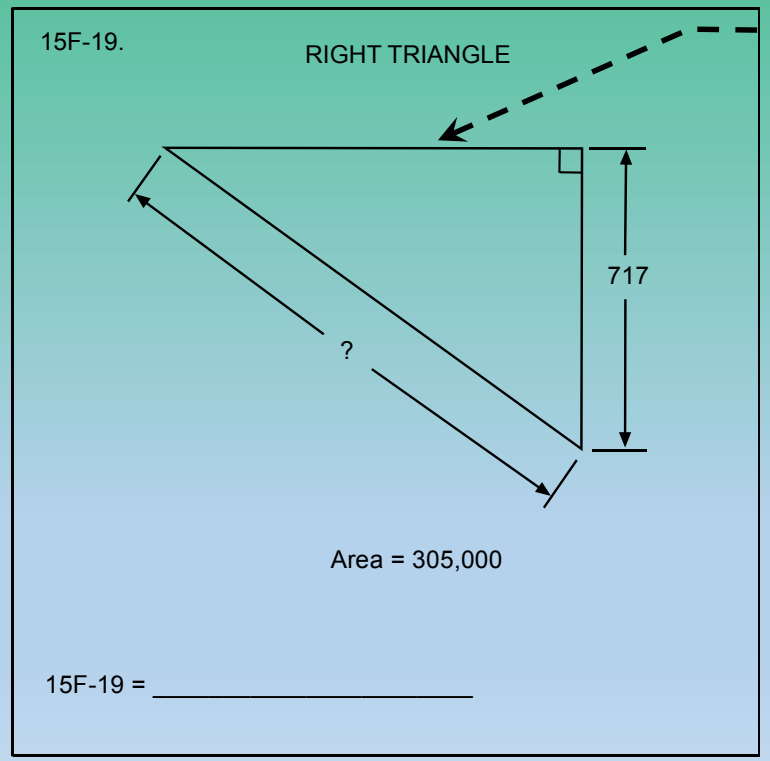
Selected Problems From 2015 HS Calculator Applications Contest



$(\frac{1}{2})(x)(717) = 305000$

$x = 850.767\ldots$

$(850.767\ldots)^2 + 717^2 = ?^2$

Andy Zapata
Azle HS

Andy Zapata

Azle ISD – 1974 to present

Azle HS – Physics teacher

Married – 4 children & 2 grandchildren

Co-founded Texas Math and Science Coaches Association (TMSCA)

Current president of TMSCA

Coached all 4 UIL math & science events + slide rule

Current UIL Elem/JH number sense, mathematics and calculator consultant

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Each year Dr. David Bourell writes at least nine UIL high school Calculator Application contests for competition. There are 21 stated problems and 14 geometry drawings. The stated and geometry problems range in difficulty from basic arithmetic to differential and integral calculus. The problems on the last page are repeated problems from past year's contest material. I've selected some of the problems that have appeared from this past year's competition to show how they are worked. My solutions might not be unique, and in fact they are the work of other coaches, but they are accurate solutions – in that they yield answers that agree with the answers that Dr. Bourell gave.

I will confess that my knowledge of the math topic, calculus, is rudimentary; and I will also admit that when I saw some of the solutions that eluded me, I had several "aha" moments.

If you have not purchased a copy of the "UIL Calculator Applications Contest Manual" by Dr. Bourell; **you need to do so**.

In any case, I hope these particular solutions will be of help to you so that you can pass them on to the students you coach – since there really is no sense in keeping this information to yourself.

z.

Page 1 Problems

15A-6. How many times can 487 be subtracted from 19,921 maintaining a positive remainder? ---------- 6=____integer

$$\frac{19921}{487} = 40.9055\ldots$$

40

15A-7. Josh bought three tax-free items that cost $1.59, $4.48 and $3.29. How much change did he get back if he paid with a $20 bill? 7=$_____

$$20 - (1.59 + 4.48 + 3.29)$$

10.64

Page 1 Problems

15B-8. What is b if $e^x = 2^{bx}$? ----------8=_____

$\ln(e^x) = \ln(2^{bx})$ $x \cdot \ln(e) = bx \cdot \ln(2)$ $b = \frac{\ln e}{\ln 2}$ **1.44**

15E-8. Chantal walks 3 blocks in 2.7 min. If a block is 330 ft long, what is her average walking speed? ----------8=_____mph

$$\frac{3\,\text{blocks}}{1} \times \frac{330\,\text{ft}}{\text{block}} \div \frac{2.7\,\text{min}}{1} \times \frac{60\,\text{sec}}{\text{min}} \times \frac{15}{22}$$

converts ft/min to mph

4.17

Page 1 Problems

15A-9.

SQUARE

Area = ?

15A-9 = _____

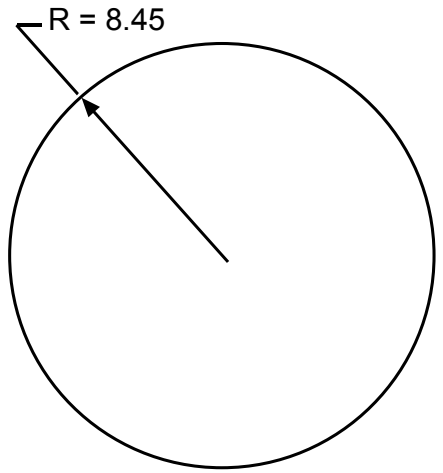
Area = $(356)^2$

127000

Page 1 Problems

15B-9.

CIRCLE



Area = ?

15B-9 = _____

Area = $\pi \cdot (8.45)^2$

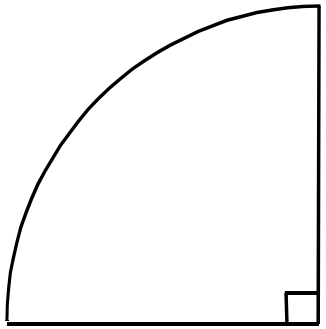
224

Page 1 Problems

15C-9.

QUARTER CIRCLE

Perimeter = ?



