Scaling Stated Problems

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Attention All Attendees:

Thank you for registering your attendance for **EACH SESSION**:

http://www.uiltexas.org/academics/capital-conference/online

Electronic handouts are available there too.

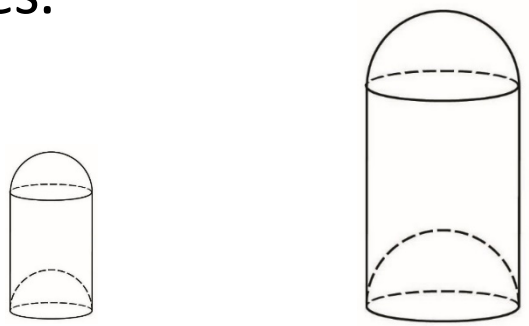
Scaling Stated Problems

UIL Contest Manual for the Calculator Applications Contest, pp. 53-7

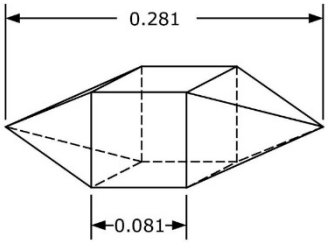
Problem #46 on all UIL tests is a scaling problem.

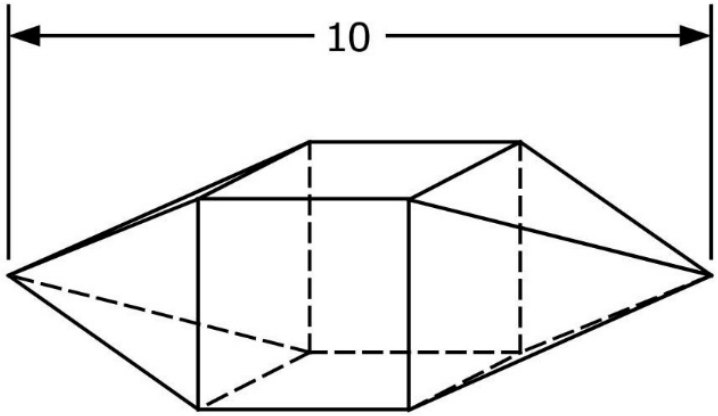
Background

- Scaling deals with geometrically similar figures.
- The advantage of using scaling principles is that the calculation is much simpler than a “brute force” approach.
- The simplification arises from elimination of the necessity to calculate the constant of proportionality.
- This applies to 2D and 3D objects as well as general proportionalities.



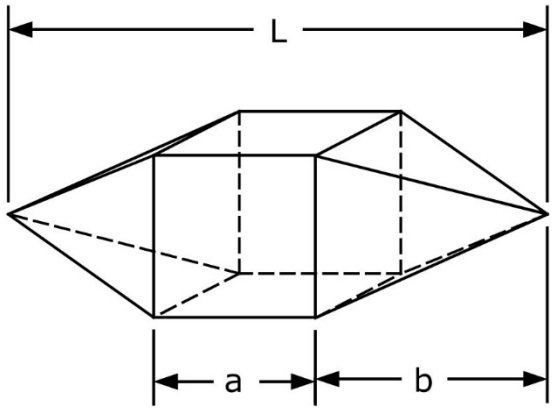
Example





Calculate the volume of the large figure.

Example – The Hard Way



$$V = 2\left[\frac{1}{3}a^2 b\right] + a^3$$

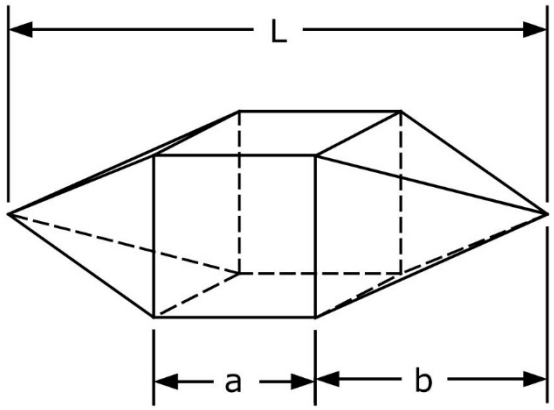
Let $\alpha = \frac{a}{L} = \frac{0.081}{0.281} = 0.288$ and $\beta = \frac{b}{a} = \frac{0.1}{0.081} = 1.235$

