UIL Number Sense Contest

Problems #41-60 and #61-80 from the Sequence Chart

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First, some interesting thoughts and ideas

Right Triangles -- Pythagorean Triples -- Use in Trigonometry

1. **$s^2 + m^2 = h^2$ (interesting labels ---- $a^2 + b^2 = c^2$)**

2. **Area of right triangle $A = (s \times m)/2$ (interesting labels ---- $A = (b \times h)/2$)**
   $A = 1/2(a \times b \times \sin c)$

3. **Altitude of right triangle $h = (a \times b)/c$ (interesting labels --- $a = (s \times m)/h$)**

4. **Given m & n --- a triple can be created by $m^2 - n^2$, 2mn, & $m^2 + n^2$ provided:**
   - **m & n are relatively prime integers**
   - **m > n**
   - **m is even and n is odd or vice versa**

5. **Special note: the product of the integral sides of a right triangle is divisible by 60.**

6. **A 30-60-90 triangle has side ratios of x, $\sqrt{3}$ x, & 2**

7. **A 45-45-90 triangle has side ratios of x, x, $\sqrt{2}$**

8. **Pythagorean triples can be used to determine acute, obtuse, or right triangles.**

9. **Trig: sin = opp/hyp = y/r**
   cos = adj/hyp = x/r
   tan = opp/adj = y/x

Sample problems

1. The legs of a right $\triangle$ are 5 and 12. The length of the altitude to the hypotenuse is

2.The leg opposite the 60° angle in a right triangle is $\sqrt{12}$. The hypotenuse is

3. The hypotenuse of an isoscles right triangle is $\sqrt{32}$ cm . The sum of the lengths of the two legs is

4. The legs of a right $\triangle$ are 8 and 15. The length of the altitude to the hypotenuse is

UIL High School Number Sense Test Problem Sequencing
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Problems 41 - 60 *

1) Laws of Exponents
2) Right Triangle Problems
3) Coordinate Geometry Problems
4) Regular Polygon Problems
5) Inequalities
6) Applications of Theorems from Geometry
7) Direct and Inverse Variation
8) Sequences & Series (Finite & Infinite)
9) Complex Numbers
10) Logarithms & Logarithmic Equations
11) Factorials, Permutations, & Combinations
12) Probability/Odds
13) Conics
14) Binomial Theorem (Expansion)
15) Base System Problems Using Operations
16) Roots of equations
17) Polygonal numbers

Problems 61 - 70 *

1) Volume & Surface Area
2) Greatest Integer
3) Application of Remainder Theorem
4) Trigonometry
5) Determinants
6) Matrices
7) Vectors
8) Composite Functions
9) Bases Involving Decimals or Fractions
10) Polar/Rectangular Coordinates

Problems 71 -80 *

1) Function domains and ranges
2) Modular Arithmetic
3) Limits
4) Derivatives
5) Slopes of Tangent Lines
6) Horizontal & Vertical Asymptotes
7) Determining Critical Values
8) Maximum & Minimum Problems
9) Definite Integration
10) Inverse functions

Any questions on any of these?

(41) $48^2 - 58^2 =$ _____

(42) $504_7 + 305_7 + 534_7 =$ _____ $_7$

(43) Find k, given 5, 4, 9, 13, 22, ..., 57, k, 149, ... _____

(44) $5^{(-3)} =$ _____ (decimal)

(45) The vertex of $y = 4x^2 - 5x - 3$ is (h, k). h = _____

(46) The midpoint between the points (— 5,4) and (3,— 5) is (h, k). Find h + k. _____

(47) The smallest root of $(x + 3)^2 = \frac{1}{4}$ is _____

(48) If 6 apps cost $12.24, then 9 apps cost $ _____

(49) $991^2 =$ _____

***(50)** $\sqrt[3]{542018} =$ _____

(51) Let $(1 + 2i) \times (3 - 4i) = a + bi$. Find a + b. _____

(52) $i \times i \times i \times i \times i \times i =$ _____

(53) If 4, 18, and x are the sides of a triangle, then $x + 5 >$ _____

(54) $4\log 10^5 =$ _____

(55) $\frac{3}{4} + \frac{1}{2} + \frac{1}{3} + ... + \frac{8}{81} + ... =$ _____

(56) $1 + 3 + 6 + 10 + 15 + ... + 78 + 91.$ _____

(57) $74^2 + 33^2 =$ _____

(58) $(504_6 - 405_6)(2_6) =$ _____ $_6$

(59) Find the sum of all positive integers x such that $3x - 6 \leq 10.$ _____

***(60)** $7 \times 14 \times 21 \times 28 =$ _____

Any questions on any of these?

(61) 0.454545... base 8 = _____ base 10 (fraction)

(62) $(6x^2 + x - 7) \div (x + 1)$ has a remainder of _____

(63) X varies inversely as Y. If X = 16 when Y = 4. find Y when X = 12. Y = _____

(64) The simplified coefficient of the x^4y^2 term in the expansion of $(x + 3y)^6$ is _____

(65) $f(x) = 3 - 5\cos(\pi x + 1)$. The amplitude is _____

(66) $\cos^2(\frac{5\pi}{6}) =$ _____

(67) $\sec^2(\frac{7\pi}{6}) =$ _____

(68) $f(x) = 5x^2 - 4$. $g(x) = 5 + 4x + x^2$. $f(g(-1)) =$ ____

(69) $10^{11} \div 12$ has a remainder of _____

***(70)** $\pi^5 \times e^4 =$ _____

(71) If $3.2^{(x+1)} = 64$ then $3.2^{(x)} =$ _____

(72) $\lim_{x \to \infty} \frac{3\cos(x)}{x} =$ _____

(73) Let $f(x) = x^3 - 3x^2 - 2x + 1$. Find $f'(1)$. _____

(74) If $x < 0$ and $|5x + 4| = 18$ then x = _____

(75) A pair of dice is rolled. The probability of rolling a four on one die but not on both is _____

(76) If $14^4 \div 4 = (4^x)(49^y)$, then x + y = _____

(77) If $f(x) = 5 - \frac{4x - 5}{4}$ then $f^{-1}(8) =$ _____

(78) (0.571428571428571428...) ÷ (0.222...) = _____

(79) 12.5% of a mile = _____ yards

***(80)** $(504.2018)^3 =$ _____