Area = 7.54

15C-9 = _____

$(\frac{1}{4})\pi r^2 = 7.54$

$r = 3.098 \ldots$

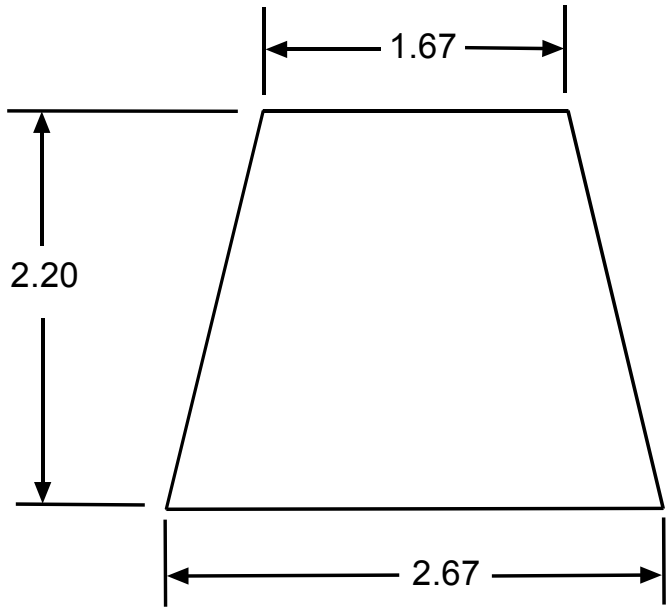
$P = (\frac{1}{4})2\pi(3.098 \ldots) + 2(3.098 \ldots)$

11.1

Page 1 Problems

15C-10.

ISOSCELES TRAPEZOID



Area = ?

15C-10 = _____

Area = (½) (1.67 + 2.67) (2.2)

4.77

Page 2 Problems

15A-17. The "Ceremonial" South Pole is a monument marking the location of the South Pole. Because it sits on almost 2 mi of shifting ice, the actual location moves about 10 meters/yr. How long would it take the actual location to shift 100 yd from the Ceremonial South Pole? ----------17=_____yr

$$\left(\frac{100\,\text{yd}}{1}\right)\left(\frac{36\,\text{in}}{\text{yd}}\right) \times \left(\frac{2.54\,\text{cm}}{\text{in}}\right)\left(\frac{1\,\text{m}}{100\,\text{cm}}\right) \times \left(\frac{1\,\text{yr}}{10\,\text{m}}\right)$$

9.14

OR

$$\left(\frac{100\,\text{yd}}{1}\right)\left(\frac{.9144\,\text{m}}{\text{yd}}\right)\left(\frac{1\,\text{yr}}{10\,\text{m}}\right)$$

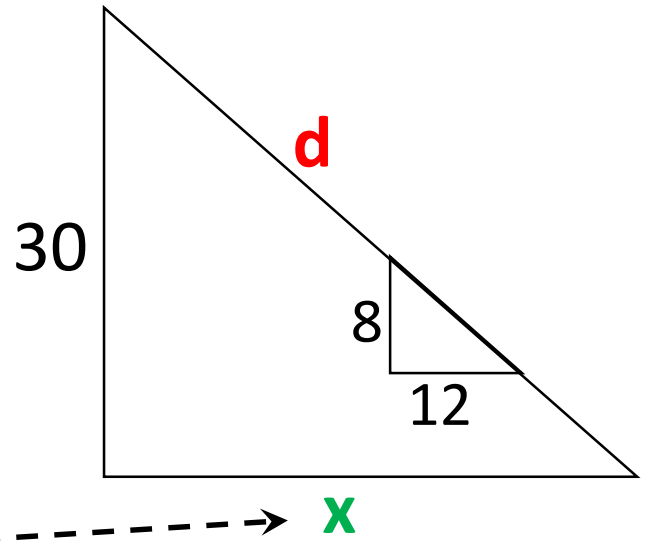
9.14

Page 2 Problems

15B-17. Steps are 8 in tall and 12 in deep. If the top step in a straight stairway to a door is 30 ft above the ground, how far from the bottom of the door does the stairway start? ----------17=_____ ft

Similar Triangles

$$\frac{8\,\text{in}}{12\,\text{in}} = \frac{(30\,\text{ft})(12\,\text{in}/\text{ft})}{x}$$



$x = 45$ ft

$d^2 = 30^2 + 45^2$

54.1

Page 2 Problems

15E-18. Frank is charged with buying hamburger buns for a large cookout. He needs buns for 357 hamburgers. At the store, he learns that he can buy an 8-pack for $1.69 or a 36-pack for $5.99. What is the least amount of money he can spend on the buns? ----------18=$_____

$$\frac{357\,\text{buns}}{8\,\text{buns/package}} = 44.6\,\text{pkgs} \rightarrow \text{Need 45 pkgs} \qquad (45\,\text{pkgs})\left(\frac{\$1.69}{\text{pkg}}\right) = \$76.05$$

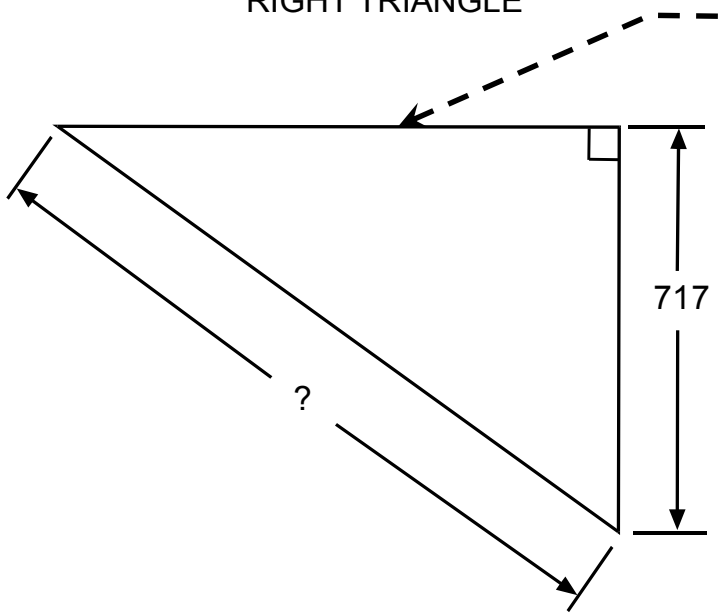
$$\frac{357\,\text{buns}}{36\,\text{buns/package}} = 9.92\,\text{pkgs} \rightarrow \text{Need 10 pkgs} \qquad (10\,\text{pkgs})\left(\frac{\$5.99}{\text{pkg}}\right) = \$59.90$$

$59.90

Page 2 Problems

15F-19.

