1. Evaluate: $(3)^3 \div (3+6) - 3! \times \sqrt{9}$

(A) -17 (B) -15 (C) -9 (D) 18 (E) 27

2. $888_9 + 555_6 + 222_3 =$ _____ 12

(A) 582 (B) 689 (C) 969 (D) 1111 (E) 1169

3. 70 miles per hour is equivalent to _____ inches per second.

(A) 840 (B) 1056 (C) 1232 (D) 1680 (E) 6160

4. Which of the following equations has a graph of a parabola that intersects the y-axis at only one point and the x-axis at only one point? $y =$ _____ .

(A) $.5x^2 - 2x + 1$ (B) $x^2 - 4x - 5$ (C) $|2x - 4| + 1$ (D) $2 \pm \sqrt{x}$ (E) $12(x)^{-1}$

5. Tryce Ikle can get to school in 12 minutes riding his bike at an average of 15 miles per hour (mph). How many minutes would it take him to walk to school if he walks at 4 mph?

(A) 31 (B) 32 (C) 45 (D) 48 (E) 72

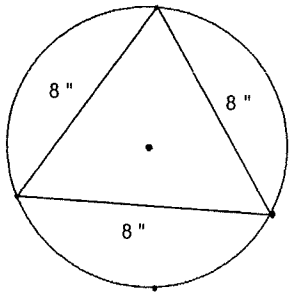
6. If $x + y = 5$ and $xy = 1$ then $x^3 + y^3 = ?$

(A) 128 (B) 124 (C) 122 (D) 115 (E) 110

7. The area of a rectangle is 300 cm^2. The ratio of its length to its width is 4:3. The perimeter of the rectangle is:

(A) 125 cm (B) 112 cm (C) 100 cm (D) 70 cm (E) 35 cm

8. Find the radius of the circle. (nearest tenth)



(A) 3.8 " (B) 5.7 " (C) 4.6 " (D) 5.2 " (E) 4.1 "

9. $\angle P$ is supplementary to $\angle Q$ and $\angle R$ is complementary to $\angle S$. If $m\angle P = 75°$ and $m\angle Q = 3 \times m\angle R$, then $m\angle S = ?$

(A) 75 ° (B) 55 ° (C) 35 ° (D) 20 ° (E) 15 °

10. Noah Sense is making a trapezoid using pennies. The bottom base is a row of 15 pennies. The next row above the base row contains 1 less penny and each successive row contains 1 less penny. He continues until the top base of the trapezoid has only 3 pennies. How much money does he need to form the trapezoid of pennies?

(A) \$1.20 (B) \$1.17 (C) \$1.14 (D) \$1.10 (E) \$1.05

11. Determine the type of conic section this equation $x^2 + 2xy + y^2 - 6x - 6y + 9 = 0$ will produce.

(A) point (B) parabola (C) line (D) hyperbola (E) ellipse

12. The roots of the equation $x^3 - bx^2 + 23x + d = 0$ are -1, 9, and R. Find R.

(A) -14 (B) -9 (C) 4 (D) 14 (E) 15

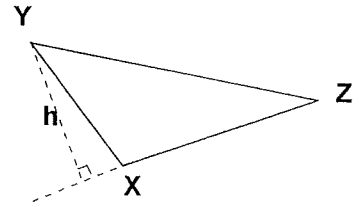
13. Determine the period of the function $y = 3 - 2\cos(\frac{x}{4} + \pi)$

(A) $\frac{\pi}{2}$ (B) π (C) 4π (D) $\frac{11\pi}{2}$ (E) 8π

14. How many points of intersection are there for the curves $r = \sin 2\theta$ and $r = 2\sin\theta$?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 5

15. A triangle is drawn as shown. Find the height, h, if YZ = 18", m∠YZX = 30 °, and XZ = 12".



(A) 15" (B) 12" (C) 10" (D) 9" (E) 8"