$$V = 2\left[\frac{1}{3}\left(\frac{a}{a}\right)^2 \frac{b}{a}\right] a^3 + a^3 = \left[\frac{2}{3}\beta + 1\right] a^3$$

$$V = \left[\frac{2}{3}\beta + 1\right]\left(\frac{a}{L}\right)^3 L^3 = \left[\left(\frac{2}{3}\beta + 1\right)\alpha^3\right] L^3 = KL^3 = (0.04367)L^3$$

$$V_L = (0.04367)(10)^3 = 43.7$$

Example - Scaling



$$V_s = 2\left[\frac{1}{3}(0.081)^2(0.1)\right] + (0.081)^3 = 0.0009688$$

$$\mathrm{V} = KL^3$$

$$\frac{V_L}{V_s} = \frac{KL_L^3}{KL_S^3} = \frac{L_L^3}{L_S^3}$$

$$V_L = \left[\frac{L_L}{L_S}\right]^3 V_S = \left[\frac{10}{0.281}\right]^3 (0.0009688)$$

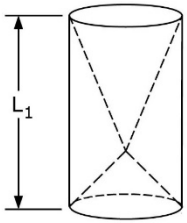
$$\mathrm{V} = 43.7$$

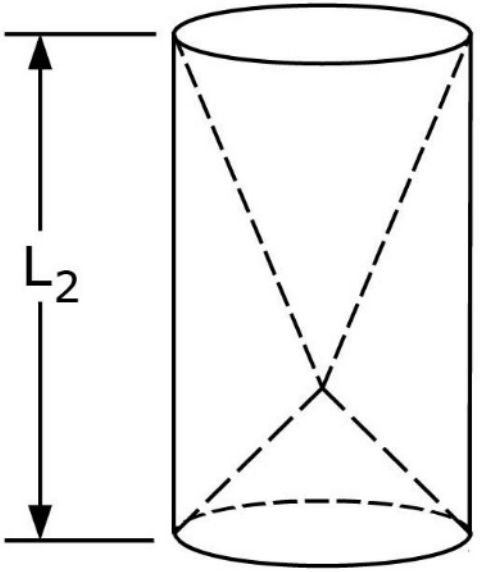
Scaling Rule - Volume

$$\mathrm{V} = KL^3$$

$$\mathrm{V} \propto L^3$$

$$\frac{V_2}{V_1} = \left[\frac{L_2}{L_1}\right]^3$$



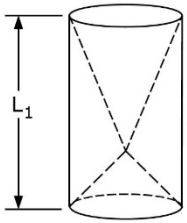


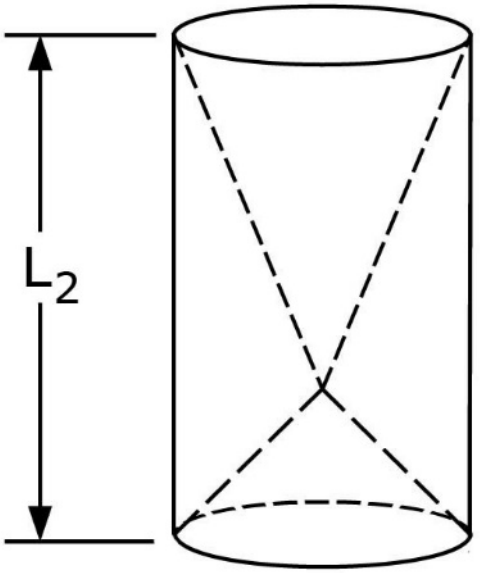
Scaling Rule - Area

$$A = KL^2$$

$$A \propto L^2$$

$$\frac{A_2}{A_1} = \left[\frac{L_2}{L_1}\right]^2$$



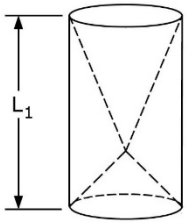


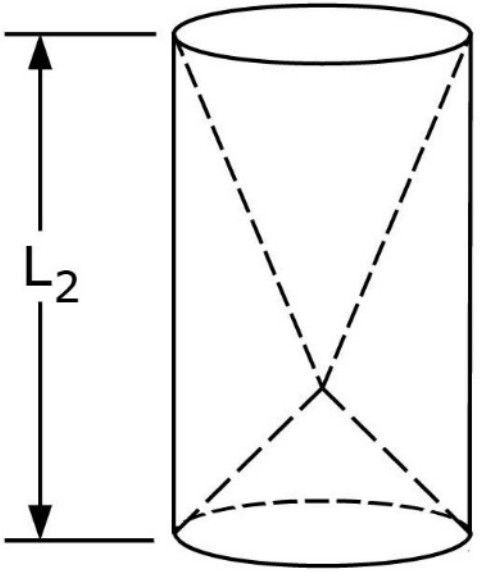
Scaling Rule - Lines

$$\mathrm{d} = KL$$

$$\mathrm{d} \propto L$$

$$\frac{d_2}{d_1} = \left[\frac{L_2}{L_1}\right]$$





Scaling - Lines

A tree 10 ft tall has a trunk circumference of 18 in. What is the height of a tree with a 25 in diameter trunk? (ft)

$$\frac{H_2}{H_1} = \left[\frac{C_2}{C_1}\right]$$

$$\frac{H_2}{10\ ft} = \left[\frac{\pi(25\ in)}{18}\right]$$

$$H_2 = 43.6\ ft$$

Scaling - Area

A wall map of Texas is scaled at 1:1,580,000. If the area of the State is 267,339 mi^2, what is the area of the map? (in^2)

$$\frac{A_2}{A_1} = \left[\frac{L_2}{L_1}\right]^2$$

$$\frac{A_2}{267{,}339\ mi^2} = \left[\frac{1}{1{,}580{,}000}\right]^2$$