RIGHT TRIANGLE



Area = 305,000

15F-19 = _____

$(½)(x)(717) = 305000$

$x = 850.767\ldots$

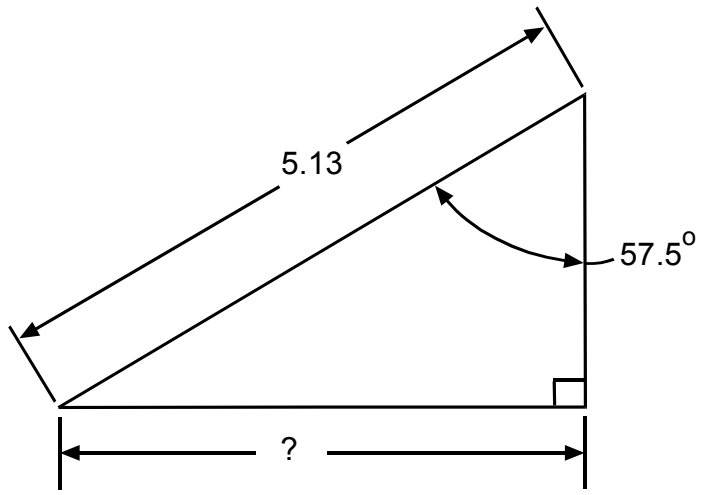
$(850.767\ldots)^2 + 717^2 = ?^2$

1110

Page 2 Problems

15B-19.

RIGHT TRIANGLE



15B-19 = _____

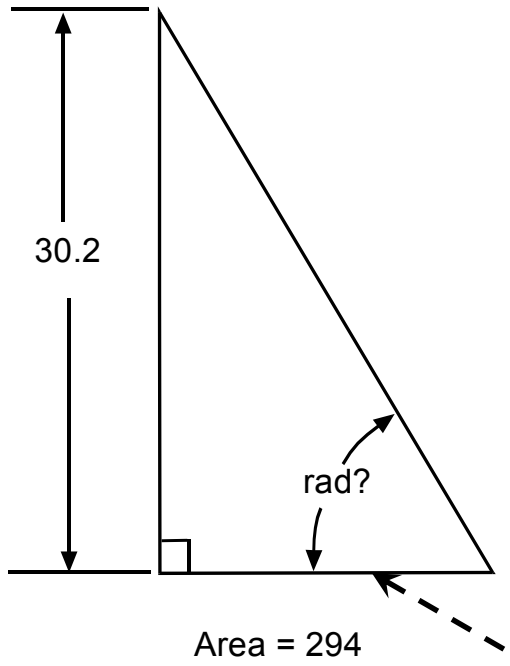
$$\sin 57.5° = \frac{?}{5.13}$$

4.33

Page 2 Problems

15C-19.

RIGHT TRIANGLE



15C-19 = _____

(½) (b)(30.2) = 294

b = 19.470 . . .

$$\tan ? = \frac{30.2}{19.470...}$$

? = 57.189 . . . °

.998 rad

Page 3 Problems

15A-27. The red sandstone Stone of Scone is rectangular, 26.2 in by 16.75 in by 10.5 in, and its weight is approximately 336 lbs. What is the percent difference in its density and the handbook value for sandstone, 2.65 g/cm^3? ----------27=_____ %(SD)

(336 lb) / [(26.2)(16.75)(10.5)] in^3 ➔ converting 336 lbs ≈ 152.407 . . . kg

(.033074982 kg/ in^3) x (1000 g/kg) x (1 in/2.54 cm)^3 = 2.018359236 g/cm^3

$$\%\text{Difference} = 100 \times \left[\frac{\text{2nd Number}}{\text{1st Number}} - 1 \right] \qquad 100\,\% \times \left[\frac{2.65}{2.0183\ldots} - 1 \right]$$

1.312947642 {3SD} – 1 = . 312947642 {2SD} x 100%

31 {2 SD}

Page 3 Problems

15E-28. David has a 20-in long sling and wants to sling a stone at Goliath. His arm rotation adds another 15 in to the radius of rotation. At what RPM must David spin the sling if the release velocity of the stone is 30 mph?---------28=_____ RPM

$$(30\,\text{mph})\left(\frac{22}{15}\right) = 44 \text{ ft/s x (12 in/ft)} = 528 \text{ in/s}$$

linear speed = (angular speed) x (radius) ➔ $v = (\omega)(r)$

$$\omega = \frac{v}{r} \rightarrow \omega = \frac{528\,\text{in}}{(20\,\text{in} + 15\,\text{in})} = 15.0857\ldots \text{ rads/s}$$

$$(15.0857\ldots \text{rads/sec})\left(\frac{1\,\text{rev}}{2\pi\,\text{rads}}\right)\left(\frac{60\,\text{sec}}{1\,\text{min}}\right)$$

144 rpm

Page 3 Problems

15F-26. Randy and Walter buy lunch. They spend $18.75 and $16.45, respectively, on their meal, but they decide to split the check in half. If they add on an 18% tip, how much of Randy's lunch did Walter pay for? ----------26=$_____

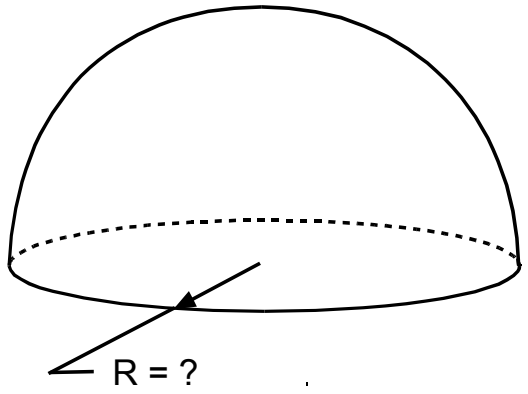
$$\frac{\$18.75 + \$16.45}{2} = \$17.60$$

$$\frac{(\$18.75 - \$17.60)(1.18)}{2} = \boxed{1.36}$$

Page 3 Problems

15E-29.

HEMISPHERE



Volume = 80.7

15E-29 = _____

$$V_{Sphere} = \frac{4}{3}\pi r^3$$

$$\left(\frac{1}{2}\right)\left(\frac{4}{3}\right)\pi r^3 = 80.7$$

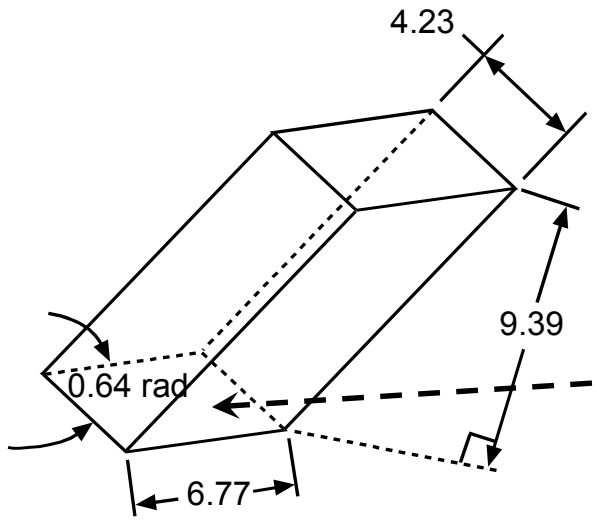
$$r = \sqrt[3]{\frac{3(80.7)}{2\pi}}$$

3.38

Page 3 Problems

15C-29.