16. Use the Fibonacci characteristic sequence ... -1.5, p, q, 3, r, ... to Find $p + q + r$.

(A) .75 (B) 2.25 (C) 6.75 (D) 8.25 (E) 11.25

17. The directrix for the parabola $-8y = x^2$ is $y =$ _____.

(A) $2\sqrt{2}$ (B) 2 (C) $\frac{\sqrt{2}}{2}$ (D) $-\frac{1}{2}$ (E) $-\frac{1}{8}$

18. If $A = \begin{bmatrix} 2 & 3 \\ 4 & x \end{bmatrix}$ and $B = \begin{bmatrix} y & 1 \\ -1 & -1 \end{bmatrix}$ then $AB = \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix}$. Find $x + y$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

19. Let $f(x) = \begin{cases} x & \text{if } x \leq 0 \\ x^2 & \text{if } 0 < x \end{cases}$. Which of the following statements is a false statement.

(A) f is continuous at 0 (B) the right hand derivative at 0 is 0
(C) the left hand derivative at 0 is 1 (D) f is not differentiable at 0 (E) $f(-1) = f(1)$

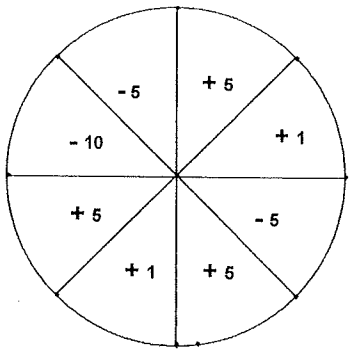
20. Let $\frac{6x^2}{5} - \frac{3xy}{2} + \frac{19y^2}{5} - 4 = 0$. What is the angle of rotation from it's parent function? (nearest degree)

(A) 8° (B) 11° (C) 15° (D) 23° (E) 30°

21. Two non negative numbers x and y exist such that the sum of the numbers is 12 and that the product of one number and the square of the other number is a maximum. What is the maximum product?

(A) 256 (B) 245 (C) 216 (D) 175 (E) 128

22. Betty Wheel spins the Wheel of Fun. The wheel consists of eight congruent sectors as shown. What is the mathematical expectation on any one spin?



(A) -3 (B) $-.375$ (C) $+.25$ (D) $+1.25$ (E) $+5$

23. The probability of scoring less than 200 on this test is 75%. What are the odds of a student scoring greater than or equal to 200 on this test?

(A) 1 to 3 (B) 1 to 4 (C) 1 to 8 (D) 3 to 1 (E) 3 to 8

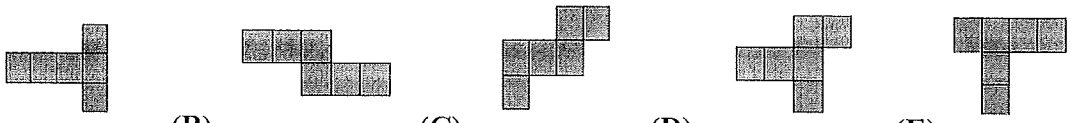
24. Berry Kold Creamery has four flavors of ice cream: vanilla, pistachio, black walnut, and strawberry. The daily sundae has three scoops of ice cream. How many variations of sundaes are there?

(A) 35 (B) 12 (C) 24 (D) 6 (E) 20

25. Vector $v = (8, 6, -2)$ and vector $u = (-4, x, 1)$. Find x if the dot product of vectors u and v is 2.

(A) 24 (B) 6 (C) 3 (D) -2 (E) -7

26. Which of the following nets when folded will not form a cube?



(A) (B) (C) (D) (E)

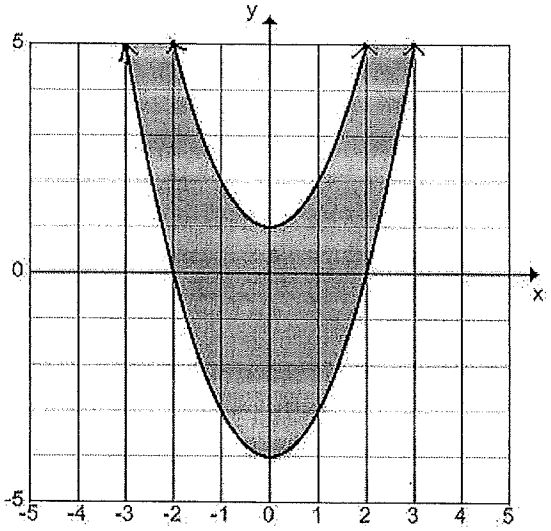
27. Melody Toone's music store sells a new CD for 125% above the wholesale cost. The store will buy the CD back in used condition for 40% of the selling price. How much profit will the store make if the selling price was $19.99?