$$A_2 = 1.071x10^{-7} mi^2 \left\{\frac{5280\ ft}{mi}\right\}^2 \left\{\frac{12\ in}{ft}\right\}^2 = 430\ in^2$$

Scaling - Area

The material cost of an empty 12 oz water bottle is $0.03. How much does an empty 2 liter water bottle cost? Assume that the thickness is constant and does not scale with size. ($)

$$A \propto L^2 \ \ and \ \ L \propto V^{1/3}, so\ A \propto V^{2/3}$$

$$Cost \propto Capacity \propto At \propto V^{2/3}$$

$$\frac{Cost_2}{\$0.03} = \left[\frac{2\ liter}{12\ oz}\right]^{2/3} \left[\left\{\frac{1.0567\ qt}{1\ liter}\right\}\left\{\frac{32\ oz}{1\ qt}\right\}\right]^{2/3}$$

$$Cost_2 = \$0.10$$

Scaling - Area

A recipe calls for 2 cups of flour to make 3 dozen 2 in diameter cookies. How much flour is needed to make 5 dozen 3 in diameter cookies? Assume cookie dough is rolled to constant thickness regardless of cookie size. (cups)

$$C \propto Nd^2$$

$$\frac{C_2}{2\ cups} = \left[\frac{5\ dozen}{3\ dozen}\right]\left[\frac{3\ in}{2\ in}\right]^2$$

$$C_2 = 7.50\ cups$$

Scaling - Area

If the cloth cost for a pair of 18 in waist blue jeans costs $12, what is the cloth cost for a 40 in waist pair of jeans?

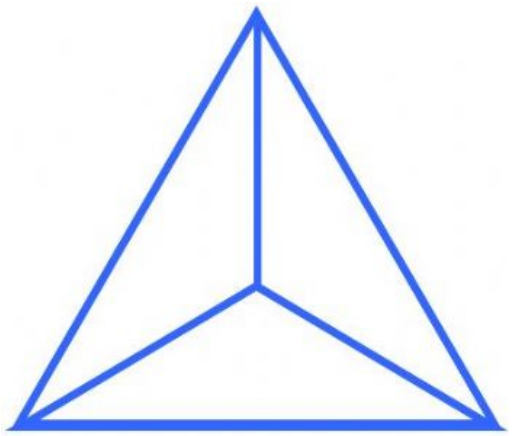
$$Cost \propto A \propto L^2$$

$$\frac{Cost_2}{Cost_1} = \left[\frac{L_2}{L_1}\right]^2 \qquad \frac{Cost_2}{\$12} = \left[\frac{40\ in}{18\ in}\right]^2$$

$$Cost_2 = \$59.26$$

Scaling - Volume

Four equilateral triangles are drawn on a sheet of paper and folded to make a pyramid of volume 8 in^3. If the paper was reduced on a copier by 47%, what is the volume of the resulting smaller pyramid? (in^3)



$$\frac{V_2}{V_1} = \left[\frac{L_2}{L_1}\right]^3$$

$$\frac{V_2}{8\ in^3} = \left[\frac{0.53}{1}\right]^3$$

$$V_2 = 1.19\ in^3$$

Scaling - Volume

A 3D printed artwork is 14 in long and weighs 4 lb 3 oz. How long is the same artwork built larger that weighs 17 lbs?