PARALLELOGRAM SOLID



Volume = ?

15C-29 = _____

Either place calculator in “radian” mode or change 0.64 rads to degrees.

$A_{Face} = (6.77)(4.23)\sin(.64)$

$A_{Face} = 17.1019\ldots$

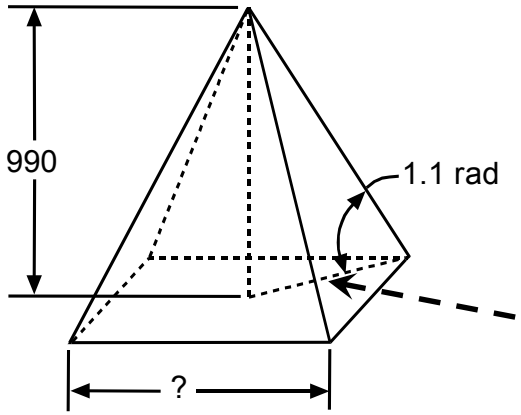
$V = (17.109\ldots)(9.39)$

161

Page 3 Problems

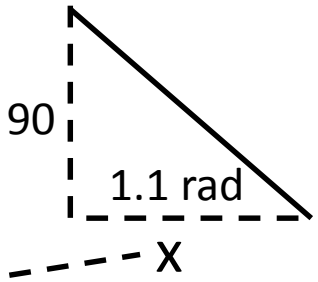
15E-30.

SQUARE PYRAMID



15E-30 = _____

Either place calculator in “radian” mode or change 1.1 rads to degrees.



$$\tan 1.1 = \frac{990}{x}$$

$x = 503.87\ldots$ {½ square’s diagonal}

$$? = \frac{(2)(503.87\ldots)}{\sqrt{2}}$$

713

Page 4 Problems

15E-36. Uriel's keys sometimes slip out of her hands. She grabs them after they have fallen 15 in. If her reaction time is 200 ms, what is the time between when they slip out of her hand and when she grabs them? ----------36= _____ msec

$y = y_0 + v_0 t + \frac{1}{2} a t^2$

Where y is final vertical height; y_0 is initial vertical height; v_0 is initial vertical speed; t is time; a is acceleration. Note in this case a is really g – the acceleration due to gravity.

$$0 = \frac{15\,\text{in}}{12\,\text{in}/\text{ft}} + 0 + \left(\tfrac{1}{2}\right)\left(-32.174\,\text{ft}/\text{sec}^2\right)\left(t^2\right)$$

Note that in this equation we are stating that the final location the keys are caught is "0" and that the initial location of the keys is 15 inches above, the initial speed is "0" and that the acceleration due to gravity is negative because of the downward direction.

t = .27875… sec x (1000 msec / sec)

279

Page 4 Problems

15H-37. For a catalytic reaction to occur, the chemicals must be exposed to 1 m^2 of catalyst surface. A 1 in^3 block of catalyst is ground to powder. Assuming no material loss and spherical particles, what average particle diameter is needed?

----------37= _____ µm

$1\ in^3 \times (2.54\ cm / in)^3 \times (1\ m / 100\ cm)^3 = .000016387\ m^3$

$$\frac{V_{Sphere}}{SA_{Sphere}} = \frac{\left(\frac{4}{3}\right)(\pi)\left(r^3\right)}{4\pi r^2} = \frac{r}{3} = \frac{.000016387\,m^3}{1\,m^2}$$

$r = .00004961\ldots$

$$2r = .000098322\ldots m \times \frac{1\,\mu m}{10^{-6}\,m}$$

98.3

Page 4 Problems

15F-36. Farmer Gilmore thought he had 25.5 acres of land, but a surveyor measured the rectangular plot to be 892.52 ft by 1285.88 ft. What is the percent error in the farmer's estimate? ----------36=_____%(SD)

$$\frac{(5280\ \text{ft})^2}{640\ \text{acres}} = 43560\ \text{ft}^2/\text{acre}$$

(25.5 acres) x (43560 ft^2/acre) = 1110780 ft^2 {3 SD}

(892.52 ft) x (1285.88 ft) = 1147673.67 . . . ft^2 {5 SD}

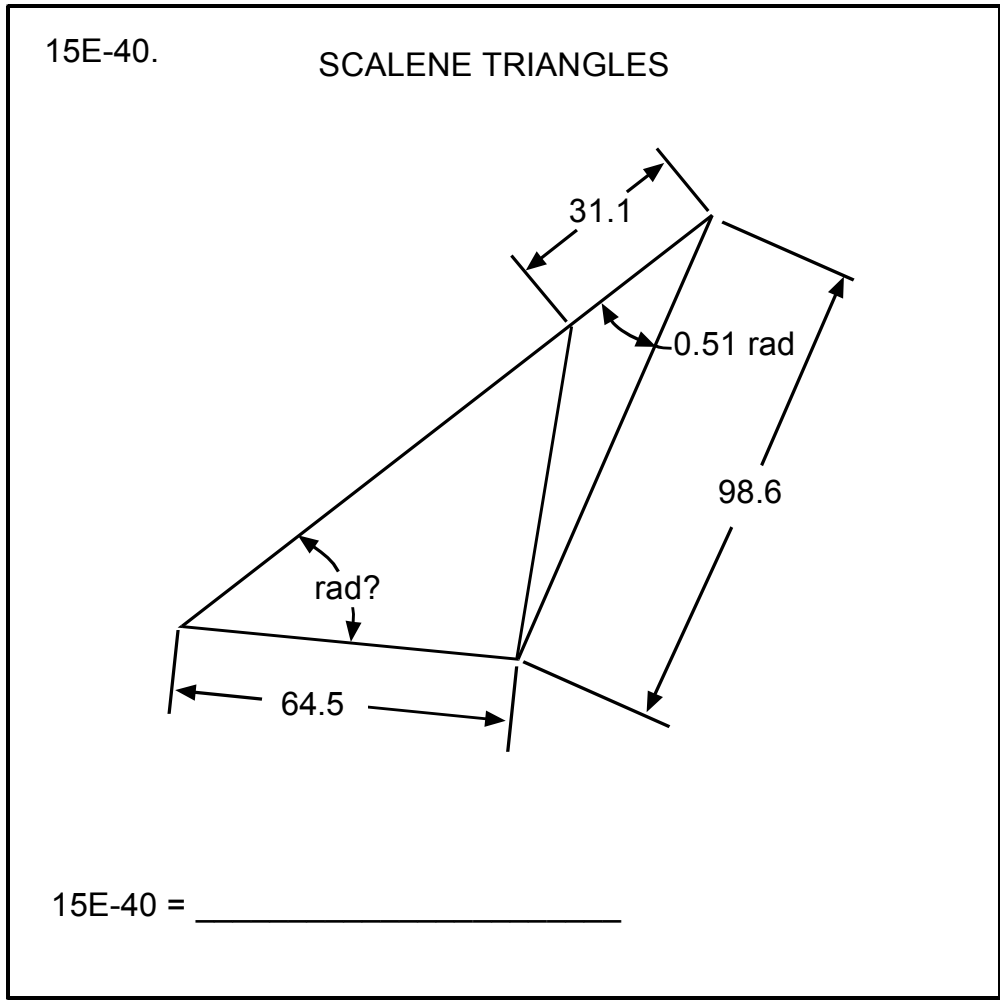
$$\%\text{Error} = 100 \times \left[\frac{\text{approximate}}{\text{exact}} - 1\right] \qquad \%\text{Error} = 100 \times \left[\frac{11107\ldots(3\text{SD})}{11476\ldots(5\text{SD})} - 1\right]$$

