UIL Number Sense Contest

Basic Ideas, Shortcuts and Problems #1-20 from the Sequence Chart

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Special Numbers -- What Pops into Your Mind?

3375

1024

1331

361

2.236...

1.414...

3.141...

2.718...

1.618

Notes:

Mental Math -- How fast can you work these?

1. **12.5% of 48 =**

2. **$30 \div 0.1666... =$**

3. **$45 \times 0.\overline{7} =$**

4. **75% of 80 is**

5. **$24 \div 0.375 =$**

6. **$33\frac{1}{3}$% of 54 =**

Notes:

Math Magic (Number Sense Tricks)

A. Memorize the first 35 squares, the first 15 cubes, and the square roots of 2, 3, 5, 6, 7, 8, & 10.

B. Know the "One-sies" equivalents. (Fractions-Decimals-Percents)

C. $\frac{3}{5} + \frac{5}{3} =$ **? (Is it a trick? Is it magic? See proof)**

D. Find the average of 25, 36, and 47 using a focus number.

E. LCM (24, 42) is ?

F. Write 0.1222... as a fraction.

G. $(37 \times 13 + 19) \div 8$ **has a remainder of ?**

H. $35 \times 35 =$ **?** $35 \times 45 =$ **?** $35 \times 55 =$ **?** $35 \times 65 =$ **?**

I. $\frac{13}{16} \times 13 =$ **?**

J. $53 \times 47 =$ **?**

K. Change 234 base 5 to base 10.

L. $36^2 + 57^2 =$ **?**

Math Magic (solutions and tricks)

C. $\frac{3}{5} + \frac{5}{3} = 2\frac{4}{15}$ **(Is it magic ?)**

$\frac{a}{b} + \frac{b}{a}$ **Proof**

Let $x = \frac{a}{b} + \frac{b}{a}$

$x = \frac{(a^2 + b^2)}{ab}$ **(common denominator)**

$x - 2 = \frac{(a^2 + b^2)}{ab} - 2$ **(subtract 2 from both sides)**

$x - 2 = \frac{(a^2 + b^2 - 2ab)}{ab}$ **(common denominator)**

$x - 2 = \frac{(a - b)^2}{ab}$ **(binomial square)**

$x = 2 + \frac{(a - b)^2}{ab}$ **(solve for x)**

D. The average of 25, 36, and 47 is 36.

Using 35 as a focus number,
add 10 to 25; subtract 1 from 36; subtract 12 from 47
$-10 + 1 + 12 = 3$.
Since 3 divided by three numbers is 1, then 35 + 1 = 36.

E. LCM (24, 42) = 168

use GCF(24, 42) which is 6
$24 \div 6 = 4$ and $4 \times 42 = 168$

F. 0.1222... = 11/90

12 — 1 = 11 and there is 1 repeater (hence the 9)
and 1 non-repeater (hence the 0)

G. $(37 \times 13 + 19) \div 8$ has a remainder of 4

37 ÷ 8 has remainder of 5, 13 ÷ 8 has remainder of 5,
and 19 ÷ 8 has remainder of 3
So, $5 \times 5 + 3 = 28$ and 28 ÷ 8 has remainder of 4

H. $35 \times 35 = 1225$ $35 \times 45 = 1575$ $35 \times 55 = 1925$ $35 \times 65 = 2275$

a5 × b5 = a × b + the integer portion of (a + b) ÷ 2
then put either 25 or 75 on the end depending on whether
(a + b) is even or odd

I. $\frac{13}{16} \times 13 = 10\frac{9}{16}$

numerator ---> 16 — 13 = 3, and $3^2 = 9$
whole number ---> 13 — 3 = 10

J. $53 \times 47 = 2491$
difference of squares $(50 + 3)(50 - 3) = 50^2 - 3^2 = 2491$

K. 234 base 5 to base 10 = 69

$2 \times 25 + 3 \times 5 + 4 \times 1 = 69$

L. $36^2 + 57^2 = 4545$

note 3 + 7 = 10 and 6 - 5 = 1
so $(3^2 + 6^2) \times 101 = 4545$

SHORTCUTS

I. Multiplying numbers ending in 5

A. First digits are equal: 1) always ends in 25
2) multiply first digit by first digit plus 1

Ex: 35 x 35 = 3 x (3 + 1) and ends in 25 = 1225
65 x 65 = 6 x (6 + 1) and ends in 25 = 4225

B. First digits differ by 1: 1) always ends in 75
2) multiply smallest first digit by largest first digit plus 1

Ex: 45 x 35 = 3 x (4 + 1) and ends in 75 = 1575
65 x 75 = 6 x (7 + 1) and ends in 75 = 4875