$$m = \rho V \propto L^3$$

$$\frac{m_2}{m_1} = \left[\frac{L_2}{L_1}\right]^3$$

$$\frac{17\ lb}{\left(4\frac{3}{16}\ lb\right)} = \left[\frac{L_2}{14\ in}\right]^3$$

$$L_2 = 22.3\ in$$

Scaling - Volume

It takes 38 blows to inflate a balloon to 14 in. How many more blows are needed to inflate it to 18 in?

$$V_T = NV_b \propto NL^3$$

$$\frac{V_{T2}}{V_{T1}} = \frac{N_2 V_b}{N_1 V_b} = \frac{N_2}{N_1} = \left[\frac{L_2}{L_1}\right]^3$$

$$\frac{N_2}{38\ blows} = \left[\frac{18\ in}{14\ in}\right]^3$$

$$N_2 = 80.8\ blows \quad so \quad \Delta N = 80.8 - 38 = 42.8\ blows$$

Scaling - Volume

Five pounds of pebbles (0.1 in long) has a certain total surface area. What mass of 0.01 in sand is needed to provide the same total surface area? (lb)

$$m_T = NV_1 \propto NL^3 \text{ or } \mathrm{N} \propto \frac{m_T}{L^3}$$

$$A_T = NA_1 \propto NL^2 \propto \left[\frac{m_T}{L^3}\right] L^2 \propto \frac{m_T}{L}$$

$$\frac{A_2}{A_1} = \left[\frac{m_{T2}}{L_2}\right]\left[\frac{L_1}{m_{T1}}\right] = \left[\frac{m_{T2}}{0.01\ in}\right]\left[\frac{0.1\ in}{5\ lbs}\right] = 1$$

$$m_{T2} = 0.500\ lb$$

Scaling - General

The pitch (frequency) of a bell is inversely proportional to its diameter. An octave higher pitch has double the frequency of the lower note. If a 5-in diameter bell is pitched at middle C, what is the diameter of a bell pitched at high C (one octave higher)?

$$F \propto 1/d$$

$$\frac{2F_1}{F_1} = \left[\frac{1}{d_2}\right]\left[\frac{5\ in}{1}\right]$$

$$\frac{F_2}{F_1} = \left[\frac{1}{d_2}\right]\left[\frac{d_1}{1}\right] = \frac{d_1}{d_2}$$

$$d_2 = 2.50\ in$$

Scaling - General

Sonny reads by a 100 watt light 2.5 ft from his book. How close should a 50 watt light be to his book? Light intensity is inversely proportional to the square of distance.

$$I \propto \frac{I_o}{d^2}$$

$$\frac{I_2}{I_1} = 1 = \left[\frac{I_{o2}}{d_2^2}\right]\left[\frac{d_1^2}{I_{o1}}\right]$$

http://loristephensllc.com/home/child-writing-with-books-and-desk-lamp/

$$1 = \left[\frac{50\ watt}{d_2^2}\right]\left[\frac{(2.5\ ft)^2}{100\ watt}\right]$$

$$d_2 = 1.77\ ft$$

Scaling – Practice Problem

A Chihuahua dog is 7 in tall and weighs 2.5 lbs. What does a 25 in tall Great Dane dog weigh? (lb)

https://www.quora.com/Are-Great-Danes-and-Chihuahuas-different-species

Ans = 114 lb

Scaling – Practice Problem

A 85-lb girl needs 2 ft^2 of material to make a hat. How much does a woman weigh if she needs 3 ft^2 of material to make her hat? (lb)

http://absfreepic.com/free-photos/drawing-of-a-girl-wearing-hat.html

Ans = 156 lb

Scaling – Practice Problem

A scone recipe uses 3/4 cup butter and makes 8 4-in long scones. For a banquet, 450 scones are needed, but to save money, the size was reduced to 3 in. If a package of butter is 2 cups, how many packages are needed?

https://www.tastemade.com/videos/apple-butter-scones

Ans = 9 integer

The End