(A) $3.11 (B) $4.89 (C) $8.88 (D) $11.88 (E) $16.88

28. A tangent and a secant intersect at point A in the exterior of a circle. The measures of the two intercepted arcs are 75° and 50°. What is the measure of angle A formed by the tangent and the secant?

(A) 125° (B) 62.5° (C) 37.5° (D) 25° (E) 12.5 °

29. Which of the following system of inequalities would be best represented by the shaded region shown?



(A) $y \geq x^2 + 4$ $y \geq x^2 - 1$ (B) $y \leq x^2 + 4$ $y \geq x^2 - 1$ (C) $y \leq x^2 - 4$ $y \geq x^2 + 1$ (D) $y \geq x^2 - 4$ $y \leq x^2 + 1$ (E) $y \geq x^2 - 4$ $y \leq x^2 - 1$

30. A porch is 3 feet high. A ramp is built to reach from the porch to the ground with an angle of elevation of 15°. How far from the base of the porch does the ramp touch the ground? (nearest inch)

(A) 11' 7" (B) 11' 2" (C) 10' 6" (D) 9' 11" (E) 8' 5"

31. Mr. White and his dog walked 1 mile at an average speed of $3\frac{1}{3}$ mph and returned home the same route at an average speed of $2\frac{1}{2}$ mph. What was their average speed for the entire walk?

(A) $2\frac{11}{12}$ mph (B) $2\frac{6}{7}$ mph (C) $2\frac{8}{9}$ mph (D) $2\frac{5}{6}$ mph (E) $2\frac{3}{4}$ mph

32. The slope of the line tangent to the curve $y = x^3 - 5x + 6$ at $x = 1$ is -2. The point of intersection of the tangent line and the curve is:

(A) $(-2, 8)$ (B) $(-1, 2)$ (C) $(-1, 11)$ (D) $(-2, 0)$ (E) $(-3, -6)$

33. Fifty-Fifty High School has five male teachers and five female teachers. How many ways are there to form a committee of three female teachers and two male teachers?

(A) 20 (B) 25 (C) 50 (D) 80 (E) 100

34. Find the product of all of the solutions of $16^{x^2+x+4} = 32^{x^2+x}$.

(A) -16 (B) -6 (C) 2 (D) 4 (E) 8

35. The average of five tests is 85. If two test scores have 5 points removed from each, 1 test score has 20 points added, and the remaining two remain the same, the new average is:

(A) 84 (B) 85 (C) 86 (D) 87 (E) 88

36. Kandy Heart had a box of valentines. She gave $\frac{2}{3}$ of them to her classmates. She gave 5 of the remaining valentines to her brothers and sisters. She had 3 left over for her father, her mother, and herself. How many valentines were in the original box?

(A) 12 (B) 18 (C) 24 (D) 30 (E) 36

37. One of Eratosthenes of Cyrene's main contributions to mathematics involved a method for finding _____.

(A) quadratic solutions (B) slopes of line (C) prime numbers
(D) diagonals of polygons (E) complex numbers

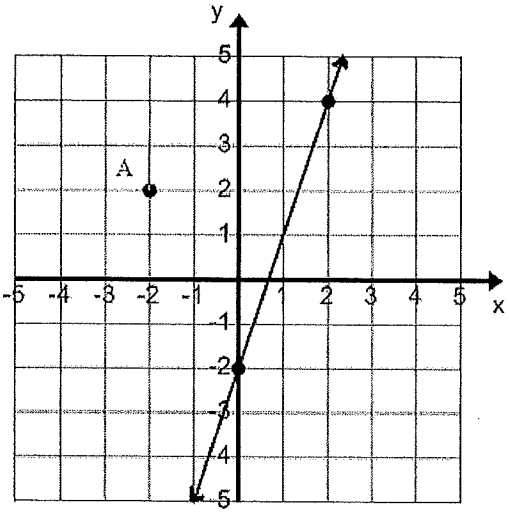
38. Line $6x - 5y = 4$ is perpendicular to line $3x - ay = 1$. What is the value of a?