%Error = (.96785… {3SD} − 1) x 100

(-.032146… {2SD}) x 100

-3.2

Page 4 Problems



Place calculator in “radian” mode and use Law of Sines.

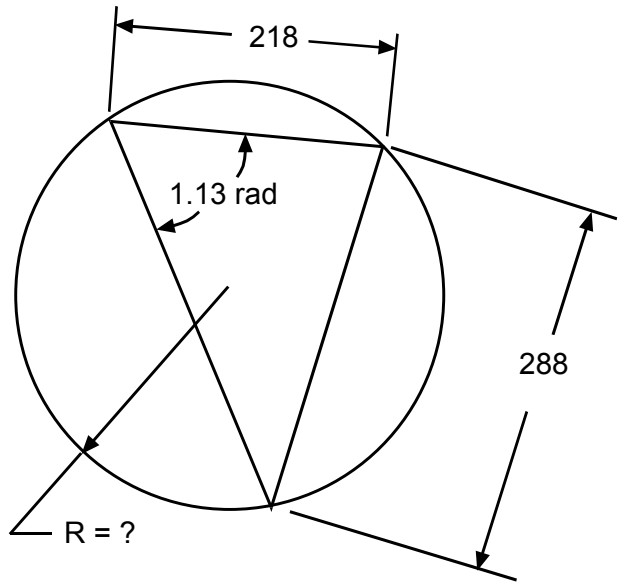
$$\frac{\sin .51}{64.5} = \frac{\sin ?}{98.6}$$

? = .842

Page 4 Problems

15A-39.

SCALENE TRIANGLE AND
CIRCUMSCRIBED CIRCLE



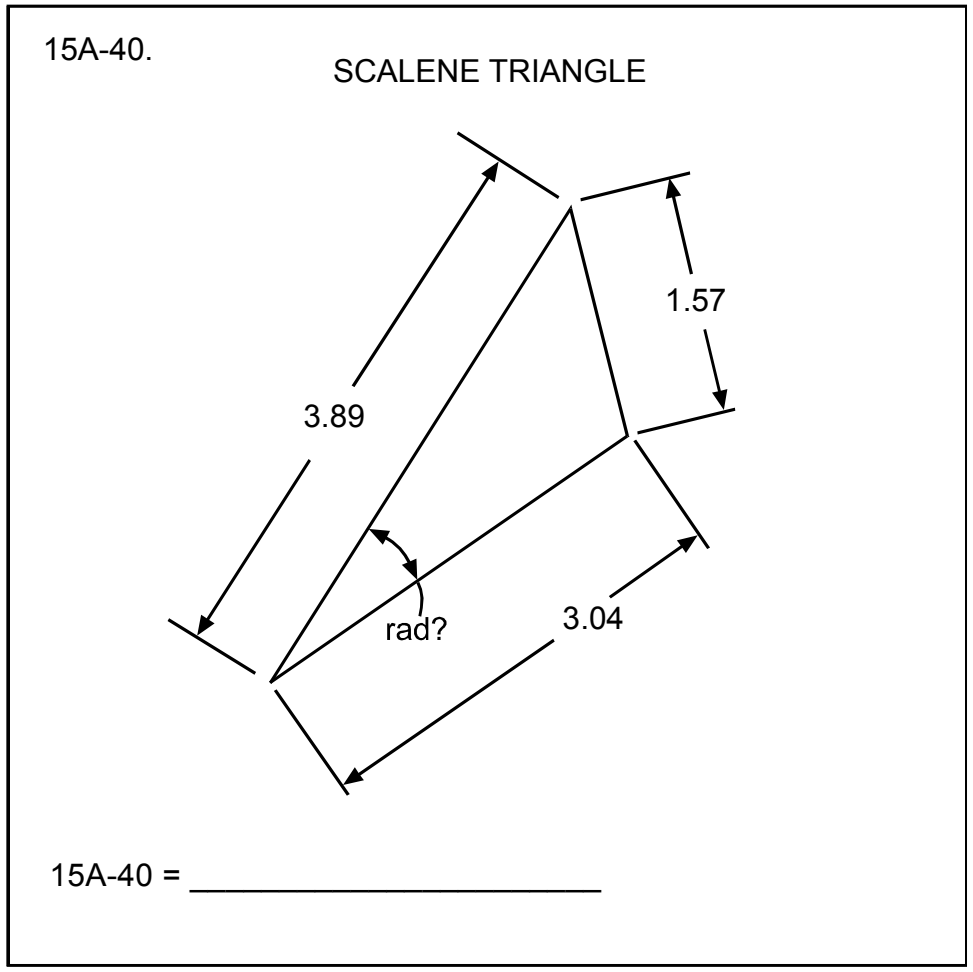
15A-39 = _____

Place calculator in "radian" mode.

$$\text{Radius} = \frac{288}{2\sin 1.13}$$

159

Page 4 Problems



Place calculator in “radian” mode and use Law of Cosines.