C. First digits differ by an even number: 1) always ends in 25
2) add first digits and divide by 2
3) multiply first digits and add quotient from step 2

Ex: 65 x 25 = 6 x 2 + ((6 + 2)/2) and ends in 25 = 6 x 2 + 4 and ends in 25 = 1625
35 x 95 = 3 x 9 + ((3 + 9)/2) and ends in 25 = 3 x 9 + 6 and ends in 25 = 3325

D. First digits differ by an odd number: 1) always ends in 75
2) add first digits and divide by 2
3) multiply first digits and add integer part of quotient

Ex: 85 x 55 = 8 x 5 + (int((8 + 5)/2) and ends in 75 = 8 x 5 + 6 and ends in 75 = 4675
35 x 65 = 3 x 6 + (int((3 + 6)/2) and ends in 75 = 3 x 6 + 4 and ends in 75 = 2275

II. Multiplying by 11 or Teens

A. Multiply by 11: 1) bring down units digit
2) add two digits at a time
3) bring down first digit plus any carry

Ex: 72 x 11 = (7 + carry) & (7 + 2) & (2) = 7 & 9 & 2 = 792
84 x 11 = (8 + carry) & (8 + 4) & (4) = 8 & 12 & 4 = (8+1) & 2 & 4 = 924
134 x 11 = (1 + carry) & (1 + 3 + carry) & (3+4) & 4 = 1 & 4 & 7 & 4 = 1474

B. Multiply by teens: 1) multiply units digit of the teen times units digit
2) multiply units digit of the teen times other digits and add back plus carry
3) bring down first digit plus any carry

Ex: 72 x 13 = (7 + C) & (3 x 7 + 2) & (3 x 2) = 7 & 23 & 6 = (7 + 2) & 3 & 6 = 936
164 x 12 = (1 + C) & (2 x 1 + 6 + C) & (2 x 6 + 4 + C) & (2 x 4) = 1968

III. Multiplying by 25 or 75

A. Multiply by 25: 1) divide by 4
2) last two digits 00, 25, 50, or 75 depends on the remainder

Ex: 64 x 25 = 64 ÷ 4 = 16 R 0 & and remainder digits = 1600
57 x 25 = 57 ÷ 4 = 14 R 1 & add remainder digits = 1425

B. Multiply by 75: 1) divide by 4
2) last two digits 00, 25, 50, or 75 depends on the remainder
3) multiply results by 3

Ex: 64 x 75 = 64 ÷ 4 = 16 R 0 & add remainder digits = 1600 x 3 = 4800
57 x 75 = 57 ÷ 4 = 14 R 1 & add remainder digits = 1425 x 3 = 4275

IV. Dividing by 25

A. Divide by 25: 1) multiply by 4
2) place decimal so the answer has 2 decimal places

Ex: 64 ÷ 25 = 64 x 4 = 256 & place decimal = 2.56
57 ÷ 25 = 57 x 4 = 228 & place decimal = 2.28

V. Multiplying by numbers when first or last digits total 10

A. Multiply when units digits total 10 and first digits are equal:
1) multiply first digit times first digit plus 1
2) multiply units digits

Ex: 43 x 47 = 4 x (4 + 1) & 3 x 7 = 4 x 5 & 3 x 7 = 2021
72 x 78 = 7 x (7 + 1) & 2 x 8 = 7 x 8 & 2 x 8 = 5616

B. Multiply when first digits total 10 and units digits are equal:

1) multiply first digits and add the units digit
2) square the units digit

Ex: 27 x 87 = 2 x 8 + 7 & 7 x 7 = 16 + 7 & 49 = 2349
43 x 63 = 4 x 6 + 3 & 3 x 3 = 24 + 3 & 9 = 2709

VI. Multiplying by difference of squares

A. Algebra: $a^2 - b^2 = (a + b)(a - b)$:

1) easiest to see shortcut by examples

Ex: 53 x 47 = (50 + 3) x (50 — 3) = $50^2 - 3^2$ = 2500 — 9 = 2491
28 x 32 = (30 — 2) x (30 + 2) = $30^2 - 2^2$ = 900 — 4 = 896

VII. Least Common Multiple

A. LCM(a,b) = a ÷ GCF x b: 1) find the greatest common factor (GCF)
2) divide one number by the GCF
3) multiply quotient times the other number

Ex: LCM(8,14) --- GCF = 2 --- 8 ÷ 2 = 4 ---> 4 x 14 = 56 ---> LCM(8,14) = 56
LCM(24,99) --- GCF = 3 --- 24 ÷ 3 = 8 ---> 8 x 99 = 792 ---> LCM(24,99) = 792