(A) -3.6 (B) -2.5 (C) 2.5 (D) 3 (E) 5

39. Two circles, $(x-2)^2 + (y-5)^2 = 25$ and $(x-6)^2 + (y-13)^2 = 65$, intersect at two points. Find the equation of the line passing through the two points of intersection.

(A) $2x + y = 27$ (B) $2x + 2y = 19$ (C) $x - y = 17$ (D) $x + 2y = 17$ (E) $x - 2y = 27$

40. Line AB is parallel to the line shown. Which of the following points could be point B?



(A) $(-7, -5)$ (B) $(-6, -10)$ (C) $(4, 21)$ (D) $(2, 13)$ (E) $(5, -1)$

41. Six boys and twelve girls are in the senior class. Half the boys and 25% of the girls wear glasses. What is the probability that a student chosen randomly is a boy, wears glasses, or both?

(A) $16\frac{2}{3}\%$ (B) 25% (C) $33\frac{1}{3}\%$ (D) 50% (E) $66\frac{2}{3}\%$

42. Find the unit vector in the same direction as (8 , 15).

(A) $(\frac{8}{17}, \frac{15}{17})$ (B) (9 , 2) (C) (25 , 32) (D) $(8\sqrt{17}, 15\sqrt{17})$ (E) $(\frac{8}{15}, \frac{15}{8})$

43. The point (3 , 4) is rotated 60 degrees clockwise about the origin. The coordinates of the point after the rotation is _____. (closest approximation)

(A) (5 , — .6) (B) (— 2 , 4.6) (C) (4.6 , 2) (D) (.6 , 5) (E) (5 , — 2)

44. One of the roots of $ax^2 + bx + c = 0$ is $2 - 3i$. Find $b^2 - 4ac$, when $a = 1$.

(A) — 9 (B) — 13 (C) — 24 (D) — 30 (E) — 36

45. Evaluate: $(\log_2 8)(\log_3 9)(\log_4 4)$

(A) 6 (B) 5 (C) 4 (D) 3 (E) 2

46. Find the area, in square units, of the figure bounded by $y = x^2 - x - 2$ and below the x-axis.

(A) $1\frac{5}{6}$ (B) $2\frac{3}{4}$ (C) $3\frac{2}{3}$ (D) $4\frac{1}{2}$ (E) $6\frac{1}{3}$

47. Find the value of $\sin(\text{Arcsin}\ \frac{1}{2} - \text{Arccos}\ \frac{1}{2})$.

(A) $-\frac{\sqrt{3}}{2}$ (B) $-\frac{1}{2}$ (C) 0 (D) $\frac{1}{2}$ (E) $\frac{\sqrt{3}}{2}$

48. A scout troop leaves their vehicles and travels on a hike of 2 km on a bearing of 45° to Camp Fife for a swim. Then they travel 3 km on a bearing of 135° to the scout lodge for lunch. What is the least distance they will have to hike to return to their vehicles? (nearest tenth)

(A) 5.0 km (B) 4.3 km (C) 3.9 km (D) 3.6 km (E) 2.5 km

49. Let $f(x) = \frac{x-2}{3x+5}$. Find $f'(-1)$.

(A) $-1\frac{1}{2}$ (B) $-1\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $1\frac{1}{4}$ (E) $2\frac{3}{4}$

50. Two legs of a triangle have lengths of 10 cm and 15 cm with an included angle of 30°. Find the area of the triangle.

(A) 75 cm^2 (B) 65 cm^2 (C) 55 cm^2 (D) 40.4 cm^2 (E) 37.5 cm^2

51. How many asymptotes does this function have? $f(x) = \frac{x^2+6x+8}{x^2-6x+8}$.

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

52. Find the digit in the ten-thousandths place of the series $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$, when $x = \pi$.