$1.57^2 = 3.89^2 + 3.04^2 - 2(3.89)(3.04) \cos ?$

.386

Page 5 Problems

15B-46. A cookie recipe calls for 2 cups oatmeal and makes 4 dozen 3-in cookies. How many cups of oatmeal are needed to make 8 dozen 4-in cookies?

----------46=_____ cups

$$\frac{(4)(3^3)}{2} = \frac{(8)(4^3)}{x}$$

Scaling problem: see paged 53 – 57 in UIL Calculator Applications Contest Manual

9.48

Page 5 Problems

15C-47. The mean earth temperature increase relative to the value in 1970 is given by (1970, 0°C), (1980, 0.17°C), (1990, 0.30°C), (2000, 0.41°C), (2010, 0.58°C). Estimate the year in which the earth's temperature will have increase by 1°C.----------47=_____ integer

Linear Regression problem: see paged 57 – 59 in UIL Calculator Applications Contest Manual

list 1:	1970	1980	1990	2000	2010
list 2:	0	.17	.30	.41	.58

linear regression: → $y_1(x)$ Solve $y_1(x) = 1$

2040

Page 5 Problems

15B-47. Tracey ran a mile each day for a week. Her times were 9 min 20 s, 9 min 8 s, 8 min 45 s, 8 min 18 s, 8 min 2 s, 7 min 55 s, and 7 min 40 s. After how many more days will her time be estimated to drop just below 7 min?

----------47= dy integer

list 1:	1	2	3	4	5	6	7
list 2:	9 1/3	9 8/60	8 ¾	8 18/60	8 1/30	7 55/60	7 2/3

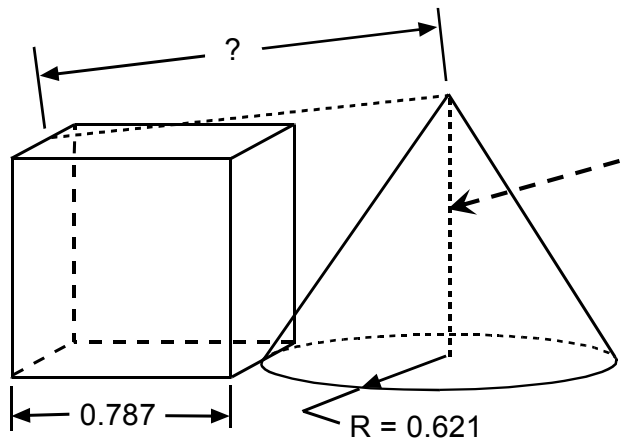
linear regression: → Solve $y_1(x) = 7.00$

x = 8.97 → Day 9 → 9 – 7 **2**

Page 5 Problems

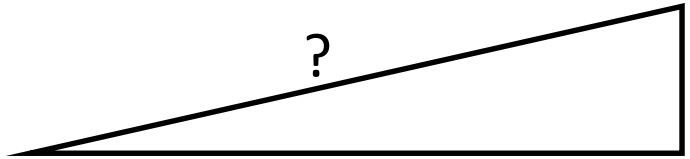
15C-49

CUBE AND CONE



Volume(Cube) = Volume(Cone)

15C-49 = _____



$(1/3)(h)(\pi)(.621)^2 = (.787)^3$

$h = 1.20701\ldots$

$h - .787 = .42001$

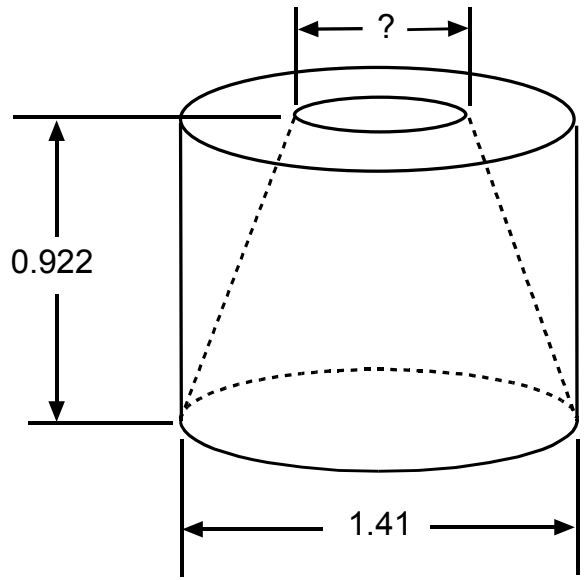
$(1.408)^2 + (.42001\ldots)^2 = ?^2$

1.47

Page 5 Problems

15D-50.

CYLINDER WITH FRUSTUM CAVITY



Volume = 0.676

15D-50 = _____

large radius = (1.41 / 2); small radius x 2 = ?

large radius = .705

$V_{Cavity} = V_{Cylinder} - V_{Frustum}$

$.676 = \pi(.705^2)(.922) - (\pi/3)(.922)($

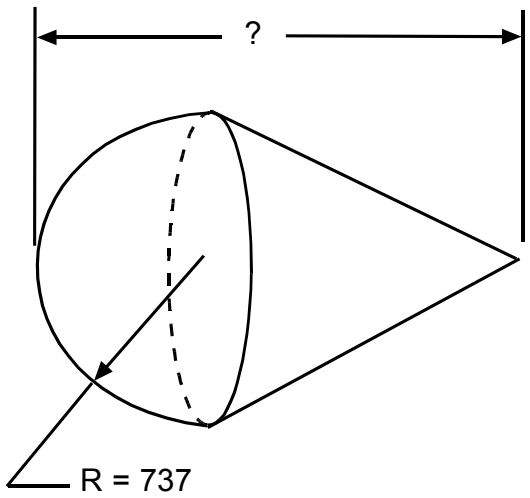
$(.705^2 + r^2 + .705r)$

$r = .294 \rightarrow 2r =$ **.588**

Page 5 Problems

15D-49.

CONE AND HEMISPHERE



Hemisphere Surface Area = Cone Surface Area

15D-49 = _____

$SA_{Cone} = (\pi)(\text{radius})(\text{slant height})$

$SA_{Sphere} = (4)(\pi)(\text{radius})^2$

$(\pi)(737)(s) = 2(\pi)(737)^2$

$s = 1474$

$737^2 + (\text{cone height})^2 = 1474^2$

cone height = 1276.5 . . .

1276.5 . . . + 737 =

2010

Page 6 Problems

15A-56. Calculate b if the slope of the curve $y = 3x^3+bx^2-104x+850$ at $x = 2$ equals 37.