VIII. Division by 9

A. xyz divided by 9: 1) add x plus y plus z and put sum over 9 (be sure to reduce)
2) add x plus y plus carry
3) bring down x plus carry

Ex. 201 ÷ 9 = (2 + C) & (2 + 0 + C) & (2 + 0 + 1)/9 = 22 3/9 = 22 1/3
1240 ÷ 9 = (1 + C) & (1 + 2 + C) & (1 + 2 + 4 + C) & (1 + 2 + 4 + 0)/9 = 137 7/9

IX. Multiplying numbers close to 100

A. Numbers close to and below 100:

1) A = 100 minus first number and B = 100 minus second number
2) subtract A from the second number (or vice versa)
3) multiply A and B

Ex. 96 x 99 --> A = 4 & B = 1 --> 99 — 4 (or 96 — 1) = 95 --> 4 x 1 = 4 --> 96 x 99 = 9504
92 x 97 --> A = 8 & B = 3 --> 97 — 8 (or 92 — 3) = 89 --> 8 x 3 = 24 --> 92 x 97 = 8924

B. Numbers close to and above 100:

1) A = first number minus 100 and B = second number minus 100
2) add A to the second number (or vice versa)
3) multiply A and B

Ex. 106 x 103 --> A = 6 & B = 3 --> 6 + 103 (or 3 + 106) = 109 --> 6 x 3 = 18 --> 10918
112 x 105 --> A = 12 & B = 5 --> 12 + 105 (or 5 + 112) = 117 --> 12 x 5 = 60 --> 11760

X. Repeating decimals converted to fractions

A. All digits repeat:

1) the number of digits that repeat is the number of 9's in the denominator
2) one set of the repeating digits is the numerator (be careful to reduce)

Ex: $0.\overline{13}$ --> two repeaters means two 9's --> 13/99
0.341341341... --> three repeaters means three 9's --> 341/999

B. Some digits repeat and some don't:

1) the number of digits that repeat is the number of 9's in the denominator
2) the number of non-repeating digits is the number of 0's in the denominator
3) subtract the non-repeating digits from the number before repetition starts for the umerator

Ex: 0.12424... --> two repeaters and one non-repeater means two 9's and one 0
--> 124 — 1 = 123 --> 0.12424... = 123/990

$0.12\overline{35}$ --> two repeaters and two non-repeaters means two 9's and two 0's
--> 1235 — 12 = 1223 --> 0.12353535... = 1223/9900

UIL High School Number Sense Test Problem Sequencing

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Problem 1 - 20 *

1) Addition, subtraction, multiplication, & division of Integers, Mixed Numbers, Fractions, and Decimals
2) Order of Operations
3) Use of the Distributive Property
4) Comparison of Fractions and Decimals
5) Multiplication Short-Cuts
6) Squaring Numbers
7) Conversion Problems (either way):
Percent/Fractions, English/Metric,
Roman Numerals/Arabic Numerals,
Measurement units
(length, weight, capacity, time)
8) Greatest Common Divisor (GCD) and Least Common Multiple (LCM)
9) Percent Problems
10) Mean, Median, & Mode
11) Sums of Integers
12) Remainder Problems
13) Consumer Type Problems
14) Number Theory Problems Involving:
Prime Numbers, Divisors, Sums of Divisors, etc.

***** A type of problem from a particular section could appear later in the test.**
Example: A base problem could appear as problem #55, but should not appear earlier than problem #21.

Any questions on any of these?

(1) $5418 + 8145 =$ _____

(2) $504 \times 8 =$ _____

(3) $5042018 \div 9$ has a remainder of _____

(4) $5 \times 4 \div 2^0 + 1 - 8 =$ _____

(5) $29^2 =$ _____

(6) $5420 \div 18 =$ _____ (mixed number)

(7) $5\frac{1}{4} - 1\frac{4}{5} =$ _____ (mixed number)

(8) $5.4 \div 2.5 =$ _____ (decimal)

(9) The negative reciprocal of 3.5 is _____

***(10)** $20 + 18 \times 504 =$ _____

(11) $24 \times 38 - 24 \times 14 =$ _____

(12) The GCD of 85 and 102 is _____

(13) $4 \times 8 - 12 + 16 \div 20 =$ _____

(14) The LCM of 102 and 85 is _____

(15) Simplify to lowest terms: $\frac{144}{234}$. _____

(16) The arithmetic mean of 5, 4, 20, and 18 = _____

(17) 20% of 60 less 40 is _____

(18) The largest prime number less than 95 is _____

(19) $11 \times 504 =$ _____

***(20)** $81547 \div 347 =$ _____