(A) 0 (B) 1 (C) 4 (D) 6 (E) 9

53. Let E = {0,2,4,6,8}. Two elements of set E are selected at random without replacement. What is the probability that the mean of the two numbers selected is an odd number ?

(A) 30% (B) 40% (C) 50% (D) 60% (E) 70%

54. I'm an unhappy deficient number but a number that is lucky to be prime. Which of the following numbers am I?

(A) 11 (B) 23 (C) 37 (D) 71 (E) 91

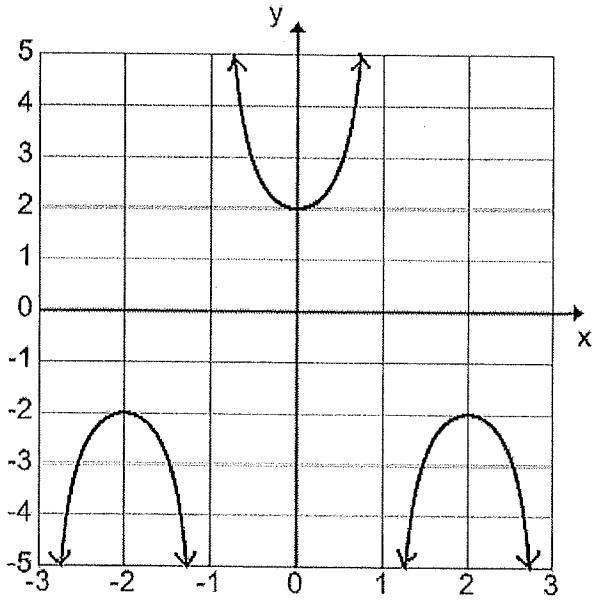
55. Find the remainder when $f(x) = x^3 + 2x^2 - 3x - 4$ is divided by $x - 5$.

(A) 156 (B) 120 (C) 1 (D) -29 (E) -64

56. If $a_1 = -3$, $a_2 = 1$ and $a_n = (a_{n-1})(a_{n-2})$, where $n \geq 3$, then a_5 equals:

(A) 9 (B) 3 (C) -1 (D) -3 (E) -27

57. The equation y = _____ will produce this graph.



(A) 2 sec $(2\pi x)$ (B) 2 csc $(\frac{x}{2} + \pi)$ (C) 4 sec $(\frac{x}{2} + \pi)$ (D) 4 csc $(\frac{\pi}{2}x)$ (E) 2 sec $(\frac{\pi}{2}x)$

58. Point A (2 , — 4) lies in the x-y plane. Point A is reflected across the line y = — x to point B. Point B is reflected across the x-axis to point C. Point C is reflected across the line y = x to point D. Find the coordinates of the point D.

(A) (2 , 4) (B) (2 , 2) (C) (— 2 , 4) (D) (4 , 4) (E) (— 2, — 4)

59. Which equality axiom of multiplication is demonstrated by $(a)(a)^{-1} = 1$?

(A) Identity (B) Associative (C) Inverse (D) Commutative (E) Distributive

60. The value of $(0.08333...)^{-1} \div (0.0625)^{-1} \times (.0555...)$ is:

(A) $13\frac{1}{2}$ (B) $10\frac{2}{3}$ (C) $\frac{1}{24}$ (D) $\frac{2}{27}$ (E) $\frac{3}{32}$

University Interscholastic League
MATHEMATICS CONTEST
HS • Invitation B • 2008
Answer Key

1. B	21. A	41. D
2. B	22. B	42. A
3. C	23. A	43. A
4. D	24. E	44. E
5. C	25. B	45. A
6. E	26. E	46. D
7. D	27. A	47. B
8. C	28. E	48. D
9. B	29. D	49. E
10. B	30. B	50. E
11. C	31. B	51. D
12. C	32. A	52. D
13. E	33. E	53. D
14. B	34. A	54. C
15. D	35. D	55. A
16. C	36. C	56. A
17. B	37. C	57. E
18. E	38. A	58. A
19. E	39. D	59. C
20. C	40. B	60. C