----------56=_____

The first derivative of a polynomial expression yields the slope of the line described by the polynomial.

$$y' = 9x^2 + 2bx - 104$$

Substituting and solving for "b"

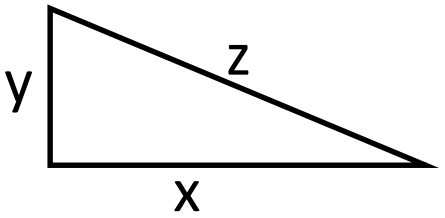
$$37 = 9(2^2) + 2b(2) - 104$$

26.3

Page 6 Problems

15E-57. A 4-ft by 8-ft sheet of plywood is leaned against a wall with the 4-ft edges touching the floor and wall. The piece starts sliding to the floor, but contact with the wall is maintained. If the end touching the wall moves downward with a constant velocity of 1.5 ft/s, how far above the floor is it when the opposite end is sliding along the floor at 1 ft/s?----------57=_____ ft

Given $x = 8$ ft & $\frac{dy}{dt} = -1.5\,\text{ft/s}$ Find y when $\frac{dx}{dt} = 1\,\text{ft/s}$



$$x^2 + y^2 = 8^2 \rightarrow 2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$$

$$\rightarrow x\frac{dx}{dt} = -y\frac{dy}{dt}$$

$$\left(\sqrt{8^2 - y^2}\right)(1\,\text{ft/s}) = -y(-1.5\ \text{ft/s})$$

4.44

Page 6 Problems

15A-58. Calculate t given that $A_3 = 88$, **B** = $\begin{bmatrix} 52 & 10 & 8 \\ 9 & 7 & 4 \\ 8 & 4 & -38 \end{bmatrix}$ and C = $\begin{bmatrix} -3 \\ 7 \\ t \end{bmatrix}$

and **A** = **BC**.----------58=_____

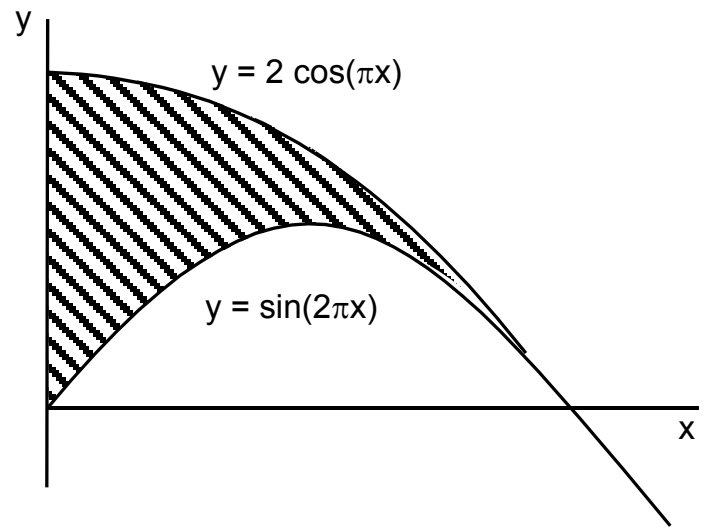
8(-3) + 4(7) + (-38)(t) = 88

-2.21

Page 6 Problems

15D-59.

Radians



Hatched Area = ?

15D-59 = _____

$y_1(x) = 2\cos(\pi x)$ & $y_2(x) = \sin(2\pi x)$

Find the intersection point fo the two graphs.

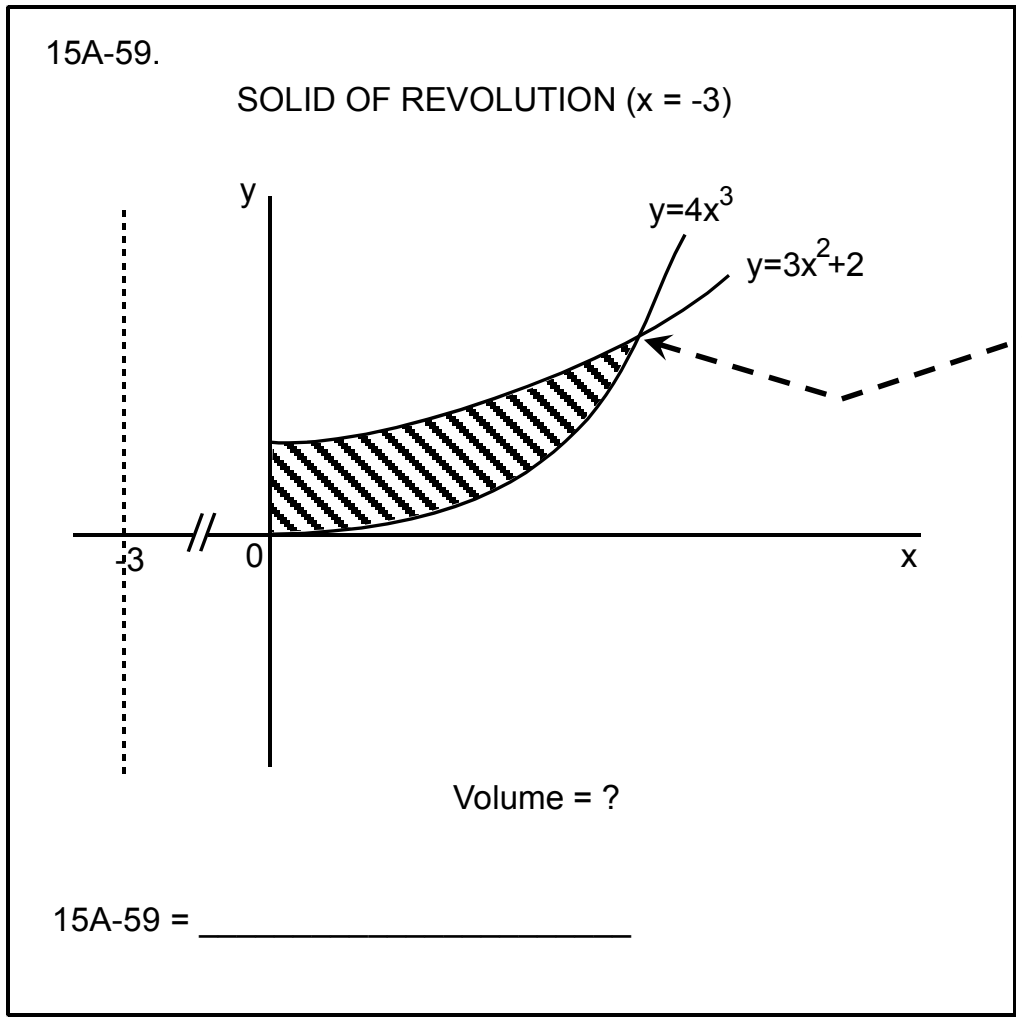
intersection is at $x = .5$

Integrate from 0 to .5 to find the area between the two graphs.

$$\int_0^{0.5} [y_1(x) - y_2(x)]$$

$$\int_0^{0.5} [2\cos(\pi x) - \sin(2\pi x)]$$ **.318**

Page 6 Problems



Use the "Shell" method to solve.

