Consumer type problems
6. Substitution problems
7. Solving simple equations
8. Square roots/cube roots
9. Greatest common divisor (GCD) & least common multiple (LCM)
Junior Problem Guidelines

Problems 21 – 40 (continued)
10. Number theory - prime numbers and divisors
11. Perimeter/area of: square, rectangle, circle
12. Ratio/proportion
13. Inverses
14. Multiplication of 101, 111

Problems 41 – 60
1. Sets
2. Word problems
3. Pythagorean theorem
4. Sequences
Junior Problem Guidelines

Problems 41 – 60 (continued)
5. Volume/surface area of rectangular solid/cube
6. Base systems: conversions and basic operations
7. Area of: parallelogram, rhombus, trapezoid, circle
8. Solving inequalities
9. Basic geometry facts
10. Remainder problems

Problems 61 – 80
1. Repeating decimals
2. More number theory
3. Powers of numbers
Junior Problem Guidelines

Problems 61 – 80 (continued)

4. Volume of: circular cylinder, cone, sphere
5. Sequences & series
6. Factorial
7. Coordinate geometry
8. Probability/odds
9. More percent type problems: Advanced
10. More remainder type problems
11. More multiplication short-cuts

Please note that problem types in one category should not be found in a previous category. However problem types of one category can be found in subsequent categories.

For example one would not expect to see the problem $\frac{4}{9} + \frac{2}{9}$ in problems 1 – 20. However one can see this problem $\left(\frac{4}{9} + \frac{2}{9}\right)^2$ in problems 61 – 80.
Sample Problems

(1) \[25 \times 8 = \text{______________}\]

\[8 \div 4 = 2\]

\[2 \times 100 = 200\]
Sample Problems

(2) \[ 75 \times 23 = \] 

\[ 75 = \frac{300}{4} \]

\[ 23 \div 4 = 5.75 \]

\[ 5.75 \times 300 \]

\[ 1725 \]
Sample Problems

(3) \( \text{XXIX} = \underline{\text{________}} \) Arabic Number

M = 1000; D = 500; C = 100; L = 50; X = 10; V = 5; I = 1

\[ \text{XXIX} = 20 + 9 \]

= 29
Sample Problems

(4) \( \frac{11}{8} - \frac{5}{8} = \boxed{\frac{3}{4}} \) (fraction)
Sample Problems

(5) \(0.121212\ldots = \frac{12}{99}\) (fraction)

\[
\begin{align*}
121212\ldots &= \frac{12}{99} \\
\frac{12}{99} &\div \frac{3}{3} \\
4 &= \frac{4}{33}
\end{align*}
\]
Sample Problems

(6) 1 + 2 + 3 + … + 9 = __________

1 + 2 + 3 + … + n = \frac{n(n+1)}{2}

SUM = \frac{9(9+1)}{2} = 45

Note: You should know the formulas for sums of odd and even integers also!
Sample Problems

(7) \(4 \frac{1}{3} \times 4 \frac{2}{3} = \underline{\quad}\) (mixed number)

\(\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}\) Write this down

4 \(\times (4 + 1) = 20\) Write this down

\(20 \frac{2}{9}\)
Sample Problems

**(8)** \( 6 \frac{1}{3} \times 3 \frac{1}{3} = \underline{\hspace{2cm}} \) (mixed number)

\[
\frac{1}{3} \times \frac{1}{3} = \frac{1}{9} \quad \text{Write this down}
\]

\[
6 \times 3 + (6 + 3) \frac{1}{3} = 21 - \text{Write this down}
\]

\[21 \frac{1}{9}\]
Sample Problems

(9) \(12 \div 4 \times 3 = \) ________________

\[12 \div 4 = 3 \quad 3 \times 3 \quad 9\]

(10) \(12 + 4^2 \times 3 = \) ________________

\[4^2 = 16 \]
\[16 \times 3 = 48 \]
\[12 + 48 \quad 60\]
Sample Problems

(11) \( 26 \times 86 = \) 

Since one’s digits are the same
And ten’s digits add up to 10

\[ 6 \times 6 = 36 \] – write this down \[ _{36} \]

\( (2 \times 8) + 6 = 22 \) – write this down for the finished answer

\[ 2236 \]
Sample Problems

(12) What is the area of a square with diagonal 14? _____

\[
\text{Area} = \frac{(\text{diagonal})^2}{2}
\]

\[
A = \frac{14^2}{2} \quad 98
\]
Sample Problems (Estimation)

*(13)  69 + 79 + 199 = _________________

70 + 80 + 200

350

330 – 364
Sample Problems (Estimation)

*(14)  624 \times 240 = \underline{\phantom{000000000}}

Recall \( \frac{5}{8} = .625 \)

\[ \frac{5000}{8} \times 240 = 150000 \]

142,272 – 157,248
Sample Problems (Estimation)

*(15) 167 \times 359 + 33 = \underline{\hspace{2cm}}

\frac{1}{6} \approx .167

\frac{1000}{6} \times 360 + 0 = 56987 - 62985

= 60000
Sample Problems (Estimation)

*(16) \[ \sqrt{224} \times \sqrt{325} = \] _________________

Recall: \(15^2 = 225\) and \(18^2 = 324\)

\[ 15 \times 18 = 270\]

\[257 \ - \ 283\]
Sample Problems (Practice)

(1) $25 \times 32 = \underline{____________}$

(2) $1 + 2 + 3 + \ldots + 19 = \underline{__________}$

(3) $97 \times 93 = \underline{________________}$

(4) What is the area of a square with diagonal 8? \underline{______________}

(5) DCLX = \underline{________________}(Arabic Number)

(6) $17 \times 97 = \underline{________________}$

(7) $113^2 \div 9$ has a remainder of \underline{_______________}

(8) $77 \times 73 = \underline{________________}$

*(9) $119 \times 165 = \underline{________________}$

*(10) $119 \times 251 = \underline{________________}$

800; 190; 9021; 32; 660; 1649; 7; 5621; 18654-20616; 28376-31362