First find point of intersection for the two graphs

Intersection is at x = 1.1368612 . . . .

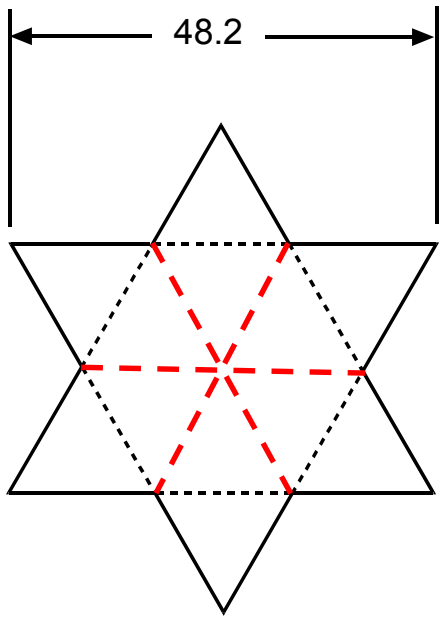
$$\text{Volume} = (2\pi)\int_{0}^{1.1368612}(x+3)\left[\left(3x^2+2\right)-4x^3\right]dx$$

45.5

Page 6 Problems

15A-60.

ROTATED CONGRUENT
EQUILATERAL TRIANGLES



Total Area = ?

15A-60 = _____

Note that drawing some auxiliary lines creates 12 equilateral triangles.

$$\text{Total Area} = 12 \times \left[\frac{\left(\frac{48.2}{3}\right)^2 \left(\sqrt{3}\right)}{4} \right]$$

1340

Page 7 Problems

15A-68. A person can jump 5.5 ft vertically on earth. For the same effort, defined as identical initial velocity, how far could they jump on the moon, if the gravitational acceleration is 16.9% that of earth? ----------68=_____ ft

$$\text{New Height} = \frac{5.5}{.165}$$

32.5

Page 7 Problems

15H-66. A motorcycle dare devil rides his motorcycle up a 25^o ramp at 65 mph. The ramp was built using 12 sheets of 8-ft long plywood. What is the horizontal distance from the end of the ramp to the spot on the ground where the dare devil lands?

---------- 66=_____ ft

$$96 \sin 25^o + [(22/15)(65) \sin 25^o](t) - (½)(32.174)(t^2) = 0$$

$$t = 3.2746\ldots$$

Horizontal distance = $[(22/15)(65) \cos 25^o]$ (3.2746 …)

283

Page 7 Problems

15D-68. A projectile is fired from Athens TX to Tyler, 36.1 mi away, at a release angle of 46°. What is the projectile maximum elevation during flight?

----------68=_____ mi

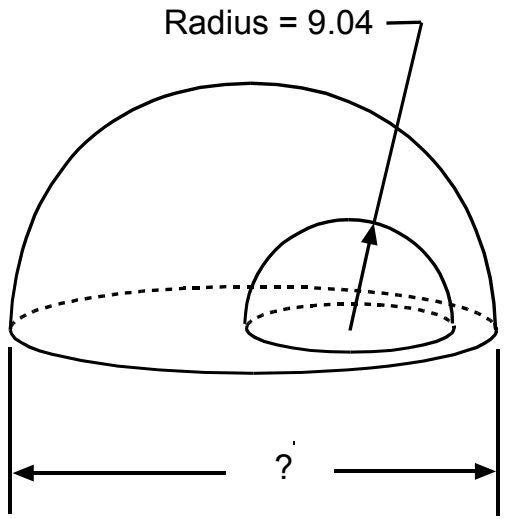
$$\text{height} = \frac{(36.1)\tan 46^\circ}{4}$$

9.35

Page 7 Problems

15F-69.

HEMISPHERES



Total Surface Area = 4,200

15F-69 = _____

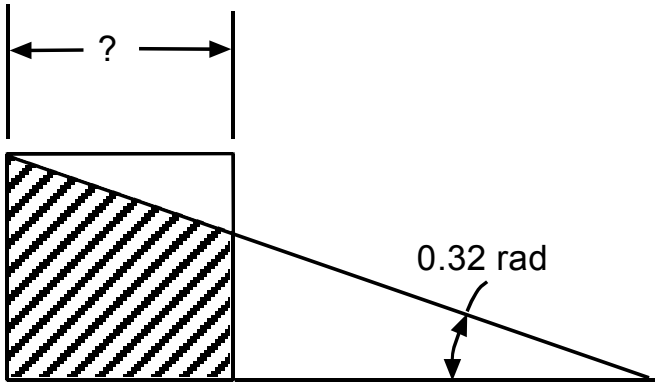
$$\text{diameter} = (2)\sqrt{\frac{4200 - \pi\left(9.04^2\right)}{3\pi}}$$

40.9

Page 7 Problems

15E-69.

SQUARE AND RIGHT TRIANGLES



Hatched Area = 2.77

15E-69 = _____

Place calculator in "radian" mode.

$?^2 - (.5)(?^2)[\tan(.32)] = 2.77$

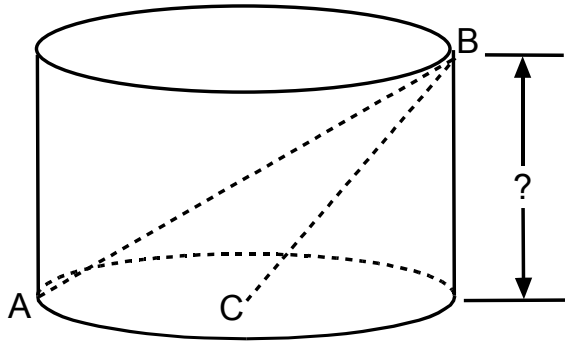
1.82

Page 7 Problems

15A-70.

CYLINDER

AB = 0.602 BC = 0.407



C = Center

15A-70 = _____

$(\text{radius})^2 + ?^2 = (.407)^2$

$(2 \times \text{radius})^2 + ?^2 = (.602)^2$

radius = .25609…

$(.25609\ldots)^2 + ?^2 = (.407)^2$

.316